Holographic Monopole Catalysis of Baryon Decay

Ho-Ung Yee

ICTP, Trieste

12 March 2009

Institut Jozef Stefan, Ljubljana

Reference : JHEP 0808:018,2008 (arXiv:0804.1326 [hep-th])
with Deog Ki Hong, Ki-Myeong Lee, Cheonsoo Park

Monopole catalysis

Callan and Rubakov Effect

In the presence of magnetic monopole, baryon number can be violated by anomaly

$$\partial_\mu B^\mu \sim \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

Monopole catalysis

Callan and Rubakov Effect

In the presence of magnetic monopole, baryon number can be violated by anomaly

$$\partial_\mu B^\mu \sim \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

Because it is from anomaly, this effect survives at low energy without being decoupled

Monopole catalysis

Callan and Rubakov Effect

In the presence of magnetic monopole, baryon number can be violated by anomaly

$$\partial_\mu B^\mu \sim \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

Because it is from anomaly, this effect survives at low energy without being decoupled
However, at low energy QCD, baryons are described by topological solitons called Skyrmions

Monopole catalysis

Callan and Rubakov Effect

In the presence of magnetic monopole, baryon number can be violated by anomaly

$$\partial_\mu B^\mu \sim \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

Because it is from anomaly, this effect survives at low energy without being decoupled
However, at low energy QCD, baryons are described by topological solitons called Skyrmions

Q : How can the topological solitons can decay in the presence of magnetic monopole ???

Monopole catalysis of Skyrmion decay

Callan and Witten showed that

it is possible for the Skyrmions to decay in the presence of magnetic monopole

- Skyrmions are topological solitons from $\pi_3(SU(2)) = Z$ with the Skyrme action

$$L = \frac{f_\pi^2}{4}\mathrm{tr}\left(U^{-1}\partial_\mu U\right)^2 + \frac{1}{32e^2}\mathrm{tr}\left[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U\right]^2$$

- There is a topologically conserved current

$$B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U\right)$$

  Naturally, the topological baryon number $B = \int d^3x B^0$ is always conserved in any finite time processes.

Monopole catalysis of Skyrmion decay

Callan and Witten showed that

it is possible for the Skyrmions to decay in the presence of magnetic monopole

- Skyrmions are topological solitons from $\pi_3(SU(2)) = Z$ with the Skyrme action

$$L = \frac{f_\pi^2}{4}\mathrm{tr}\left(U^{-1}\partial_\mu U\right)^2 + \frac{1}{32e^2}\mathrm{tr}\left[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U\right]^2$$

- There is a topologically conserved current

$$B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U\right)$$

Naturally, the topological baryon number $B = \int d^3x B^0$ is always conserved in any finite time processes.

How about the magnetic monopole ?

- Recall that the $SU(2)$ valued group field U transforms under chiral symmetry $U(2)_L \times U(2)_R$ as

$$U \to U_L U U_R^\dagger$$

- Electromagnetism is

$$Q = \frac{B}{2} + I_3$$

where B is from $U(1)_V$ and I_3 is the third component of $SU(2)_V$. We weakly gauge these symmetries to introduce electromagnetism

- Therefore, our topological baryon current B^μ should also respect this weakly gauged symmetry to be gauge invariant. For example,

$$\partial_\mu \to D_\mu$$

How about the magnetic monopole ?

- Recall that the $SU(2)$ valued group field U transforms under chiral symmetry $U(2)_L \times U(2)_R$ as

$$U \to U_L U U_R^\dagger$$

- Electromagnetism is

$$Q = \frac{B}{2} + I_3$$

where B is from $U(1)_V$ and I_3 is the third component of $SU(2)_V$. We weakly gauge these symmetries to introduce electromagnetism

- Therefore, our topological baryon current B^μ should also respect this weakly gauged symmetry to be gauge invariant. For example,

$$\partial_\mu \to D_\mu$$

How about the magnetic monopole ?

