19. april 2019

Fizika jedra in osnovnih delcev

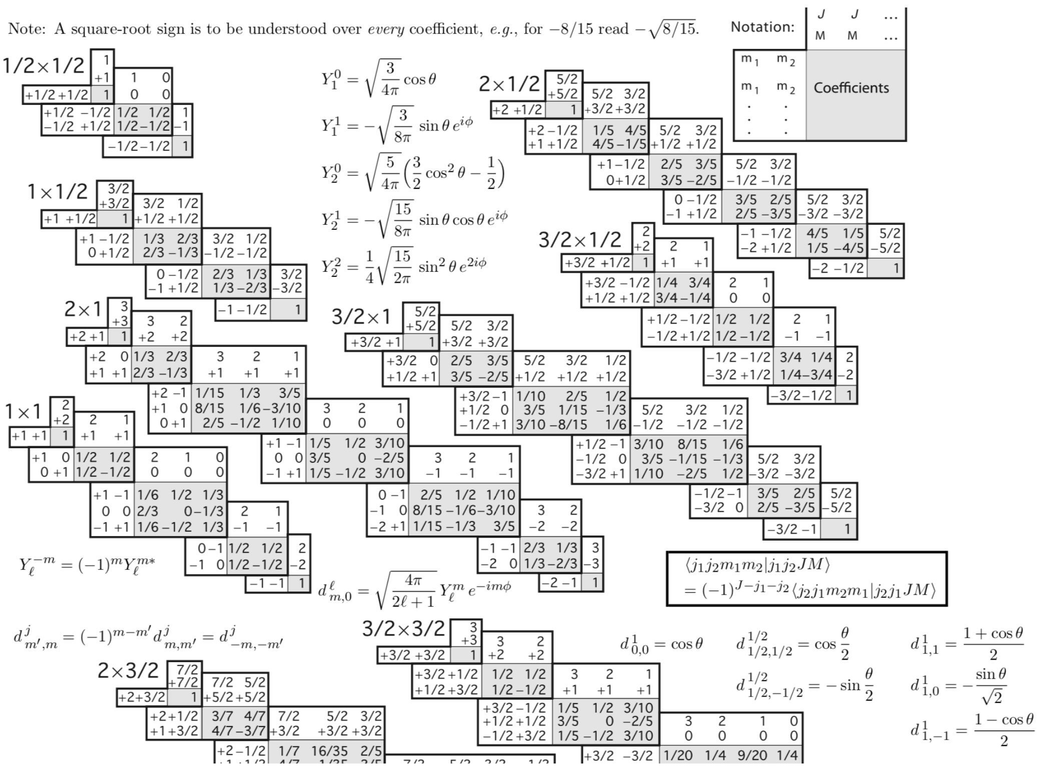
1. KOLOKVIJ

1. Izračunaj presek za sipanje elektronov z gibalno količino $p_e = 120\,\mathrm{MeV}/c$ na jedru ${}^{9}_{4}\mathrm{Be}$ pod kotom $98^\circ < \vartheta < 102^\circ$. Gostota naboja v jedru je $\bar{\rho}(r) \propto re^{-r/R}$, kjer je $R = 0.3\,\mathrm{fm}$. V pomoč ti bo integral
$$\int_0^\infty x^2 \sin x\, e^{-ax}\, dx = \frac{6a^2-2}{(1+a^2)^3}, \qquad a > 0.$$

2. Izotop stroncija ${}^{90}_{38}\mathrm{Sr}$ lahko uporabljamo kot vir energije s specifično močjo $0.54\,\mathrm{W/g}$. Sr razpada preko β^- v izotop ${}^{90}_{39}\mathrm{Y}$ z razpolovnim časom 28.8 let, ki nato hipoma razpade preko β^-:
$${}^{90}_{39}\mathrm{Y} \to {}^{90}_{40}\mathrm{Zr}\; e^- \bar{\nu}_e.$$
Oceni razliko mas jeder ${}^{90}_{39}\mathrm{Y}$ in ${}^{90}_{40}\mathrm{Zr}$, če predpostaviš, da je sproščena energija v razpadu ${}^{90}_{38}\mathrm{Sr}$ zanemarljiva. Zanemari tudi odriv jedra. Kolikšna je najverjetnejša kinetična energija elektronov v razpadih Y?

