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Analytic presentation of a solution of the Schrödinger equation*

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Abstract High-precision approximate analytic expressions for energies and wave functions are found for arbitrary physical potentials. The Schrödinger equation is cast into the nonlinear Riccati equation, which is solved analytically in first iteration of the quasi-linearization method (QLM). The zeroth iteration is based on general features of the exact solution near the boundaries. The approach is illustrated on the Yukawa potential. The results enable accurate analytical estimates of effects of parameter variations on physical systems.

We find an accurate analytic presentation of wave functions and energies for an arbitrary physical potential $U(r)$. We use the quasi-linearization method (QLM) suggested recently for solving the Schrödinger equation after conversion to Riccati form [1, 2]. In QLM the nonlinear terms of the differential equation are approximated by a sequence of linear expressions. The QLM is iterative but not perturbative and gives stable solutions to nonlinear problems without depending on the existence of a smallness parameter.

Substitution of the expression

$$y(r) = \frac{\chi'(r)}{\chi(r)}$$

converts the radial Schrödinger equation

$$\left[-\frac{1}{2m}\frac{d^2}{dr^2} + U(r)\right]\chi(r) = E\chi(r)$$

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