

## Fizika jedra in osnovnih delcev

### 2. KOLOKVIJ

- Za barion  $\Omega_c^0$ , ki je osnovno stanje kvarkov  $ssc$  s tirno vrtilno količino  $\ell = 0$  in spinom  $1/2$ , zapiši valovno funkcijo  $|\Omega_c^0(S_z = +1/2)\rangle$ . Izračunaj magnetni moment  $\Omega_c^0$ , če je  $m_s = 400$  MeV, masa  $m_c$  je zelo velika v primerjavi z  $m_s$ , naboja sta  $q_s = -e_0/3$ ,  $q_c = 2e_0/3$ .
- Ali naslednji procesi lahko potekajo v okviru Standardnega modela (preko močne, šibke, ali elektromagnetne interakcije)? Če potekajo, skiciraj dominanten Feynmanov diagram, ki prispeva k amplitudi, sicer utemelji, zakaj proces ne poteka.

- $p\pi^- \rightarrow \pi^0 n$
- $D^+(c\bar{d}) \rightarrow \tau^+\nu_\tau$
- $p\bar{n} \rightarrow pn$
- $n \rightarrow \bar{p}e^+\nu_e$
- $n \rightarrow pe^-\bar{\nu}_e$
- $n \rightarrow p\mu^-\bar{\nu}_\mu$
- $\Sigma^+(uus) \rightarrow \pi^+ n$
- $\tau \rightarrow \nu_\tau \mu^+ \mu^-$

Za sipalne procese lahko predpostaviš, da je energija trkov dovolj velika za tvorbo končnega stanja. Pri razpadih upoštevaj sledeče mase:  $m_p \approx m_n = 938$  MeV,  $m_n - m_p \approx 1.2$  MeV,  $m_\pi = 140$  MeV,  $m_D = 1860$  MeV,  $m_\mu = 106$  MeV,  $m_e = 511$  keV,  $m_\tau = 1778$  MeV,  $m_\Sigma = 1189$  MeV.

- V detektorju nevtrinov zaznavamo  $\bar{\nu}_e$  preko nabite šibke interakcije s kvarki, kjer v končnem stanju opazimo  $e^+$ . Izračunaj spinsko povprečen presek za reakcijo  $\bar{\nu}_e u \rightarrow e^+ b$ . Za težiščno energijo vzemi  $\sqrt{s} = 7$  GeV in zanemari mase vseh delcev,  $V_{ub} = 0.0039$ . Zapiši tudi kotno porazdelitev izhodnega  $e^+$  po kotu  $\vartheta$  med  $e^+$  in  $\bar{\nu}_e$ . Preveri, da dobiš enako kotno porazdelitev, če uporabiš rotacijo vhodnega stanja in funkcije  $d_{m' m}^j$ .

Dotatna naloga (0.375 točke): Določi kotno porazdelitev za  $\bar{\nu}_e u \rightarrow e^+ b$ , če upoštevaš končno maso  $m_b (= 4.2$  GeV).

Note: A square-root sign is to be understood over every coefficient, e.g., for  $-8/15$  read  $-\sqrt{8/15}$ . Notation:  $\begin{matrix} J & J & \dots \\ M & M & \dots \end{matrix}$

$Y_0^0 = \sqrt{\frac{3}{4\pi}} \cos\theta$   $2 \times 1/2 \begin{matrix} +5/2 & 5/2 & 3/2 \\ +2+1/2 & 1 & +3/2+3/2 \\ +1/2 & 1/5 & 4/5 \\ +1+1/2 & 4/5-1/3 & +1/2+1/2 \end{matrix}$   $\begin{matrix} m_1 & m_2 \\ m_1 & m_2 \\ \dots & \dots \\ \dots & \dots \end{matrix}$  Coefficients

$Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin\theta e^{i\phi}$   $\begin{matrix} +2 & 2 \\ +2+1/2 & 1 \\ +1/2 & 1/5 & 4/5 \\ +1+1/2 & 4/5-1/3 & +1/2+1/2 \end{matrix}$

