

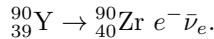
## Fizika jadra in osnovnih delcev

### 1. KOLOKVIJ

1. Izračunaj presek za sipanje elektronov z gibalno količino  $p_e = 120 \text{ MeV}/c$  na jedru  ${}^9_4\text{Be}$  pod kotom  $98^\circ < \vartheta < 102^\circ$ . Gostota naboja v jedru je  $\bar{\rho}(r) \propto r e^{-r/R}$ , kjer je  $R = 0.3 \text{ fm}$ . V pomoč ti bo integral

$$\int_0^\infty x^2 \sin x e^{-ax} dx = \frac{6a^2 - 2}{(1 + a^2)^3}, \quad a > 0.$$

2. Izotop stroncija  ${}^{90}_{38}\text{Sr}$  lahko uporabljamo kot vir energije s specifično močjo  $0.54 \text{ W/g}$ . Sr razpada preko  $\beta^-$  v izotop  ${}^{90}_{39}\text{Y}$  z razpolovnim časom  $28.8 \text{ let}$ , ki nato hipoma razpade preko  $\beta^-$ :



Oceni razliko mas jeder  ${}^{90}_{39}\text{Y}$  in  ${}^{90}_{40}\text{Zr}$ , če predpostaviš, da je sproščena energija v razpadu  ${}^{90}_{38}\text{Sr}$  zanemarljiva. Zanemari tudi odziv jedra. Kolikšna je najverjetnejša kinetična energija elektronov v razpadih Y?

3. Pri sipanju mezonov  $\pi$  in nukleonov opazimo novo resonanco  $R$ . Določi izospin  $R$ , če izmerimo razmerje presekov  $\sigma(\pi^+n \rightarrow R^+ \rightarrow \pi^+n)/\sigma(\pi^0p \rightarrow R^+ \rightarrow \pi^0p) = 1/4$ ? Nato napovej razmerje resonančnih presekov

$$\frac{\sigma(\pi^+n \rightarrow R^+ \rightarrow \Delta^+\pi^0)}{\sigma(\pi^-p \rightarrow R^0 \rightarrow \Delta^+\pi^-)},$$

kjer ima stanje  $\Delta$  izospin  $I = 3/2$  in hipernaboj  $Y = 1$ .

Nato opazujemo še sipanje  $\pi^+n \rightarrow R^+ \rightarrow \rho^+n$  pri težiščnem sipalnem kotu  $10^\circ$  med smerjo vhodnega  $\pi^+$  in  $\rho^+$  v končnem stanju. Mezon  $\rho^+$  ima spin 1. Kakšna je verjetnost, da ima  $\rho^+$  komponento spina 1 ( $\rho = |1, 1\rangle$ ) v smeri gibalne količine? Predpostavi, da ima  $R$  spin  $1/2$ , ter da je tirna vrtilna količina v končnem stanju  $\ell = 0$ . Komponenta spina vhodnega  $n$  je vzdolž gibalne količine  $\pi^+$ .

Note: A square-root sign is to be understood over every coefficient, e.g., for  $-8/15$  read  $-\sqrt{8/15}$ .

Notation: 

$J$	$J$	...
$M$	$M$	...
$m_1$	$m_2$	
$m_1$	$m_2$	Coefficients
.	.	.
.	.	.

$1/2 \times 1/2$ 

1	0
+1/2 +1/2	1 0
-1/2 +1/2	1/2 1/2
-1/2 -1/2	1/2 -1/2
-1/2 -1/2	1 -1

 $Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$   $2 \times 1/2$ 

5/2	3/2
+5/2	1
+2 +1/2	1/5 4/5
+1 +1/2	4/5 -1/5
	5/2 3/2
	+1/2 +1/2

 $Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}$

$1 \times 1/2$ 

3/2	1/2
+3/2	1
+1 -1/2	1/3 2/3
0 +1/2	2/3 -1/3
	3/2 1/2
	-1/2 -1/2

 $Y_2^0 = \sqrt{\frac{5}{4\pi}} \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$   $3/2 \times 1/2$ 

5/2	3/2
+5/2	1
+3/2 +1/2	2
+2 +1	1
+1 +1	1
	1/10 2/5 1/2
	3/5 1/5 -1/3
	5/2 3/2 1/2
	-1/2 -1/2