- Recall that the $SU(2)$ valued group field U transforms under chiral symmetry $U(2)_L \times U(2)_R$ as

$$U \to U_L U U_R^\dagger$$

- Electromagnetism is

$$Q = \frac{B}{2} + I_3$$

where B is from $U(1)_V$ and I_3 is the third component of $SU(2)_V$. We weakly gauge these symmetries to introduce electromagnetism

- Therefore, our topological baryon current B^μ should also respect this weakly gauged symmetry to be gauge invariant. For example,

$$\partial_\mu \to D_\mu$$

Conserved gauge invariant baryon current

The simplest choice

$$B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(U^{-1}D_\nu UU^{-1}D_\alpha UU^{-1}D_\beta U\right)$$

seems ok at first sight as it is gauge invariant

Conserved gauge invariant baryon current

The simplest choice

$$B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(U^{-1}D_\nu U U^{-1}D_\alpha U U^{-1}D_\beta U\right)$$

seems ok at first sight as it is gauge invariant

But, it can be checked that it is no longer conserved, that is,

$$\partial_\mu B^\mu \neq 0 \quad !!!$$

We need to add something more to make it both gauge invariant and conserved

Conserved gauge invariant baryon current

The simplest choice

$$B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(U^{-1}D_\nu UU^{-1}D_\alpha UU^{-1}D_\beta U\right)$$

seems ok at first sight as it is gauge invariant

But, it can be checked that it is no longer conserved, that is,

$$\partial_\mu B^\mu \neq 0 \quad !!!$$

We need to add something more to make it both gauge invariant and conserved

Witten found that

$$\begin{aligned} B^\mu &= \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(U^{-1}\partial_\nu UU^{-1}\partial_\alpha UU^{-1}\partial_\beta U\right) \\ &- \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu\left[3A^{EM}_\alpha\mathrm{tr}\left(Q\left(U^{-1}\partial_\beta U+\partial_\beta UU^{-1}\right)\right)\right] \end{aligned}$$

Monopole background is singular

- The B^μ is now both gauge invariant and conserved only for smooth EM potential A^{EM}
- The tricky point is that the magnetic monopole potential is singular : Dirac string

$$A^{EM} = \frac{-i}{2}(1 - \cos\theta)d\phi$$

- Interestingly, the baryon number conservation can be violated precisely at the center of the monopole

This is the low energy manifestation of Callan-Rubakov effect

- Charged pions are suppressed at the monopole core, but neutral pions can be non-zero $U = \exp(\frac{2i}{F_\pi}\pi^0\sigma^3)$. Then there is a radial flux

$$B^r = \frac{(\partial_t \pi^0)}{4\pi^2 F_\pi r^2}$$

which results in the baryon number violation

$$\frac{dB}{dt} = \frac{1}{\pi F_\pi}\left(\partial_t \pi^0\right)\Big|_{r=0}$$

Monopole background is singular

- The B^μ is now both gauge invariant and conserved only for smooth EM potential A^{EM}
- The tricky point is that the magnetic monopole potential is singular : Dirac string

$$A^{EM} = \frac{-i}{2}(1-\cos\theta)d\phi$$

- Interestingly, the baryon number conservation can be violated precisely at the center of the monopole

This is the low energy manifestation of Callan-Rubakov effect

- Charged pions are suppressed at the monopole core, but neutral pions can be non-zero $U = \exp(\frac{2i}{F_\pi}\pi^0\sigma^3)$. Then there is a radial flux

$$B^r = \frac{(\partial_t \pi^0)}{4\pi^2 F_\pi r^2}$$

which results in the baryon number violation

$$\frac{dB}{dt} = \frac{1}{\pi F_\pi}\left(\partial_t \pi^0\right)\Big|_{r=0}$$

Monopole background is singular

- The B^μ is now both gauge invariant and conserved only for smooth EM potential A^{EM}
- The tricky point is that the magnetic monopole potential is singular : Dirac string

$$A^{EM} = \frac{-i}{2}(1 - \cos\theta)d\phi$$

- Interestingly, the baryon number conservation can be violated precisely at the center of the monopole

This is the low energy manifestation of Callan-Rubakov effect

- Charged pions are suppressed at the monopole core, but neutral pions can be non-zero $U = \exp(\frac{2i}{F_\pi}\pi^0\sigma^3)$. Then there is a radial flux

$$B^r = \frac{(\partial_t \pi^0)}{4\pi^2 F_\pi r^2}$$

which results in the baryon number violation

$$\frac{dB}{dt} = \frac{1}{\pi F_\pi}\left(\partial_t \pi^0\right)\Big|_{r=0}$$

Monopole background is singular

- The B^μ is now both gauge invariant and conserved only for smooth EM potential A^{EM}
- The tricky point is that the magnetic monopole potential is singular : Dirac string

$$A^{EM} = \frac{-i}{2}(1 - \cos\theta)d\phi$$

- Interestingly, the baryon number conservation can be violated precisely at the center of the monopole

This is the low energy manifestation of Callan-Rubakov effect

- Charged pions are suppressed at the monopole core, but neutral pions can be non-zero $U = \exp(\frac{2i}{F_\pi}\pi^0\sigma^3)$. Then there is a radial flux

$$B^r = \frac{(\partial_t \pi^0)}{4\pi^2 F_\pi r^2}$$

which results in the baryon number violation

$$\frac{dB}{dt} = \frac{1}{\pi F_\pi}\left(\partial_t \pi^0\right)\Big|_{r=0}$$

What is going on, topologically ?