$Y_2^0 = \sqrt{\frac{5}{4\pi}} \left(\frac{3}{2} \cos^2\theta - \frac{1}{2}\right)$   $\begin{matrix} +1-1/2 & 2/5 & 3/5 & 5/2 & 3/2 \\ 0+1/2 & 3/5-2/5 & -1/2-1/2 & & \end{matrix}$

$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{i\phi}$   $\begin{matrix} 0 & -1/2 & 3/5 & 2/5 & 5/2 & 3/2 \\ -1+1/2 & 2/5-3/5 & -3/2-3/2 & & & \end{matrix}$

$Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2\theta e^{2i\phi}$   $3/2 \times 1/2 \begin{matrix} 2 & 2 & 1 \\ +2 & 2 & 1 \\ +3/2+1/2 & 1 & +1 & +1 \\ +3/2-1/2 & 1/4 & 3/4 & 2 & 1 \\ +1/2+1/2 & 3/4-1/4 & 0 & 0 & \end{matrix}$

$Y_3^0 = \sqrt{\frac{7}{16\pi}} \left(\frac{5}{2} \cos^3\theta - \frac{3}{2} \cos\theta\right)$   $\begin{matrix} +3/2 & 2 & 1 \\ +3/2+1/2 & 1 & +1 & +1 \\ +3/2 & 2/5 & 3/5 & 5/2 & 3/2 & 1/2 \\ +1/2 & 3/5-2/5 & +1/2 & +1/2 & +1/2 & \end{matrix}$

$Y_3^1 = \sqrt{\frac{21}{32\pi}} \sin\theta \cos^2\theta e^{i\phi}$   $\begin{matrix} +3/2 & 2 & 1 \\ +3/2+1/2 & 1 & +1 & +1 \\ +3/2 & 2/5 & 3/5 & 5/2 & 3/2 & 1/2 \\ +1/2 & 3/5-2/5 & +1/2 & +1/2 & +1/2 & \end{matrix}$

$Y_3^2 = \sqrt{\frac{35}{64\pi}} \sin^2\theta \cos\theta e^{2i\phi}$   $\begin{matrix} +3/2 & 2 & 1 \\ +3/2+1/2 & 1 & +1 & +1 \\ +3/2 & 2/5 & 3/5 & 5/2 & 3/2 & 1/2 \\ +1/2 & 3/5-2/5 & +1/2 & +1/2 & +1/2 & \end{matrix}$

$Y_3^3 = \sqrt{\frac{63}{128\pi}} \sin^3\theta e^{3i\phi}$   $\begin{matrix} +3/2 & 2 & 1 \\ +3/2+1/2 & 1 & +1 & +1 \\ +3/2 & 2/5 & 3/5 & 5/2 & 3/2 & 1/2 \\ +1/2 & 3/5-2/5 & +1/2 & +1/2 & +1/2 & \end{matrix}$

$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$   $d_{\ell m, 0}^\ell = \sqrt{\frac{4\pi}{2\ell+1}} Y_\ell^m e^{-im\phi}$   $\begin{matrix} +3/2 & 2 & 1 \\ +3/2+1/2 & 1 & +1 & +1 \\ +3/2 & 2/5 & 3/5 & 5/2 & 3/2 & 1/2 \\ +1/2 & 3/5-2/5 & +1/2 & +1/2 & +1/2 & \end{matrix}$   $\begin{matrix} (j_1 j_2 m_1 m_2 | j_1 j_2 J M) \\ = (-1)^{J-j_1-j_2} (j_2 j_1 m_2 m_1 | j_2 j_1 J M) \end{matrix}$

$d_{m' m}^j = (-1)^{m-m'} d_{m, m'}^j = d_{-m, -m'}^j$   $3/2 \times 3/2 \begin{matrix} +3 & 3 & 2 \\ +3/2+3/2 & 1 & +2 & +2 \\ +3/2 & 1/2 & 1/2 & 3 & 2 & 1 \\ +1/2+3/2 & 1/2-1/2 & +1 & +1 & +1 & \end{matrix}$   $d_{1,0}^1 = \cos\theta$   $d_{1/2, 1/2}^{1/2} = \cos\frac{\theta}{2}$   $d_{1,1}^1 = \frac{1+\cos\theta}{2}$