 $Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$

$2 \times 1$ 

3	2
+3	1
+2 +1	1
+1 +1	1
	1/3 2/3
	2/3 -1/3
	3 2 1
	+1 +1 +1

 $3/2 \times 1$ 

5/2	3/2
+5/2	1
+3/2 +1	1
+1 +1	1
	2/5 3/5
	3/5 -2/5
	5/2 3/2 1/2
	+1/2 +1/2

 $1 \times 1$ 

2	1
+2	1
+1 +1	1
0 +1	1
	1/15 1/3 3/5
	1/5 -1/5
	3 2 1
	0 0 0

 $Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi}$ 

5/2	3/2
+5/2	1
+3/2 +1	1
+1 +1	1
	1/10 2/5 1/2
	3/5 1/5 -1/3
	5/2 3/2 1/2
	-1/2 -1/2

$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$   $d_{m,0}^\ell = \sqrt{\frac{4\pi}{2\ell+1}} Y_\ell^m e^{-im\phi}$ 

3	2
+3	1
+2 +1	1
+1 +1	1
	2/5 1/2 1/10
	8/15 -1/6 -3/10
	3 2 1
	0 0 0

3	2
+3	1
+2 +1	1
+1 +1	1
	1/15 -1/3 3/5
	1/5 -1/5
	3 2 1
	-1 -1 -1

3	2
+3	1
+2 +1	1
+1 +1	1
	3/10 8/15 1/6
	1/10 -2/5 1/2
	5/2 3/2
	-3/2 -3/2

3	2
+3	1
+2 +1	1
+1 +1	1
	2/5 1/2 1/10
	8/15 -1/6 -3/10
	3 2
	-2 -2

3	2
+3	1
+2 +1	1
+1 +1	1
	1/15 -1/3 3/5
	1/5 -1/5
	3 2
	-2 -2

3	2
+3	1
+2 +1	1
+1 +1	1
	2/3 1/3
	1/3 -2/3
	3
	-2 -1

3	2
+3	1
+2 +1	1
+1 +1	1
	2/3 1/3
	1/3 -2/3
	3
	-2 -1

 $(j_1 j_2 m_1 m_2 | j_1 j_2 JM)$   
 $= (-1)^{J-j_1-j_2} (j_2 j_1 m_2 m_1 | j_2 j_1 JM)$

$d_{m',m}^j = (-1)^{m-m'} d_{m,m'}^j = d_{-m,-m'}^j$   $3/2 \times 3/2$ 

3	2
+3	1
+2 +1	1
+1 +1	1
	1/2 1/2
	1/2 -1/2
	3 2 1
	+1 +1 +1

 $d_{0,0}^1 = \cos \theta$   $d_{1/2,1/2}^{1/2} = \cos \frac{\theta}{2}$   $d_{1,1}^1 = \frac{1 + \cos \theta}{2}$

$2 \times 3/2$ 

7/2	5/2
+7/2	1
+5/2 +1/2	1
+3/2 +1/2	1
	3/7 4/7
	7/2 5/2 3/2
	+3/2 +3/2

 $d_{1/2,-1/2}^{1/2} = -\sin \frac{\theta}{2}$   $d_{1,0}^1 = -\frac{\sin \theta}{\sqrt{2}}$

$1 \times 1$ 

2	1
+2	1
+1 +1	1
0 +1	1
	1/7 16/35 2/5
	1/7 -16/35 2/5
	2 1 0
	0 0 0

 $d_{1,-1}^1 = \frac{1 - \cos \theta}{2}$

$$1. \quad p_e = 120 \text{ MeV}/c$$

$$Z = 4$$

$$\vartheta \in [98^\circ; 102^\circ]$$

$$\bar{\rho} = A r e^{-r/R}$$

$$R = 0,3 \text{ fm}$$

$$\frac{d\sigma}{d\vartheta} \frac{1}{2} d\Omega (98^\circ; 102^\circ)$$

normalizacija  $\bar{\rho} = \int \bar{\rho} dV = \int \bar{\rho} 4\pi r^2 dr \stackrel{!}{=} 1$

$$A \int_0^\infty dr \cdot 4\pi r^3 e^{-r/R} = A \int_0^\infty du \cdot 4\pi R^4 u^3 e^{-u}$$

$$= A \cdot 4\pi R^4 \Gamma(4)$$

$$= A \cdot 4\pi R^4 \cdot 6 \stackrel{!}{=} 1$$

$$\Rightarrow \bar{\rho} = \frac{1}{24\pi R^3} \frac{r}{R} e^{-\frac{r}{R}}$$

$\frac{1}{4}^+$

običajna funkcija:

$$F(\vec{q}) = \bar{F}(q) = \frac{4\pi}{q} \int_0^\infty dx \cdot x \sin(qx) \bar{\rho}(x)$$

$$= \frac{4\pi}{q \cdot 24\pi R^3} \int_0^\infty dx \cdot x \cdot \sin(qx) \frac{x}{R} e^{-\frac{x}{R}}$$

$$= \frac{1}{6q^4 R^4} \int_0^\infty dx \cdot x^2 \sin x \cdot e^{-\frac{x}{qR}}$$

$$= \frac{1}{6(qR)^4} \frac{\frac{6}{(qR)^2} - 2}{\left(1 + \frac{1}{(qR)^2}\right)^3} = \frac{3 - (qR)^2}{3(1 + (qR)^2)^3}$$