3. Pri sipanju mezonov π in nukleonov opazimo novo resonanco R. Določi izospin R, če izmerimo razmerje presekov $\sigma(\pi^+ n \to R^+ \to \pi^+ n)/\sigma(\pi^0 p \to R^+ \to \pi^0 p) = 1/4$? Nato napovej razmerje resonančnih presekov
$$\frac{\sigma(\pi^+ n \to R^+ \to \Delta^+ \pi^0)}{\sigma(\pi^- p \to R^0 \to \Delta^+ \pi^-)},$$
kjer ima stanje Δ izospin $I = 3/2$ in hipernaboj $Y = 1$.

   Nato opazujemo še sipanje $\pi^+ n \to R^+ \to \rho^+ n$ pri težiščnem sipalnem kotu 10° med smerjo vhodnega π^+ in ρ^+ v končnem stanju. Mezon ρ^+ ima spin 1. Kakšna je verjetnost, da ima ρ^+ komponento spina 1 ($\rho = |1,1\rangle$) v smeri gibalne količine? Predpostavi, da ima R spin 1/2, ter da je tirna vrtilna količina v končnem stanju $\ell = 0$. Komponenta spina vhodnega n je vzdolž gibalne količine π^+.



(1.) $p_e = 120 \text{ MeV}/c$

$Z = 4$

$\vartheta \in [98°; 102°]$

$\bar{\rho} = A\, r\, e^{-r/R}$

$R = 0{,}3 \text{ fm}$

$\frac{d\sigma}{d\vartheta}$ $d\Omega\,(98°; 102°)$

normalizacija $\bar{\rho}$: $\int \bar{\rho}\, dV = \int \bar{\rho}\, 4\pi r^2\, dr \overset{!}{=} 1$

$$A \int_0^\infty dr \cdot 4\pi r^3 e^{-r/R} = A \int_0^\infty du \cdot 4\pi R^4 u^3 e^{-u}$$

$$= A \cdot 4\pi R^4\, \Gamma(4)$$

$$= A \cdot 4\pi R^4 \cdot 6 \overset{!}{=} 1$$

$$\Rightarrow \bar{\rho} = \frac{1}{24\pi R^3} \frac{r}{R} e^{-\frac{r}{R}}$$

1/4+

oblikovna funkcija:

$$F(\vec{q}) = F(q) = \frac{4\pi}{q} \int_0^\infty dr\; r \sin(qr)\, \bar{\rho}(r)$$

$$= \frac{4\pi}{q \cdot 24\pi R^3} \int_0^\infty dr \cdot r \cdot \sin(\underbrace{qr}_{x}) \frac{r}{R} e^{-\frac{r}{R}}$$

$$= \frac{1}{6 q^4 R^4} \int_0^\infty dx\; x^2 \sin x\; e^{-\frac{x}{qR}}$$

$$= \frac{1}{6(qR)^4} \frac{\frac{6}{(qR)^2} - 2}{\left(1 + \frac{1}{(qR)^2}\right)^3} = \frac{3 - (qR)^2}{3\left(1 + (qR)^2\right)^3}$$

1/4+

$=$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 Z^2 (mc^2)^2 (\hbar c)^2}{4 (pc)^4} \quad \frac{|F(q)|^2}{\sin^4 \frac{\vartheta}{2}}$$

$$q^2 = 4 \frac{p^2}{\hbar^2} \sin^2 \frac{\vartheta}{2} \; , \quad (qR)^{\cancel{2}} = \frac{\cancel{4}\,2\,(pc)^{\cancel{2}} R^{\cancel{2}}}{\hbar c} \sin \frac{\vartheta}{2}$$

$$= \frac{2 \cdot 120 \text{ MeV} \cdot 0{,}3 \text{ fm}}{200 \text{ MeV fm}} \sin 50^\circ$$