$2 \times 3/2 \begin{matrix} +7/2 & 7/2 & 5/2 \\ +2+3/2 & 1 & +5/2+5/2 \\ +2+1/2 & 3/7 & 4/7 & 7/2 & 5/2 & 3/2 \\ +1+3/2 & 4/7-3/7 & +3/2 & +3/2+3/2 & & \end{matrix}$   $d_{1/2, -1/2}^{1/2} = -\sin\frac{\theta}{2}$   $d_{1,0}^1 = -\frac{\sin\theta}{\sqrt{2}}$

$d_{1, -1}^1 = \frac{1-\cos\theta}{2}$

1.  $\Omega_c^0$  (ssc)

but proton,  $u \rightarrow s$   
 $d \rightarrow c$

$$|\Omega_c^0(\uparrow)\rangle = \frac{1}{\sqrt{2 \cdot 3}} (|ssc\rangle (2|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle) + |scs\rangle (2|\uparrow\downarrow\uparrow\rangle - |\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle) + |css\rangle (2|\downarrow\uparrow\uparrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\uparrow\uparrow\downarrow\rangle))$$

$$\mu_p = \frac{4\mu_u - \mu_d}{3} \Rightarrow \mu_\Omega = \frac{4\mu_s - \mu_c}{3}$$

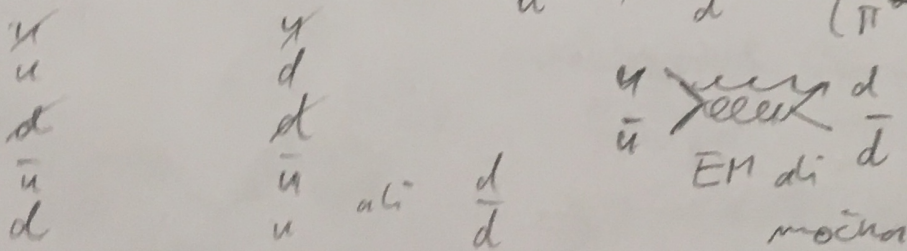
$$\mu_s = \frac{-e\hbar}{6m_s}, \quad \mu_c = \frac{e\hbar}{3m_c} \approx 0$$

$$\mu_\Omega = \frac{4}{3} \mu_s = \frac{-4e\hbar}{3 \cdot 6m_s} = -\frac{2}{9} \frac{m_p}{m_s} \frac{e\hbar}{m_p} \mu_N$$

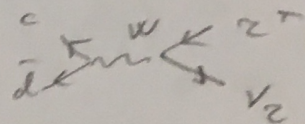
$$= -\frac{4}{9} \frac{m_p}{m_s} \mu_N = -1.0 \mu_N$$

2.  $pp\pi^- \rightarrow \pi^0 n$

$u \rightarrow d$  (proton  $\rightarrow$  neutron)  
 $\bar{u} \rightarrow \bar{d}$  ( $\pi^+ \rightarrow \pi^0$ )



$D^+ \rightarrow z^+ V_c \equiv c\bar{d} \rightarrow z^+ V_c$

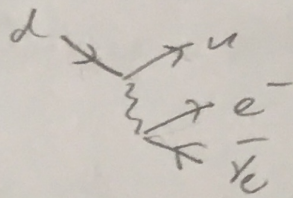


$p\bar{n} \rightarrow pn$  barionsko šteto



•  $n \rightarrow \bar{p} e^+ \gamma_e$  barionsko št.!