$\frac{1}{4}^+$

=

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 z^2 (mc^2)^2 (\hbar c)^2}{4(pc)^4} \frac{|F(q)|^2}{\sin^4 \frac{\theta}{2}}$$

$$q^2 = 4p^2 \sin^2 \frac{\theta}{2} \quad , \quad (qR)^2 = \frac{2(pc)^2 R^2}{\hbar c} \sin^2 \frac{\theta}{2}$$

$$= \frac{2 \cdot 120 \text{ MeV} \cdot 0,3 \text{ fm}}{200 \text{ MeV} \text{ fm}} \sin^2 50^\circ$$

$$= \cancel{0,355} \quad 0,276 \quad (+)$$

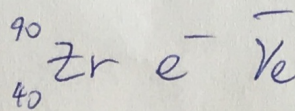
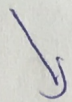
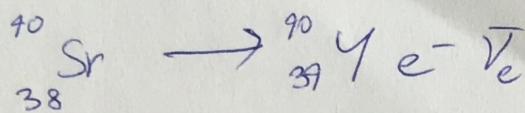
$$\frac{d\sigma}{d\Omega} = 1,91 \cdot 10^{-8} \text{ fm}^2$$

$$d\sigma = \frac{d\sigma}{d\Omega} \cdot d\Omega = \frac{d\sigma}{d\Omega} \cdot \underbrace{2\pi \sin \theta}_{(+)} d\theta, \quad d\theta = \frac{4\pi}{180}$$

$$= \underline{\underline{8,24 \cdot 10^{-9} \text{ fm}^2}} = 8,24 \cdot 10^{-39} \text{ m}^2$$

$$= 8,24 \cdot 10^{-11} \text{ b} = \underline{\underline{82,4 \text{ nb}}}$$

2.



$$A = 90$$

$$t_{1/2} = 28,8 \text{ let}, \quad \tau = \frac{t_{1/2}}{\ln 2}$$

$$Q = \frac{P}{m} = \frac{\frac{N}{\tau} \langle T_e \rangle}{N \cdot m_{\text{Sr}}} = \frac{\langle T_e \rangle}{m_{\text{Sr}} t_{1/2}} \ln 2 \Rightarrow \langle T_e \rangle = \frac{Q A \tau t_{1/2}}{\ln 2}$$

1/4

Povprčna  $\bar{E}$  elektrona:  $\langle \epsilon \rangle = \frac{\epsilon_0}{2} \left( 1 + \frac{5}{2\epsilon_0^2} \right)$

$$\langle T_e \rangle = 12,9 \text{ MeV} = 0,66 \text{ MeV} = 1,06 \cdot 10^{-13} \text{ J}$$

$$\langle T_e \rangle = \text{MeV} \frac{\epsilon_0}{2} \left( 1 + \frac{5}{2\epsilon_0^2} \right) - \text{MeV}$$

$$= \text{MeV} \left( \frac{\epsilon_0}{2} + \frac{5}{4\epsilon_0} - 1 \right)$$

$$\text{MeV} \left( \frac{\epsilon_0}{2} + \frac{5}{4\epsilon_0} - 1 \right) = \frac{Q A \tau t_{1/2}}{\ln 2}$$

1/4

$$\frac{\epsilon_0}{2} + \frac{5}{4\epsilon_0} - 1 - \frac{2Q A \tau t_{1/2}}{\text{MeV} \ln 2} = 0$$

$$\epsilon_0^2 - \tilde{q} \epsilon_0 + \frac{5}{2} = 0$$

$$\tilde{q} = 2 \left( 1 + \frac{Q A \tau t_{1/2}}{\text{MeV} \ln 2} \right)$$

$$\epsilon_0 = \frac{\tilde{q} \pm \sqrt{\tilde{q}^2 - 4 \cdot \frac{5}{2}}}{2}$$

$$= \begin{cases} 395 \leftarrow \\ 0,63 \times \end{cases}$$

1/4

$$\Rightarrow E_0 = \Delta m c^2 = \epsilon_0 m_e c^2$$

$$= 20 \text{ MeV}$$

$$= 2 \left( 1 + 12,9 \cdot 10^{-1} \right) = 4,58$$

najv. energija

$$E_{\text{max}} = \frac{\epsilon_0}{2} + \frac{1}{2\epsilon_0}$$

$$T_{\text{max}} = (E_{\text{max}} - 1) m_e c^2 = \left( \frac{\epsilon_0}{2} + \frac{1}{2\epsilon_0} - 1 \right) m_e c^2$$

