UNIVERSITY OF LJUBLJANA
FACULTY OF MATHEMATICS AND PHYSICS
PHYSICS DEPARTMENT

Jernej Mravlje

Anyons in the fractional quantum Hall effect

Seminar

Adviser: Anton Ramšak

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Abstract

The collective excitations of matter in 2D can obey statistics which is neither fermionic nor bosonic. When such quasi-particles are interchanged the many-body wavefunction representing them is multiplied with a phase factor which can differ from ± 1. With reference to *any* statistics they are subjected to they are dubbed *anyons*. Anyons are crucial for the understanding of the fractional quantum Hall effect (FQHE). We describe in simple terms how anyonic behaviour can arise and what is its relevance to the explanation of the FQHE. We also present the phenomenology of the FQHE to some extent.

Contents

1 Introduction

In the solid-state physics significant simplification is often achieved by inventing new particles describing the collective behaviour of the large number of electrons that occupy the energy bands in solids. The most known example are probably holes in p-type semiconductors. The missing electrons in a nearly full band dynamically behave like positively charged holes. This is related to a reverse sign of the Hall voltage when the measurements are performed in a p-type semiconductor. The Hall voltage is the transverse voltage that develops from sideways acceleration of a moving particle by a magnetic field. It is classically proportional to the current of particles. The ratio between the Hall current and voltage is termed Hall conductance.

In 1980 the measurements have revealed the quantization of the Hall conductance at low temperatures [1]. The Hall current does not increase continuously with Hall voltage but in steps. Later also plateaus at intermediate values of Hall voltages were measured [2]; the phenomenon is known as the fractional quantum Hall effect (FQHE). The quasi-particles relevant to the physics of the FQHE were predicted theoretically and confirmed experimentally to carry fractional charge and obey unusual – *anyonic* – statistics [3, 4, 5].

The idea, that the excitations carry fractional quantum numbers is not limited to the FQHE. In 1976 Jackiw and Rebbi explored the soliton excitations in the quantum field theory and showed that they have a fermion number 1/2, *ie.* they correspond to half a particle [6]. In the area of solid state physics Su, Schrieffer and Heeger explained the unexpected lack of magnetic response of the charged excitations in experimental data for polyacetylene by using the same ideas [7, 8, 9].

In this seminar we will, rather than analysing the variety of the circumstances that lead to charge fractionalization (we refer to reviews [10, 11, 12]), concentrate on the FQHE. First, we will show, where the usual reasoning leading solely to fermions and bosons goes wrong, second, we will construct a model for anyon in terms of regular physics and deduce its statistic properties, and third, introduce the phenomenology of the FQHE and show where the anyons come into play in its explanation.

2 Spin and statistics

Usually the particles are either fermions or bosons [13]. Because the elementary particles are indistinguishable (identical) the exchange of particles results in the same physical state. Therefore the many-particle wave-function can suffer at most a phase change

$$\Psi(2,1,3,4,...) = \exp(i\alpha)\Psi(1,2,...,N),$$

where the real phase α is defined as the *statistics* of the particles and the integers represent the full sets of quantum numbers corresponding to each particle. Performing the exchange twice we are led back to the initial state

$$\Psi(1,2,...,N) = \exp(i2\alpha)\Psi(1,2,...,N) \tag{1}$$

which imposes a constraint $\exp(i2\alpha) = 1$. The immediate consequence of Eq. (1) is that there are only two types of particles, fermions with $\alpha = \pi(2n+1)$ and bosons with $\alpha = 2n\pi$, represented by totaly antisymmetric, and totaly symmetric wavefunctions, respectively. Furthermore, there is a connection between spin s and statistics

$$\alpha = 2\pi s \tag{2}$$

which was proven by Pauli [14], and which states that for fermions $s = (2n+1)/2$ while for bosons $s = n$ with integer n. This all goes well in hand also with the known text-book derivation that the spin is quantized in half integers [15]. Although the text-book derivation looks general enough [1] it is, however, limited to 3D. In the argument the ladder operators which are constructed from noncommuting generators of the angular momentum algebra are used. Such an argument obviously fails in 2D because there is only one generator of the angular momentum algebra which obviously commutes with itself. Therefore the quantum mechanics allows the particles living in a 2D world to have a spin which is any real number. The question that appears immediately is what is the statistics of such particles.