Recall that our Skyrmions are topological with integer baryon charge

$$B \in \pi_3\,(SU(2)) = Z$$

What is going on, topologically ?

Recall that our Skyrmions are topological with integer baryon charge

$$B \in \pi_3\left(SU(2)\right) = Z$$

What is going on ???

What is going on, topologically ?

Recall that our Skyrmions are topological with integer baryon charge
$B \in \pi_3(SU(2)) = Z$

What is going on ???

Solution :

- In the presence of magnetic monopole, one is allowed to perform singular gauge transformation moving Dirac string from one place to another
- Under this singular gauge transformations, topological baryon number $\pi_3(SU(2)) = Z$ is not conserved. Different $\pi_3(SU(2)) = Z$ can be connected by our allowed gauge transformations, and thus **it is not physical**
- Gauge invariant physical baryon number is given by above, and it is not wholly given by topological numbers, and can disappear fractionally at the monopole core

Holographic QCD

- Holographic QCD is an alternative approach to low energy strongly coupled QCD based on large N limit
- In large N limit, the theory becomes classical with new variables called master fields
- The renormalization group survives in the large N limit. Asymptotic freedom and dimensional transmutation persist. One needs to renormalize the theory after fixing the renormalization scale.
- One ends up with a classical theory of master fields with the features of renormaliztion group, especially the formalism is invariant under shifts in the renormalization point, kind of general covariance in energy direction
- A 5 dimensional classical theory of master fields with 5'th direction roughly corresponding to energy scale is one natural realization of these aspects

Holographic QCD

- Holographic QCD is an alternative approach to low energy strongly coupled QCD based on large N limit
- In large N limit, the theory becomes classical with new variables called master fields
- The renormalization group survives in the large N limit. Asymptotic freedom and dimensional transmutation persist. One needs to renormalize the theory after fixing the renormalization scale.
- One ends up with a classical theory of master fields with the features of renormaliztion group, especially the formalism is invariant under shifts in the renormalization point, kind of general covariance in energy direction
- A 5 dimensional classical theory of master fields with 5'th direction roughly corresponding to energy scale is one natural realization of these aspects

Holographic QCD

- Holographic QCD is an alternative approach to low energy strongly coupled QCD based on large N limit
- In large N limit, the theory becomes classical with new variables called master fields
- The renormalization group survives in the large N limit. Asymptotic freedom and dimensional transmutation persist. One needs to renormalize the theory after fixing the renormalization scale.
- One ends up with a classical theory of master fields with the features of renormaliztion group, especially the formalism is invariant under shifts in the renormalization point, kind of general covariance in energy direction
- A 5 dimensional classical theory of master fields with 5'th direction roughly corresponding to energy scale is one natural realization of these aspects

Holographic QCD

- Holographic QCD is an alternative approach to low energy strongly coupled QCD based on large N limit
- In large N limit, the theory becomes classical with new variables called master fields
- The renormalization group survives in the large N limit. Asymptotic freedom and dimensional transmutation persist. One needs to renormalize the theory after fixing the renormalization scale.
- One ends up with a classical theory of master fields with the features of renormaliztion group, especially the formalism is invariant under shifts in the renormalization point, kind of general covariance in energy direction
- A 5 dimensional classical theory of master fields with 5'th direction roughly corresponding to energy scale is one natural realization of these aspects

Holographic QCD

- Holographic QCD is an alternative approach to low energy strongly coupled QCD based on large N limit
- In large N limit, the theory becomes classical with new variables called master fields
- The renormalization group survives in the large N limit. Asymptotic freedom and dimensional transmutation persist. One needs to renormalize the theory after fixing the renormalization scale.
- One ends up with a classical theory of master fields with the features of renormaliztion group, especially the formalism is invariant under shifts in the renormalization point, kind of general covariance in energy direction
- A 5 dimensional classical theory of master fields with 5'th direction roughly corresponding to energy scale is one natural realization of these aspects

We need further inputs

Q : How to find this 5 dimensional theory? What principles should we consider ? How precisely can we construct it?