$$= \cancel{0{,}355} \; 0{,}276$$

+

$$\frac{d\sigma}{d\Omega} = 1{,}91 \cdot 10^{-8} \text{ fm}^2$$

+

$$d\sigma = \frac{d\sigma}{d\Omega} \cdot d\Omega = \frac{d\sigma}{d\Omega} \cdot \underbrace{2\pi \sin \vartheta \, d\vartheta} \; , \quad d\vartheta = \frac{4\pi}{180}$$

$$= 8{,}24 \cdot 10^{-9} \text{ fm}^2 = 8{,}24 \cdot 10^{-39} \text{ m}^2$$

$$= 8{,}24 \cdot 10^{-11} \text{ b} = 82{,}4 \text{ nb}$$

(2.) $\,^{90}_{38}Sr \longrightarrow \,^{90}_{39}Y\, e^- \bar{\nu}_e$

$\searrow \,^{90}_{40}Zr\, e^- \bar{\nu}_e$

$A = 90$

$t_{1/2} = 28{,}8$ let, $\quad \tau = \frac{t_{1/2}}{\ln 2}$

$$q = \frac{P}{m} = \frac{\frac{N}{\tau}\langle T_e\rangle}{N \cdot m_{sr}} = \frac{\langle T_e\rangle}{m_{sr}\, t_{1/2}} \ln 2 \Rightarrow \langle T_e\rangle = \frac{q A u\, t_{1/2}}{\ln 2}$$

1/4

Povprečna E elektrona: $\langle \varepsilon\rangle = \frac{\varepsilon_0}{2}\left(1 + \frac{5}{2\varepsilon_0^2}\right)$

$\langle T_e\rangle = 1{,}29\, m_e c^2 = 0{,}66$ MeV

$= 1{,}06 \cdot 10^{-13}$ J

$$\langle T_e\rangle = m_e c^2 \frac{\varepsilon_0}{2}\left(1 + \frac{5}{2\varepsilon_0^2}\right) - m_e c^2$$

$$= m_e c^2 \left(\frac{\varepsilon_0}{2} + \frac{5}{4\varepsilon_0} - 1\right)$$

$$m_e c^2\left(\frac{\varepsilon_0}{2} + \frac{5}{4\varepsilon_0} - 1\right) = \frac{q A u\, t_{1/2}}{\ln 2}$$

1/4

$$\frac{\varepsilon_0}{\cancel{2}} + \frac{5}{\cancel{2}\varepsilon_0} \underbrace{- 2 - \frac{2 q A u\, t_{1/2}}{m_e c^2 \ln 2}}_{} = 0$$

$$\varepsilon_0^2 - \tilde{q}\,\varepsilon_0 + \frac{5}{2} = 0 \qquad \tilde{q} = 2\left(1 + \frac{q A u\, t_{1/2}}{m_e c^2 \ln 2}\right)$$

$$\varepsilon_0 = \frac{\tilde{q} \pm \sqrt{\tilde{q}^2 - 4\cdot\frac{5}{2}}}{2} = \begin{cases} 3{,}95 \leftarrow \\ 0{,}63 \times \end{cases}$$

1/4

$$= 2\left(1 + \frac{0{,}54\, \cancel{eV} \cdot 90 \cdot 1{,}66\cdot 10^{-27} \cancel{kg} \cdot 28{,}8\cdot 3{,}15\cdot 10^{7} \cancel{s}}{10^{-3} \cancel{kg}\, \cancel{s} \cdot 1{,}6\cdot 10^{-19} \cdot \ln 2 \cdot 511\cdot 10^{3} \cancel{eV}}\right)$$

$$= 2\left(1 + \frac{0{,}54\cdot 0{,}9\cdot 1{,}66\cdot 2{,}88\cdot 3{,}15}{1{,}6\cdot \ln 2\cdot 0{,}511} 10^{2-27+1+7+3+19-6}\right)$$

$$= 2\left(1 + 12{,}9\cdot 10^{-1}\right) = 4{,}58$$

$\Rightarrow E_0 = \Delta m c^2 = \varepsilon_0 m_e c^2$

$= \underline{\underline{2{,}0 \text{ MeV}}}$

najv. energija $\varepsilon_{max} = \frac{\varepsilon_0}{2} + \frac{1}{2\varepsilon_0}$ $\quad T_{max} = (\varepsilon_{max} - 1) m_e c^2 = \left(\frac{\varepsilon_0}{2} + \frac{1}{2\varepsilon_0} - 1\right) m_e c^2$

$= \underline{0{,}71 \text{ MeV}}$

1/4

3.