It turns out that also the statistics α in 2D takes the intermediate value between the fermion and boson cases and that the relation (2) remains valid. The wave functions describing such particles are not singly valued, so that the implications of Eq. (1) do not hold. For such particles $\exp(2i\alpha) \neq 1$. To see how the fractional statistics is possible we study the process of interchanging the positon of two particles. We define the interchange of particles A and B by means of a rotation of particle A around particle B followed by a translation of both (Fig 1b)). The net effect of performing the interchange of the particles twice is that particle A is taken a full loop around particle B. The phase (2) generated by this process can depend on loop in general. When the loop is moved continuously, the phase remains unchanged if it is an intrinsic property of the particles. In 3D we can shrink the loop continuosly to a point by moving it out of the plane (see Fig 1 b). Hence, the phase should be trivial, $\exp(i2\alpha) = 1$. The situation is entirely different in the 2-dimensional space since the loop is constrained to a plane. It is impossible to shrink the loop performed by particle A without crossing the position of particle B. Consequently, the parameter α is an arbitrary real number intrinsic to particles.

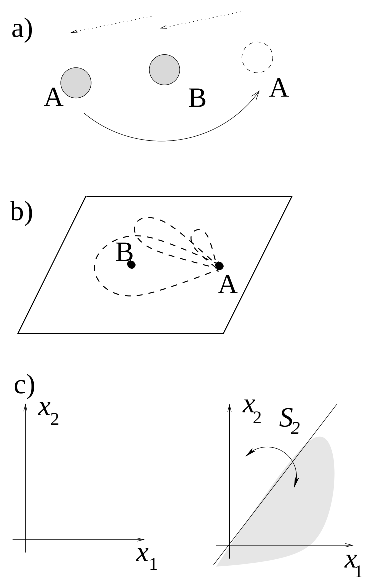

Figure 1: (a) The exchange of two particles performed by a rotation of particle A around particle B followed by a translation of both. (b) Continous deformation of a loop to a point in 3D. (c) Configurational space of two distinguishable particles X^2 (left) and configurational space (shadowed) of two *identical* particles X^2/S_2 (right) confined to move on a line.

The fractional statistics was also derived riguorously [16] . The derivation involves the identification of correct configurational space of identical particles. If the one-particle configuration space is X, the configurational space for N particles is naively constructed as Cartesian product $\mathcal{C}_{\text{wrong}} = X^N$. However, as the vectors $x = (x_1, x_2, ...x_N) \in X^N$ and $x' = (x_2, x_1, x_3, ...x_N) \in X^N$ determine the same configuration (the particles are indistinguishable), they should correspond to the *same point* in configurational space [17]. The correct configurational space is therefore $\mathcal{C} = X^N/S_N$ where the action of a discrete group of permutations between N particles S_N is divided out. The quantization of the theory on $\mathcal{C}$ leads in a straightforward way to fermions and bosons in 3D and anyons in 2D and 1D.

We illustrate the idea by performing the quantization of particles for two free particles moving on line (in 1D). The configurational space is then the half-plane, (Fig 1 c). We write the

[1]Applying the lowering operator S^- n times on a state with the total angular momentum l and maximum projection $m = l$ results in a state with minimum projection $| - m >= (S^-)^n|m >$. Counting $-m = m - n$ then implies quantization of spin in half integers $l = m = n/2$.

Hamiltonian in terms of a centrum of mass coordinate $x = (x_1 + x_2)/2$ and a relative coordinate $z = x_1 - x_2$

$$H = -\frac{\hbar^2}{2m}(\frac{\partial^2}{x_1^2} + \frac{\partial^2}{x_2^2}) = -\frac{\hbar^2}{4m}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{m}\frac{\partial^2}{\partial z^2} \tag{3}$$

As the boundary of the configurational space is nontrivial the solution $\psi(x, z)$ is obtained also by noting the appropriate boundary conditions. Demanding that the probability of finding a particle in the whole space is conserved, the probability current through the boundary $x_1 - x_2 = z = 0$ must vanish

$$\psi^*(x, 0)\frac{\partial\psi}{\partial z}(x, 0) - \frac{\partial\psi^*}{\partial z}(x, 0)\psi(x, 0) = 0 \tag{4}$$

The conditition (4) is satisfied if the wave function at the boundary vanishes $\psi(x, 0) = 0$. Such solutions correspond to fermions. Another possibility is to demand that instead of the wavefunction the derivates of the wavefunction normal to the boundary vanish $\partial\psi/\partial z = 0$ and identify such solutions as bosonic. However, this does not exhaust all the possibilities. Eq. (4) is satisfied whenever

$$\frac{\partial\psi}{\partial z}(x, 0) = \eta\psi(x, 0),$$

with η as a real parameter. The general solution of the Hamiltonian (3) then is

$$\psi_{\kappa k}(x, z) = \mathcal{N}\exp(i\kappa x)(\cos kz + \frac{\eta}{\sin}kz),$$

with $\eta = 0$, $\eta^{-1} = 0$ and $\eta \in \mathcal{R}$ for bosons, fermions and anyons, respectively.