We need further inputs

Q : How to find this 5 dimensional theory? What principles should we consider ? How precisely can we construct it?

We need further inputs to have any progress...

We need further inputs

Q : How to find this 5 dimensional theory? What principles should we consider ? How precisely can we construct it?

We need further inputs to have any progress...

There is at least one example where we have a good control

$N = 4$ super Yang-Mills $\longleftrightarrow$ Type IIB SUGRA in $AdS_5 \times S^5$

Let's learn something from this well-tested example

We need further inputs

Q : How to find this 5 dimensional theory? What principles should we consider ? How precisely can we construct it?

We need further inputs to have any progress...

There is at least one example where we have a good control

$$N = 4 \text{ super Yang-Mills} \longleftrightarrow \text{Type IIB SUGRA in } AdS_5 \times S^5$$

Let's learn something from this well-tested example

- 5 dimensional theory is a gravity theory, realizing general covariance in energy direction as a renormalization group
- When the field theory is strongly coupled, the 5 dimensional theory becomes a local theory

Symmetry mapping

One lesson we learn from this example is how to map the global symmetry of QCD to the 5 dimensional theory

Symmetry mapping

One lesson we learn from this example is how to map the global symmetry of QCD to the 5 dimensional theory

Global symmetry in QCD $\longleftrightarrow$ **Local gauge symmetry in 5 dimensions**

Symmetry mapping

One lesson we learn from this example is how to map the global symmetry of QCD to the 5 dimensional theory

Global symmetry in QCD $\longleftrightarrow$ Local gauge symmetry in 5 dimensions

$$A(x^\mu, Z) = A^{(0)} + \sum_{n\geq 1} f_n(Z) A^{(n)}$$

The point is that the zero mode $A^{(0)}$ is non-normalizable due to the specific metric property of 5 dimensions, and therefore it is non-dynamical, something like external gauge potential coupled to the currents

Symmetry mapping

One lesson we learn from this example is how to map the global symmetry of QCD to the 5 dimensional theory

Global symmetry in QCD $\longleftrightarrow$ Local gauge symmetry in 5 dimensions

$$A(x^\mu, Z) = A^{(0)} + \sum_{n\geq 1} f_n(Z) A^{(n)}$$

The point is that the zero mode $A^{(0)}$ is non-normalizable due to the specific metric property of 5 dimensions, and therefore it is non-dynamical, something like external gauge potential coupled to the currents

Other massive modes $A^{(n)}$ encode the states that can be created by the global symmetry currents in the field theory

Symmetry mapping

One lesson we learn from this example is how to map the global symmetry of QCD to the 5 dimensional theory

Global symmetry in QCD $\longleftrightarrow$ Local gauge symmetry in 5 dimensions

$$A(x^\mu, Z) = A^{(0)} + \sum_{n\geq 1} f_n(Z) A^{(n)}$$

The point is that the zero mode $A^{(0)}$ is non-normalizable due to the specific metric property of 5 dimensions, and therefore it is non-dynamical, something like external gauge potential coupled to the currents

Other massive modes $A^{(n)}$ encode the states that can be created by the global symmetry currents in the field theory

Important : By turning on $A^{(0)}$, we can introduce weakly gauged external gauge potential coupled to the global currents, and study how the system responds to this external potential

More concretely,

- Corresponding to $U(2)_L \times U(2)_R$ chiral symmetry in QCD, we have $U(2)_L \times U(2)_R$ gauge symmetry in 5 dimensional holographic QCD
- Electromagnetism is an external potential coupled to $Q = \frac{B}{2} + I_3$ symmetry current, so A^{EM} is encode as the zero mode $A^{(0)}$ of the corresponding 5 dimensional gauge field to $Q = \frac{B}{2} + I_3$
- Chiral symmetry breaking is realized as a gauge symmetry breaking in 5 dimensions. Goldstone pions are realized as Wilson lines along the 5'th direction
- The upshot is that

  5 dimensional gauge theory encapsulates both the dynamical mesons such as pions and the external potential coupled to the symmetry currents into a single framework

$$A_\mu(x, Z) = A_\mu^{EM} + f_0(Z)(\partial_\mu \pi(x)) + \text{higher vector mesons}$$