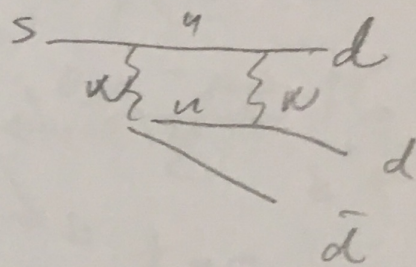
•  $n \rightarrow p e^- \bar{\nu}_e \equiv d \rightarrow u e^- \bar{\nu}_e$



•  $n \rightarrow p \mu^- \bar{\nu}_\mu$  prevažna masa n

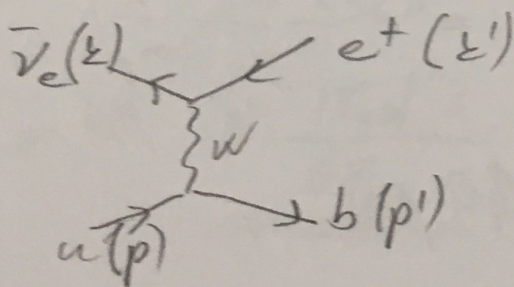
•  $\Sigma^+ (uus) \rightarrow \pi^+ n$   
 $uus \rightarrow u \bar{d} u d d$

$\equiv s \rightarrow d \bar{d}$

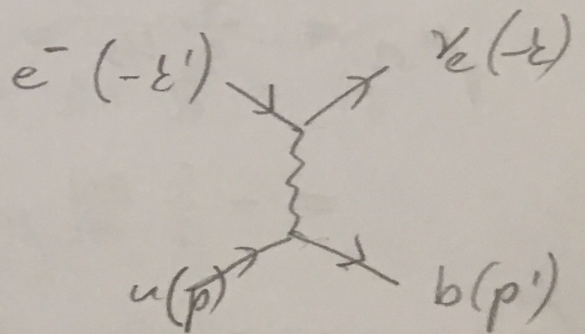


•  $\tau \rightarrow \nu_\tau \mu^+ \mu^-$  naboj

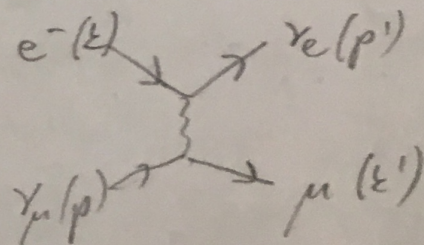
3.  $\bar{\nu}_e u \rightarrow e^+ b$



$\equiv$



na vajah pokazati, da je za



$$|M|^2 = 64 G_F^2 p' \cdot \epsilon' \epsilon \cdot p$$

~~zaradi dveh polantov črta u~~

$$|M|^2 = \frac{1}{2} 64 G_F^2 |V_{ub}|^2 (-\epsilon \cdot p') (-\epsilon' \cdot p)$$



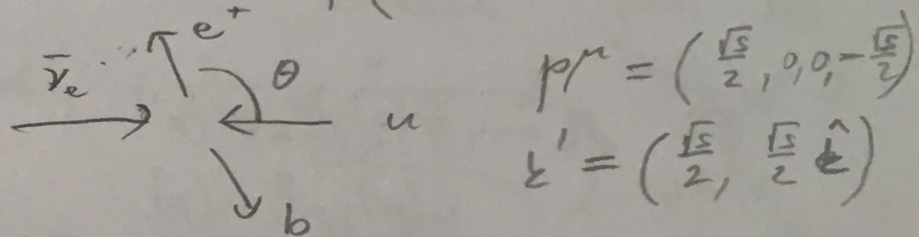
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_f}{p_i} |\mathcal{M}|^2$$

$$= \frac{1}{64\pi^2 s} \cdot \frac{3}{4} G_F^2 |V_{ub}|^2 \varepsilon \cdot p' \varepsilon' \cdot p$$

$$u \equiv (\varepsilon' - p)^2 \Rightarrow \varepsilon \cdot p' = \varepsilon' \cdot p = -\frac{u}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{32\pi^2 s} G_F^2 |V_{ub}|^2 \frac{u^2}{4}$$

$$u = -2p \cdot \varepsilon' = -2 \frac{s}{4} (1 + \cos \theta) = -\frac{s}{2} (1 + \cos \theta)$$



