Challenges for hybrid inflation: SUSY GUTs, $n_s$ & initial conditions

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(+ S. Clesse, Ringeval, J.R. in prep;
Bajc, Mazumdar, J.R., in prep)
Outline

Introduction: From the CMB to hybrid inflation

Part A: Embedding hybrid inflation in SUSY GUTs
1. Embedding the fields and superpotential?
2. Genericity of cosmic strings formation in SUSY GUTs
3. Constraints from the CMB

Part B: Non-SUSY model: is there a spectral index problem?

Part C: Two-field dynamics and initial conditions
1. Exact dynamics of 2-field inflation
2. Grids of initial conditions and trajectories in the field space
3. Effects of the potential parameters
4. SUSY/SUGRA effects? Non-renormalizable terms?

Conclusions and open questions
Intro: From CMB to inflation

WMAP observed of CMB anisotropies with highest resolution:

Image of the power-spectrum of primordial fluctuations: mostly gaussian, adiabatic

\[ P_s(k) = A_s \left( \frac{k}{k_0} \right)^{n_s - 1} \]

\[ P_T(k) = A_T \left( \frac{k}{k_0} \right)^{n_T} \]

\[ r = \frac{A_T}{A_S} \]

Inflation = easiest way to explain scale invariant power-spectrum + resolution to horizon problem, monopole problem.

3 main classes of models:

Power-law \( V(\phi) \propto \phi^p : n_s < 1, r >> \)

Exponential \( V(\phi) \propto \exp(\phi/\mu) : n_s < 1, r >> \)

Hybrid \( V(\phi) \propto 1 + (\phi/\mu)^p : n_s > 1, r << \)

[J. Martin 03]
**Hybrid models of inflation**

- Introduced to account for CMB normalization without fine-tuning (chaotic inflation requires $\lambda \sim 10^{-14}$)

Original version

$$V(\phi, \psi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda'}{2} \phi^2 \psi^2 + \frac{\lambda}{4} \left(\psi^2 - M^2\right)^2$$

SUSY version: “F-term”

$$W^F = \kappa S(\Sigma \bar{\Sigma} - M^2)$$

- **Motivations**: High Energy/Particle Physics:
  - Models to study coupling inflaton / other scalar fields (Higgs field(s) of SM or GUT, sfermions of SUSY, moduli of string theory, ... )
  - “Easily” embedded in (SUSY) GUT [Jeannerot 97, Jeannerot, J.R., Sakellariadou 03]
  - A F-term and D-term version exist in SUSY/SUGRA. Stable against radiative corrections, SUGRA corrections, SUGRA from SCFT [Copeland et al 94, Dvali et al 94, Dvali et al 04, J. R. & Sakellariadou 06]
  - P-term in extended (N=2) SUGRA [Kallosh & Linde 03]
  - In string theory, brane inflation ~ hybrid D-term model [Dvali & Tye 02]
Properties:

- Power law model coupled to a Higgs field
- Standard dynamics: inflation realized at large $\phi$ with $<\psi>=0$. At $\phi_{\text{crit}}$, $\psi$ becomes tachyonic and $<\psi>=M$.
- Inflation ends by an SSB.

Is there a topological defect problem?

- Yes if formation of monopole
- Generic formation of cosmic (super)strings at the end of (brane) SUSY version. How generic? Consequences?
  \[ \implies \text{Part A} \]

Is there a spectral index problem?

\[ \implies \text{Part B} \]

Is there an initial conditions problem?

\[ \implies \text{Part C} \]
Part A: Embedding in SUSY GUT

1. Embedding the fields and the superpotential

Embedding F-term hybrid inflation in minimal SUSY SO(10)?

$$W^F = \kappa S (\Sigma \Sigma - M^2)$$

$\Sigma$ and $\Sigma$ generically break B-L symmetry, $\Sigma = 126$, $\Sigma = \overline{126}$, (see next slide).