More concretely,

- Corresponding to $U(2)_L \times U(2)_R$ chiral symmetry in QCD, we have $U(2)_L \times U(2)_R$ gauge symmetry in 5 dimensional holographic QCD
- Electromagnetism is an external potential coupled to $Q = \frac{B}{2} + I_3$ symmetry current, so A^{EM} is encode as the zero mode $A^{(0)}$ of the corresponding 5 dimensional gauge field to $Q = \frac{B}{2} + I_3$
- Chiral symmetry breaking is realized as a gauge symmetry breaking in 5 dimensions. Goldstone pions are realized as Wilson lines along the 5'th direction
- The upshot is that

  5 dimensional gauge theory encapsulates both the dynamical mesons such as pions and the external potential coupled to the symmetry currents into a single framework

$$A_\mu(x, Z) = A_\mu^{EM} + f_0(Z)(\partial_\mu \pi(x)) + \text{higher vector mesons}$$

More concretely,

- Corresponding to $U(2)_L \times U(2)_R$ chiral symmetry in QCD, we have $U(2)_L \times U(2)_R$ gauge symmetry in 5 dimensional holographic QCD
- Electromagnetism is an external potential coupled to $Q = \frac{B}{2} + I_3$ symmetry current, so A^{EM} is encode as the zero mode $A^{(0)}$ of the corresponding 5 dimensional gauge field to $Q = \frac{B}{2} + I_3$
- Chiral symmetry breaking is realized as a gauge symmetry breaking in 5 dimensions. Goldstone pions are realized as Wilson lines along the 5'th direction
- The upshot is that

  5 dimensional gauge theory encapsulates both the dynamical mesons such as pions and the external potential coupled to the symmetry currents into a single framework

$$A_\mu(x, Z) = A_\mu^{EM} + f_0(Z)(\partial_\mu \pi(x)) + \text{higher vector mesons}$$

More concretely,

- Corresponding to $U(2)_L \times U(2)_R$ chiral symmetry in QCD, we have $U(2)_L \times U(2)_R$ gauge symmetry in 5 dimensional holographic QCD
- Electromagnetism is an external potential coupled to $Q = \frac{B}{2} + I_3$ symmetry current, so A^{EM} is encode as the zero mode $A^{(0)}$ of the corresponding 5 dimensional gauge field to $Q = \frac{B}{2} + I_3$
- Chiral symmetry breaking is realized as a gauge symmetry breaking in 5 dimensions. Goldstone pions are realized as Wilson lines along the 5'th direction
- The upshot is that

  5 dimensional gauge theory encapsulates both the dynamical mesons such as pions and the external potential coupled to the symmetry currents into a single framework

$$A_\mu(x, Z) = A_\mu^{EM} + f_0(Z)(\partial_\mu \pi(x)) + \text{higher vector mesons}$$

Holographic Baryons

Baryons can also be described within the holographic QCD

Holographic Baryons

Baryons can also be described within the holographic QCD

In 5 dimensions, we have non-Abelian 5 dimensional gauge theory of chiral symmetry

$\longrightarrow$ we have 4 space directions x^i and Z, and one can have a topological soliton with

$$B = \frac{1}{8\pi^2} \int_{R^3 \times Z} \mathrm{tr}(F \wedge F)$$

Baryons can also be described within the holographic QCD

In 5 dimensions, we have non-Abelian 5 dimensional gauge theory of chiral symmetry

$\longrightarrow$ we have 4 space directions x^i and Z, and one can have a topological soliton with

$$B = \frac{1}{8\pi^2}\int_{R^3\times Z}\mathrm{tr}(F\wedge F)$$

Note that its configuration is just like an instanton, but in our set-up it is a soliton static in time. We can use many known facts about instantons to find its properties, though.

Holographic Baryons

Baryons can also be described within the holographic QCD

In 5 dimensions, we have non-Abelian 5 dimensional gauge theory of chiral symmetry

$\longrightarrow$ we have 4 space directions x^i and Z, and one can have a topological soliton with

$$B = \frac{1}{8\pi^2}\int_{R^3\times Z} \mathrm{tr}(F\wedge F)$$

Note that its configuration is just like an instanton, but in our set-up it is a soliton static in time. We can use many known facts about instantons to find its properties, though.