1/4

3. 1)  $\frac{\sigma(\pi^+ n \rightarrow R^+ \rightarrow \pi^+ n)}{\sigma(\pi^0 p \rightarrow R^+ \rightarrow \pi^0 p)} = \frac{1}{4}$

$\pi^+ n = |11\rangle \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \right) = \frac{1}{\sqrt{3}} \left| \frac{3}{2} \frac{1}{2} \right\rangle + \sqrt{\frac{2}{3}} \left| \frac{1}{2} \frac{1}{2} \right\rangle$

$\pi^0 p = \sqrt{\frac{2}{3}} \left| \frac{3}{2} \frac{1}{2} \right\rangle - \sqrt{\frac{1}{3}} \left| \frac{1}{2} \frac{1}{2} \right\rangle$

a) R ma  $\sigma_{\pi^0 p} = \frac{3}{2}$

$\frac{\sigma_{\pi^+ n}}{\sigma_{\pi^0 p}} = \frac{\left(\frac{1}{\sqrt{3}}\right)^4}{\left(\sqrt{\frac{2}{3}}\right)^4} = \frac{1}{4}$

$\Rightarrow \boxed{I = \frac{3}{2}}$

$\frac{1}{4}^+$

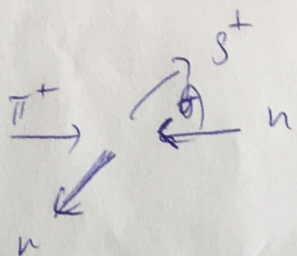
b) R ma  $I = \frac{1}{2}$

$\frac{\sigma_{\pi^+ n}}{\sigma_{\pi^0 p}} = \frac{\left(\sqrt{\frac{2}{3}}\right)^4}{\left(-\frac{1}{\sqrt{3}}\right)^4} = 4$

2)  $\frac{\sigma_{\pi^+ n \rightarrow R^+ \rightarrow \Delta^+ \pi^0}}{\sigma_{\pi^0 p \rightarrow R^0 \rightarrow \Delta^0 \pi^-}} = \frac{\left(\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{15}}\right)^2}{\left(\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{8}}{\sqrt{15}}\right)^2} = \frac{1}{8}$

$\frac{1}{4}$

3)  $\pi^+ n \rightarrow R^+ \rightarrow S^+ n$



$\left| \frac{1}{2} + \frac{1}{2} \right\rangle = R$

$R^+ = \sum_{m_1} d_{m_1 m_2}^j (-\theta) |j m_1\rangle'$   
 $R^+ = \sum_{m_1} d_{m_1 m_2}^{1/2} (-\theta) \left| \frac{1}{2} m_1 \right\rangle'$

$R^+ = d_{1/2 1/2}^{1/2} (-\theta) \left| \frac{1}{2} \frac{1}{2} \right\rangle' + d_{-1/2 1/2}^{1/2} (-\theta) \left| \frac{1}{2} -\frac{1}{2} \right\rangle'$

$= d_{1/2 1/2}^{1/2} (-\theta) \left( \sqrt{\frac{2}{3}} |11\rangle |1/2\rangle - \frac{1}{\sqrt{3}} |10\rangle |1/2\rangle \right)$

$+ d_{-1/2 1/2}^{1/2} (-\theta) \left( \frac{1}{\sqrt{3}} |10\rangle |1/2\rangle - \sqrt{\frac{2}{3}} |1-1\rangle |1/2\rangle \right)$

$$\rho = \frac{\left(\sqrt{\frac{2}{3}} d_{\frac{1}{2}\frac{1}{2}}^{1/2}(-\theta)\right)^2}{\left(d_{\frac{1}{2}\frac{1}{2}}^{1/2}(-\theta)\right)^2 + \left(d_{-\frac{1}{2}\frac{1}{2}}^{1/2}(-\theta)\right)^2}$$

$$= \frac{\frac{2}{3} \cos^2 \frac{\theta}{2}}{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}} = \frac{2}{3} \cos^2 \frac{\theta}{2}$$

$$\approx 0,66$$

1/4+