Embedding F-term hybrid inflation in minimal SUSY SO(10)? [Bajc, et al 2004]

Most general superpotential:

$$W^{SO(10)} = f(\Phi_{210}, \Sigma_{126}, \overline{\Sigma}_{126}, H_{10})$$

$$= \eta \Phi \Sigma \Sigma + m \Phi^2 + \lambda \Phi^3 + m_S \Sigma \Sigma + m_H H^2 + \Phi H (\alpha \Sigma + \overline{\alpha} \overline{\Sigma})$$

$\Phi =$ candidate for the inflaton? $W$ contains the coupling with $126.126$

- But impossible to write a gauge invariant term $M^2 \Phi$. $S =$ additional GUT singlet?

- But only way to prevent “dangerous” terms = additional global U(1) with precise charge assignment: $Q(S) \neq 0$. For $Q(\Phi)$, $Q(H)$ impossible!

Hybrid inflation not fully embedded in SUSY GUTs, only coupled?

Inflation at lower energy/symmetry than $G_{GUT}$ could give effective $W^F$?
2. How generic is the cosmic strings formation in SUSY GUTs?

SUSY GUTs models considered:

- Models based on SU(n), SO(10), E$_6$.
- + most standard phenomenological ingredients:
  - Proton life-time measurements (Super-Kamiokande) $\tau_p (p \rightarrow e^+ \pi^0) > 6 \times 10^{33}$ yr
  - Motivates:
    - Supersymmetry ($M_{\text{GUT}}$ sufficiently high)
    - $Z_2$ of R-parity unbroken at low energy (Bonus = dark matter)
  - Oscillations of solar and atmospheric neutrinos [Super-K, 98], …
    - Requires mass to neutrinos (via See-saw)
    - Requires B-L in $G_{\text{GUT}}$ and broken at high energy (Bonus = leptogenesis)
  - To explain the CMB data, solve the monopole problem, …, a phase of inflation (hybrid)
Conclusion: for all possible scheme of all GUT group studied, cosmic strings generically form at the end of hybrid inflation $\mu \approx M_{\text{infl}}^2$

[Jeannerot, J.R., Sakellariadou PRD 2003]

Higgs fields responsible break B-L in SO(10) => link with neutrino & leptogenesis

Inflaton? List the singlets at lower energy/symmetry [Bajc, Mazumdar, J.R. in prep]
3. Observations in the CMB and parameter constraints

However cosmic strings are not observed in universe:

WMAP data very well fit by inflation only.

If fit data with 1 additional parameter $\alpha$:

$$C_{l}^{tot} = \alpha C_{l}^{strings} + (1 - \alpha) C_{l}^{inflation}$$

• WMAP + SDSS: Monte-Carlo analysis with 6+2 parameters
  $\alpha < 14\%$ (95% CL), [Wyman et al. (2005)].
  + best fit with $\alpha = 4\%$ but compatible with $\alpha = 0\%$ [Fraisse (2005)].
• WMAP 3: $\alpha < 11\%$ (95% CL), [Bevis (2007)].
What constraint on inflationary parameters?

We impose:

- Normalization to CMB
  \[ \frac{\delta T}{T} \bigg|_{Q\text{-infl}} = f(M, \kappa, \phi_Q) \]
  \[ \frac{\delta T}{T} \bigg|_{Q\text{-strings}} = f(M) \]

- Resolution of the horizon problem: \( N_Q = 60 \)

- Coupling to see-saw mechanism, gravitino overproduction imposes \( \kappa < 10^{-2} \).

- WMAP3: \( A_{CS} < 11\% \) (95% CL). [Bevis et al. (2007)]

For SO(10): \( \kappa < 7 \times 10^{-7} \ll 1 \) (fine tuning ?)

or \( M < 2 \times 10^{15} \) GeV

Similar results for other G\(_{\text{GUT}}\), D-term inflation in (min and non-min) SUGRA, SCFT.