As it is topological, there is a conserved 5 dimensional topological current

$$B^M \sim \epsilon^{MNPQR}\mathrm{tr}\left(F_{NP}\wedge F_{QR}\right)$$

Baryons can also be described within the holographic QCD

In 5 dimensions, we have non-Abelian 5 dimensional gauge theory of chiral symmetry

$\longrightarrow$ we have 4 space directions x^i and Z, and one can have a topological soliton with

$$B = \frac{1}{8\pi^2}\int_{R^3\times Z} \mathrm{tr}(F\wedge F)$$

Note that its configuration is just like an instanton, but in our set-up it is a soliton static in time. We can use many known facts about instantons to find its properties, though.

As it is topological, there is a conserved 5 dimensional topological current

$$B^M \sim \epsilon^{MNPQR}\mathrm{tr}\left(F_{NP}\wedge F_{QR}\right)$$

In fact, $B = \int_{R^3\times Z} B^0$ can be shown to reduce to the Skyrmion number upon reduction to 4 dimensions

Holographic baryon number current

By integrating over 5'th dimension Z, one in fact gets 4 dimensional baryon current

$$B^\mu = \frac{1}{8\pi^2}\int dZ \epsilon^{\mu\nu\alpha\beta}\mathrm{tr}\left(F_{\nu\alpha}F_{\beta Z}\right)$$

Holographic baryon number current

By integrating over 5'th dimension Z, one in fact gets 4 dimensional baryon current

$$B^{\mu} = \frac{1}{8\pi^2} \int dZ \epsilon^{\mu\nu\alpha\beta} \mathrm{tr}\left(F_{\nu\alpha} F_{\beta Z}\right)$$

Important : This 4 dimensional current is both gauge invariant and conserved as the one Witten found before

Recall that the 5 dimensional gauge field $A(x, Z)$ contains both pion fields as well as external EM potential in a single framework

$$A_{\mu}(x, Z) = [(U^{-1} Q U)\psi_{+}(Z) + Q\psi_{-}(Z)] A_{\mu}^{EM} + \psi_{+}(Z) U^{-1} \partial_{\mu} U + \text{higher modes}$$

Holographic baryon number current

By integrating over 5'th dimension Z, one in fact gets 4 dimensional baryon current

$$B^{\mu} = \frac{1}{8\pi^2} \int dZ \epsilon^{\mu\nu\alpha\beta} \mathrm{tr}\left(F_{\nu\alpha} F_{\beta Z}\right)$$

Important : This 4 dimensional current is both gauge invariant and conserved as the one Witten found before

Recall that the 5 dimensional gauge field $A(x, Z)$ contains both pion fields as well as external EM potential in a single framework

$$A_{\mu}(x, Z) = [(U^{-1} Q U)\psi_{+}(Z) + Q\psi_{-}(Z)] A_{\mu}^{EM} + \psi_{+}(Z) U^{-1} \partial_{\mu} U + \text{higher modes}$$

The holographic baryon current for the above expansion of pions and the A^{EM} reproduces the Witten's result

Holographic version of monopole catalysis

As we have the gauge invariant, conserved baryon number current, we should be able to see the monopole catalysis phenomenon

Holographic version of monopole catalysis

As we have the gauge invariant, conserved baryon number current, we should be able to see the monopole catalysis phenomenon

We emphasize that holographic QCD set-up is more suitable for this purpose, because the external EM potential and the pions are summarized into a single framework of 5 dimensional gauge theory

Any phenomenon that involves these two, such as monopole catalysis, should finds its manifestation within our set-up

Holographic version of monopole catalysis

As we have the gauge invariant, conserved baryon number current, we should be able to see the monopole catalysis phenomenon

We emphasize that holographic QCD set-up is more suitable for this purpose, because the external EM potential and the pions are summarized into a single framework of 5 dimensional gauge theory

Any phenomenon that involves these two, such as monopole catalysis, should finds its manifestation within our set-up

In fact, we will see that physics of monopole catalysis is more transparent and simpler in holographic QCD view-point

Holographic version of monopole catalysis

As we have the gauge invariant, conserved baryon number current, we should be able to see the monopole catalysis phenomenon

We emphasize that holographic QCD set-up is more suitable for this purpose, because the external EM potential and the pions are summarized into a single framework of 5 dimensional gauge theory

Any phenomenon that involves these two, such as monopole catalysis, should finds its manifestation within our set-up

In fact, we will see that physics of monopole catalysis is more transparent and simpler in holographic QCD view-point

Monopole catalysis = Violation of Bianchi identity in 5D by an EM monopole field

Details...