1) $$\frac{\sigma(\pi^+ n \to R^+ \to \pi^+ n)}{\sigma(\pi^0 p \to R^+ \to \pi^0 p)} = \frac{1}{4}$$

$$\pi^+ n = |1\,1\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}}|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$\pi^0 p = \qquad = \sqrt{\tfrac{2}{3}}|\tfrac{3}{2}\,\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

a) R ima izospin $\frac{3}{2}$

$$\frac{\sigma_{\pi^+ n}}{\sigma_{\pi^0 p}} = \frac{\left(\frac{1}{\sqrt{3}}\right)^4}{\left(\sqrt{\frac{2}{3}}\right)^4} = \frac{1}{4} \Rightarrow \boxed{I_R = \frac{3}{2}}$$

1/4+

b) R ima $I = \frac{1}{2}$

$$\frac{\sigma_{\pi^+ n}}{\sigma_{\pi^0 p}} = \frac{\left(\sqrt{\frac{2}{3}}\right)^4}{\left(-\frac{1}{\sqrt{3}}\right)^4} = 4$$

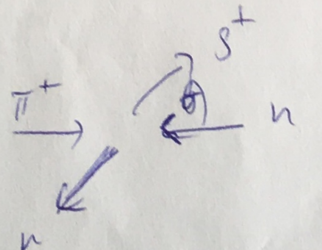
2) $$\frac{\sigma_{\pi^+ n \to R^+ \to \Delta^+ \pi^0}}{\sigma_{\pi^- p \to R^0 \to \Delta^+ \pi^-}} = \frac{\left(\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{15}}\right)^2}{\left(\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{8}}{\sqrt{15}}\right)^2} = \frac{1}{8}$$

$\Delta^+\pi^0 = \frac{1}{\sqrt{15}}|\tfrac{3}{2}\,\tfrac{1}{2}\rangle$

$\Delta^+\pi^- = \sqrt{\tfrac{8}{15}}|\tfrac{3}{2}\,-\tfrac{1}{2}\rangle$

1/4

3) $\pi^+ n \to R^+ \to S^+ n$



$\cancel{\ldots}\ |\tfrac{1}{2}\,+\tfrac{1}{2}\rangle = R$

v sistemu R': $R' = \sum_{m'} d^{j}_{m'm}(-\theta)|j\,m'\rangle'$

$$= \sum_{m'} d^{1/2}_{m'\,1/2}(-\theta)|\tfrac{1}{2}\,m'\rangle'$$

$$R' = d^{1/2}_{1/2\,1/2}(-\theta)|\tfrac{1}{2}\,\tfrac{1}{2}\rangle' + d^{1/2}_{-1/2\,1/2}(-\theta)|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle'$$

$$= d^{1/2}_{1/2\,1/2}(-\theta)\left(\sqrt{\tfrac{2}{3}}\,|1\,1\rangle|n\downarrow\rangle - \frac{1}{\sqrt{3}}|1\,0\rangle|n\uparrow\rangle\right)$$

$$+\ d^{1/2}_{-1/2\,1/2}(-\theta)\left(\frac{1}{\sqrt{3}}|1\,0\rangle|n\downarrow\rangle - \sqrt{\tfrac{2}{3}}|1\,-1\rangle|n\uparrow\rangle\right)$$

$$\rho = \frac{\left(\sqrt{\frac{2}{3}}\; d^{1/2}_{1/2\,1/2}(-\theta)\right)^2}{\left(d^{1/2}_{1/2\,1/2}(-\theta)\right)^2 + \left(d^{1/2}_{-1/2\,1/2}(-\theta)\right)^2}$$

$$= \frac{2}{3} \frac{\cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}} = \frac{2}{3}\cos^2\frac{\theta}{2}$$

1/4+

$$\cong 0{,}66$$