**Conclusion:** the coupling inflaton/Higgs is much smaller than expected.
**Part B : Is hybrid inflation ruled out by WMAP 5 ?**

Effective one field model:

\[ V(\phi, \psi) \rightarrow V(\phi) \approx \Lambda \left( 1 + \frac{\phi^2}{\mu^2} \right) \]  &  \[ \phi_{\text{crit}} \]

WMAP 5: \( n_S = 0.96 \pm 0.013 \) (1σ) but generic prediction > 1!

Hubble flow parameters

\[ \varepsilon_1 = -\frac{\ddot{H}}{H^2} \]
\[ \varepsilon_n = \frac{d \ln \varepsilon_n}{dN} \quad \text{[Schwarz et al 01, 04]} \]

Inflation if \( \varepsilon_1 < 1 \), slow roll if \( \varepsilon_n << 1 \)

For 1-field hybrid model in SR:

\[ \varepsilon_1 = \frac{1}{4\pi} \left( \frac{m_{\text{pl}}}{\mu} \right)^2 \frac{\left( \phi / \mu \right)^2}{\left[ 1 + \left( \phi / \mu \right)^2 \right]^2} \]

\[ \varepsilon_2 = \frac{1}{2\pi} \left( \frac{m_{\text{pl}}}{\mu} \right)^2 \frac{\left( \phi / \mu \right)^2 - 1}{\left[ 1 + \left( \phi / \mu \right)^2 \right]^2} \]

Numerically exact calculation in blue.

\[ m = 0.1 \quad \text{No inflation} \]
\[ m = 0.4 \quad \text{Inflation} \]
In slow roll: \[ n_S = 1 - 2\varepsilon_1(\phi_{60}) - \varepsilon_2(\phi_{60}) \]

We computed \( \varepsilon_1(\phi) \) NOT in slow roll

- \( \phi_{SSB} \) can be large to prevent the second phase of inflation
- If \( \mu < \mu_{\text{crit}} \) NO second phase of inflation. (NEW !)

**Conclusions:**

- \( n_S < 1 \) if small \( \mu \) or large \( \phi_{\text{crit}} \).
- But both suggest a large field regime!

Clesse & J.R. PRD 2009
Part C: Fine tuning of initial conditions for hybrid inflation?

- Successful = thin ($\psi=0$) band + isolated points. Problem?

- Are the isolated points of null measure? Are there some patterns?

- Can they make sense? Why is successful IC not continuous?

- What is the proportion of successful values? Can we talk about fine tuning?

- How to avoid fine tuning?

[Mendes & Liddle 2000]
2-field dynamics of hybrid inflation

Numerical integration of 2-field EoM (background):

\[
H^2 = \kappa \left[ \frac{1}{2} \left( \dot{\phi}^2 + \dot{\psi}^2 \right) + V(\phi, \psi) \right] \quad \Rightarrow \quad \ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \\
\frac{\ddot{a}}{a} = \kappa \left[ -\dot{\phi}^2 - \dot{\psi}^2 + V(\phi, \psi) \right] \quad \Rightarrow \quad \ddot{\psi} + 3H\dot{\psi} + \frac{\partial V}{\partial \psi} = 0
\]

- **Slow roll** NOT assumed (because can be transiently violated!).
- Assume all initial values \((\phi_i, \psi_i)\) and vanishing initial velocities. Allow for transplanckian values.
- Integration stopped when \(E_{\text{system}} < E_{\text{barrier}} = \lambda M^4\).
- For each initial value, we compute \(\phi(t), \psi(t), N(t), N_{\text{end}}\) and generate 2D grids of \(N_{\text{end}}(\phi_i, \psi_i)\).
- Successful inflation is defined by \(N \geq 60\). Correct normalization requires a rescaling that doesn’t affect the dynamics + normalization dependent on initial point.
- Successful initial conditions:
  - Recover “isolated points” in the unsuccessful region
  - At larger fields, inflation always successful

- Fit of unsuccessful zone:
  - Width of narrow band by comparison of the $\phi$ and $\psi$ oscillation times
    $$\psi_w \propto M \sqrt{\lambda / \lambda'}$$
  - Limit A: gradient of potential. Strongly depend on $\lambda'$.
  - Limit B: $\varepsilon_1$ isocurve

- 3 types of successful trajectories:
  A = Standard (equiv. to 1 field)
  B = Radial (purely 2 field)
  C = “miraculous”
  + D = typical failed trajectory
**Conclusion:** “miraculous” points have the right velocity to climb up the valley to become type A trajectories!