- Recall that the external EM potential is encoded in the 5D gauge field as

$$A = QA^{EM} + \psi_+(Z)U^{-1}\partial_\mu U + \cdots$$

Note that A^{EM} is a constant mode along Z

- We then put the monopole background to A^{EM} with the violation of Bianchi identity

$$-2\pi i = \int_{S^2} F^{EM} = \int_{S^2} dA^{EM} = \int_{B^3} d^2 A^{EM} \longrightarrow d^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$

- The result is having a string of monopole along Z
- In 5D, Bianchi identity is violated by A^{EM}

$$DF \equiv dF + A \wedge F - F \wedge A = Qd^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$
$$d\mathrm{tr}\left(F \wedge F\right) = 2\mathrm{tr}(DF \wedge F) = -4\pi i \mathrm{tr}(QF) \wedge \delta_3(\vec{0})$$

In components, this is

$$\partial_\mu \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\nu\alpha}F_{\beta Z})\right) = -\frac{1}{4}\partial_Z \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\right) + 4\pi i \delta^{(3)}(\vec{x})\mathrm{tr}(QF_{tZ})$$

Details...

- Recall that the external EM potential is encoded in the 5D gauge field as

$$A = QA^{EM} + \psi_+(Z)U^{-1}\partial_\mu U + \cdots$$

Note that A^{EM} is a constant mode along Z

- We then put the monopole background to A^{EM} with the violation of Bianchi identity

$$-2\pi i = \int_{S^2} F^{EM} = \int_{S^2} dA^{EM} = \int_{B^3} d^2 A^{EM} \longrightarrow d^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$

- The result is having a string of monopole along Z
- In 5D, Bianchi identity is violated by A^{EM}

$$DF \equiv dF + A \wedge F - F \wedge A = Qd^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$
$$d\mathrm{tr}\,(F \wedge F) = 2\mathrm{tr}(DF \wedge F) = -4\pi i \mathrm{tr}(QF) \wedge \delta_3(\vec{0})$$

In components, this is

$$\partial_\mu \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\nu\alpha}F_{\beta Z})\right) = -\frac{1}{4}\partial_Z \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\right) + 4\pi i \delta^{(3)}(\vec{x})\mathrm{tr}(QF_{tZ})$$

Details...

- Recall that the external EM potential is encoded in the 5D gauge field as

$$A = QA^{EM} + \psi_+(Z)U^{-1}\partial_\mu U + \cdots$$

Note that A^{EM} is a constant mode along Z

- We then put the monopole background to A^{EM} with the violation of Bianchi identity

$$-2\pi i = \int_{S^2} F^{EM} = \int_{S^2} dA^{EM} = \int_{B^3} d^2 A^{EM} \longrightarrow d^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$

- The result is having a string of monopole along Z
- In 5D, Bianchi identity is violated by A^{EM}

$$DF \equiv dF + A \wedge F - F \wedge A = Qd^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$
$$d\mathrm{tr}\,(F \wedge F) = 2\mathrm{tr}(DF \wedge F) = -4\pi i \mathrm{tr}(QF) \wedge \delta_3(\vec{0})$$

In components, this is

$$\partial_\mu \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\nu\alpha}F_{\beta Z})\right) = -\frac{1}{4}\partial_Z \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\right) + 4\pi i \delta^{(3)}(\vec{x})\mathrm{tr}(QF_{tZ})$$

Details...

- Recall that the external EM potential is encoded in the 5D gauge field as

$$A = QA^{EM} + \psi_+(Z)U^{-1}\partial_\mu U + \cdots$$

Note that A^{EM} is a constant mode along Z

- We then put the monopole background to A^{EM} with the violation of Bianchi identity

$$-2\pi i = \int_{S^2} F^{EM} = \int_{S^2} dA^{EM} = \int_{B^3} d^2 A^{EM} \longrightarrow d^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$

- The result is having a string of monopole along Z
- In 5D, Bianchi identity is violated by A^{EM}

$$DF \equiv dF + A \wedge F - F \wedge A = Qd^2 A^{EM} = -2\pi i \delta_3(\vec{0})$$
$$d\mathrm{tr}\,(F \wedge F) = 2\mathrm{tr}(DF \wedge F) = -4\pi i \mathrm{tr}(QF) \wedge \delta_3(\vec{0})$$

In components, this is

$$\partial_\mu \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\nu\alpha}F_{\beta Z})\right) = -\frac{1}{4}\partial_Z \left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\right) + 4\pi i \delta^{(3)}(\vec{x})\mathrm{tr}(QF_{tZ})$$

Details continued...