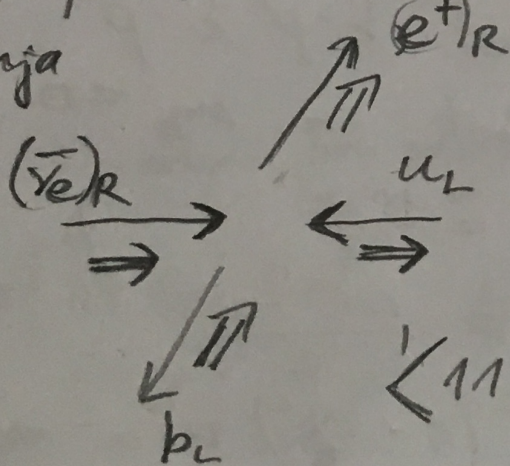
$$\frac{d\sigma}{d\Omega} = \frac{2G_F^2 |V_{ub}|^2 s}{32\pi^2} (1 + \cos \theta)^2$$

$$\sigma = \frac{2G_F^2 |V_{ub}|^2 s}{32\pi^2} \int d\Omega (1 + \cos \theta)^2$$

$$4\pi + 2\pi \cdot \frac{2}{3} = 4\pi \cdot \frac{4}{3}$$

$$\sigma = \frac{G_F^2 |V_{ub}|^2 s}{3\pi}$$

kotna porazdelitev z rotacijo vhodnega stanja



vhodno stanje  $|11\rangle$

izhodno stanje  $|11'\rangle$

$$\langle 11 | 11' \rangle = \langle 11 | \sum_{m' m} d_{m' m}^1(-\theta) |1 m'\rangle$$

$$\Rightarrow |M|^2 \sim \left| d_{11}^1(-\theta) \right|^2 = \frac{1 + \cos \theta}{2}$$



če upoštevamo končno maso  $m_b$ :

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \left( \frac{p_f}{p_i} \right) \overline{|M|^2}$$

$$\frac{p_f}{p_i} = \frac{\frac{\sqrt{s}}{2} \left( 1 - \frac{m_b^2}{s} \right)}{\frac{\sqrt{s}}{2}}$$

$$= 1 - \frac{m_b^2}{s}$$

$$(p' + \varepsilon')^2 = s$$

$$m_b^2 + 2\varepsilon' \cdot p'$$

$$\varepsilon' \cdot p' = E_{e^+} (\sqrt{s} - E_{e^+})$$

$$+ |\vec{p}_{e^+}| |\vec{p}_b|$$

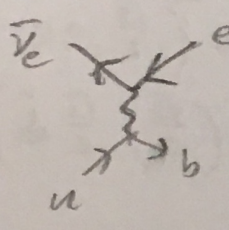
$$p_f = E_{e^+}$$

$$m_b^2 + 2 \left( p_f (\sqrt{s} - p_f) + p_f^2 \right) = s$$

$$= s$$

$$p_f = \frac{s - m_b^2}{2\sqrt{s}}$$

Za  $\overline{|M|^2}$  dobimo



$$\mathcal{M} \sim \bar{v}(e) \gamma^\mu P_L u(b) \bar{u}(p') \gamma_\mu P_L u(p)$$

spremeni se sled  
toča karakter

$$\text{Tr} \left[ (\not{p}' + m_b) \gamma_\mu P_L \not{p} \gamma_\nu P_L \right] m_b^2 + 2\sqrt{s} p_f = s$$

vendar den z  $m_b$   
ne prispeva, ker vsebuje

Tr [lito št.  $\gamma$  matrike]

$$\Rightarrow \overline{|M|^2} = 64 G_F^2 |V_{ub}|^2 \varepsilon \cdot p' \varepsilon' \cdot p$$

isto kot za  $m_b = 0$

$$u = (\varepsilon' \cdot p)^2 \Rightarrow \varepsilon' \cdot p = -\frac{u}{2}$$

$$\varepsilon \cdot p' = \frac{m_b^2 - u}{2}$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} G_F^2 |V_{ub}|^2 \left( 1 - \frac{m_b^2}{s} \right) u (u - m_b^2)$$

$$\text{in } u = -\frac{s}{2} (1 + \cos\theta)$$