**Structure:**
Composed of continuous regions. Each corresponds to a unique number of crossing of the $\phi=0$ axis.

**Quantification:**
In a square of length $M_{pl}$
17% of IC successful, 15% in the points!

Clesse, J.R.  PRD 2009

**Stat. properties:**
- Fractal: $d=1.2$
- When zoom in, new succ. regions appear.

Clesse, Ringeval, J.R.  in prep
Different set of parameters to have red spectrum in inflationary valley …

Conclusions:

• miraculous points disappeared: amount of success depends on pot. parameters.

• large field regime required in both directions.

More work required to confront to CMB fully in the two field plane.

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Robustness: Non-renormalizable and SUGRA effects

Effects of non-renorm terms => others models studied

- Smooth hybrid inflation
- Shifted hybrid inflation

Extended SUSY hybrid models with shifted valleys to avoid topological defects [Lazarides & Panagiotakopoulos 95, Jeannerot et al 00]

+ study (minimal) SUGRA effects

\[
V(\phi, \psi) = \kappa^2 \left( \frac{\psi^p}{M_{pl}^{p-2}} - M^2 - \beta \frac{\psi^4}{M_{pl}^2} \right)^2 + \kappa^2 \phi^2 \frac{\psi^q}{M_{pl}^{q-2}} \left( 1 - \beta \frac{\psi^2}{M_{pl}^2} \right)^2 + V_{SUGRA}
\]
Some results:

- Smooth inflation have up to 80%.
- Shifted inflation always below 10%.
- Large IC: still automatic if SUSY. If SUGRA, this disappears but not miraculous IC.

Clesse, J.R. PRD 2009
Conclusions

Embedding in SUSY GUTs

- If written at the GUT level, hybrid inflation requires a singlet outside of SUSY GUTs + some tuning compared to general W.
- Cosmic strings generically for at the end. CMB imposes a small coupling $10^{-6}$.

Space of initial conditions

- Space better understood. 3 types of trajectories. Isolated IC not isolated but in patterns => fine tuning?
- If restrict to subplanckian IC, 17% of IC successful. fine tuning?
- If we allow superplanckian IC, successful inflation generic. Exist new type of trajectories outside usual valley.

Confrontation to CMB

- Possible to have red spectral index.
- But this pushes original hybrid in large field regime
- More being done in 2-field space, + all parameters …

Jeannerot, J.R. Sakellariadou 03
J.R. Sakellariadou 05
Clesse, J.R. ArXiv 0809.4355,
Clesse, Ringeval, J.R. in prep
Bajc, Mazumdar, J.R., in prep
Open questions and work in progress

SUSY GUTs and orthogonal constraints

• Identifying all possibilities for the inflaton in fields present (in progress).
• See-saw constraint on energy scale/coupling of hybrid inflation? (Too) many params?
• Gauge coupling unification constraint on energy scale of hybrid inflation?

Generic nature of Cosmic Strings in GUT

• Study presence of currents in strings: requires full GUT lagrangian and coupling S to other field.
• Cosmological consequences of realistic strings.

Two-field confrontation to CMB

• Effects of initial velocities
• Statistical properties of the IC space
• Predictions of $n_s$ and $r$ in all plane.
• MCMC to confront to the data. Gives a measure in the space of initial conditions.

Bajc, Mazumdar, J.R., in prep
Peter, J.R., Sakellariadou, in prep
Clesse, Ringeval, J.R. in prep

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