By integrating over Z, we precisely get the violation of baryon number

$$\partial_\mu B^\mu = \frac{1}{32\pi^2}\left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\Big|_R - \epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\Big|_L\right) + \frac{i\delta^{(3)}(\vec{x})}{2\pi}\int dZ\,\mathrm{tr}\left(QF_{tZ}\right)$$

Details continued...

By integrating over Z, we precisely get the violation of baryon number

$$\partial_\mu B^\mu = \frac{1}{32\pi^2}\left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\Big|_R - \epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\Big|_L\right) + \frac{i\delta^{(3)}(\vec{x})}{2\pi}\int dZ\,\mathrm{tr}\left(QF_{tZ}\right)$$

- The first two pieces are baryon number violation by chiral asymmetric sphalerons realized at low energy
- The last piece is the monopole induced baryon number violation

Details continued...

By integrating over Z, we precisely get the violation of baryon number

$$\partial_\mu B^\mu = \frac{1}{32\pi^2}\left(\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\Big|_R - \epsilon^{\mu\nu\alpha\beta}\mathrm{tr}(F_{\mu\nu}F_{\alpha\beta})\Big|_L\right) + \frac{i\delta^{(3)}(\vec{x})}{2\pi}\int dZ\,\mathrm{tr}\,(QF_{tZ})$$

- The first two pieces are baryon number violation by chiral asymmetric sphalerons realized at low energy
- The last piece is the monopole induced baryon number violation

Indeed, by plugging in the mode expansion of 5D gauge field

$$A_\mu = [(U^{-1}(QA_\mu^{EM} + A_{L\mu})U)\psi_+(Z) + (QA_\mu^{EM} + A_{R\mu})\psi_-] + \psi_+ U^{-1}\partial_\mu U + \cdots$$

one finally gets

$$\partial_\mu B^\mu = \mathrm{tr}(Q\sigma^3)\frac{(\partial_t\pi)}{\pi F_\pi}\delta^{(3)}(\vec{x}) - \frac{i\delta^{(3)}(\vec{x})}{2\pi}\mathrm{tr}[QU^{-1}A_{Lt}U - QA_{Rt}]$$

The first piece is precisely monopole catalysis formula !!!

Speculations

The second piece

$$\partial_\mu B^\mu \sim -\frac{i\delta^{(3)}(\vec{x})}{2\pi}\mathrm{tr}[QU^{-1}A_{Lt}U - QA_{Rt}]$$

states that baryons can be created or annihilated in the chirally asymmetric chemical potential environment

Speculations

The second piece

$$\partial_\mu B^\mu \sim -\frac{i\delta^{(3)}(\vec{x})}{2\pi}\mathrm{tr}[QU^{-1}A_{Lt}U - QA_{Rt}]$$

states that baryons can be created or annihilated in the chirally asymmetric chemical potential environment

Recall that the non-normalizable external potentials A_{Lt} and A_{Rt} couple to the chiral current density

$$J^0_L A_{Lt} + J^0_R A_{Rt}$$

so that they represent chemical potential for chiral densities

Thank you very much

- In summary, monopole catalysis in holographic QCD is simply a violation of Bianchi identity
- The reason for this simplification is that holographic QCD unifies mesons and the external weakly gauged chiral symmetry potential in a single framework of 5D gauge theory
- What is left is a study of dynamics of monopole catalysis, such as cross sections etc, in the framework of holographic QCD

Thank you very much

Thank you very much

- In summary, monopole catalysis in holographic QCD is simply a violation of Bianchi identity
- The reason for this simplification is that holographic QCD unifies mesons and the external weakly gauged chiral symmetry potential in a single framework of 5D gauge theory
- What is left is a study of dynamics of monopole catalysis, such as cross sections etc, in the framework of holographic QCD

Thank you very much

Thank you very much

- In summary, monopole catalysis in holographic QCD is simply a violation of Bianchi identity
- The reason for this simplification is that holographic QCD unifies mesons and the external weakly gauged chiral symmetry potential in a single framework of 5D gauge theory
- What is left is a study of dynamics of monopole catalysis, such as cross sections etc, in the framework of holographic QCD

Thank you very much