3 Simple model of anyon

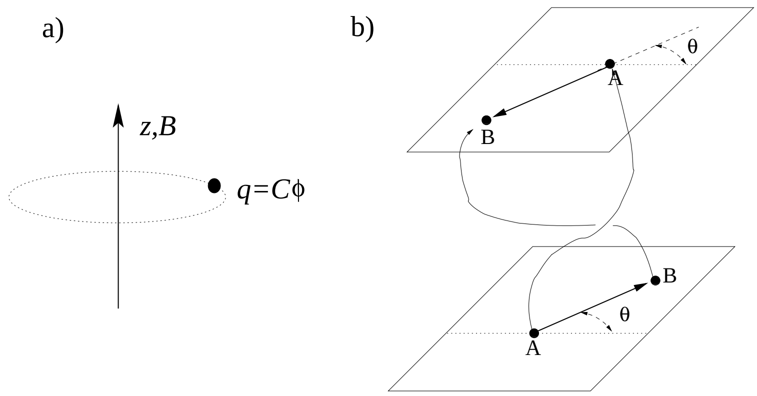

Figure 2: (a) A simple model of anyon is a a charged boson orbiting around a thin solenoid. (b) The exchange of anyons is equal to $\theta \to \theta + \pi$ in terms of relative coordinates.

A model for anyon (see Fig 2a) is a spinless particle orbiting around a thin solenoid pointed in the direction of the z-axis [18]. The charge of the particle q is taken to be proportional to the applied flux Φ

$$q = C\Phi$$

for some real constant C. Before the magnetic flux is applied the angular momentum of the particle is quantized in integers $l_z = n\hbar$. Increasing the magnetic field results in a circular electrical field as a consequence of the Maxwell equation $\vec{\nabla} \times \vec{E} = -\partial\vec{B}/\partial t$. The circular electrical field produces a torque on a particle, therefore its angular momentum is changed. To calculate the change in the angular momentum due to the change of magnetic flux we first apply the Stokes theorem

$$\int \nabla \times \vec{E} \cdot d\vec{S} = \int \vec{E} \cdot d\vec{s} = E 2\pi r = -\int \vec{B} \cdot \vec{S} = -\dot{\Phi},$$

where $d\vec{S}$ and $d\vec{s}$ are the infinitesimal elements of the surface and the curve limiting it, respectively. Considering just the z component of the angular momentum l_z we calculate further

$$\dot{l_z} = M_z = qEr = -\frac{q\dot{\Phi}}{2\pi} = -\frac{C\Phi\dot{\Phi}}{2\pi}$$

The application of the magnetic flux then results in the boost in the angular momentum,

$$\Delta l_z = -\frac{C\Phi^2}{4\pi} = -\frac{q\Phi}{4\pi}. \tag{5}$$

Taking the particles's bare angular momentum to be zero and limiting the dimensions of the solenoid and the radius of the orbite of the particle toward zero, the system can be considered as a single composite object with the spin determined by the flux passing through the solenoid. Observing Eq.(5) one also notes that for objects with charge e_0 the addition of one quantum of magnetic flux $\Phi_0 = h/e_0$ transforms the bosons to fermions (with regard to the spins of the particles) and vice-versa.

To determine the statistic properties of our model for anyon we construct a two-anyon system. The Hamiltonian of such system reads

$$H = \frac{(\vec{p}_1 - q\vec{A}_1)^2}{2m} + \frac{(\vec{p}_2 - q\vec{A}_2)^2}{2m}$$

where p_i are the momenta of both objects, and

$$\vec{A}_{1,2} = \pm \frac{\Phi}{2\pi} \hat{z} \times \frac{\vec{r}}{r^2}$$

are the vector potentials due to the presence of another object ($\vec{r} = \vec{r}_1 - \vec{r}_2$). Writing the Hamiltonian in terms of the centrum of mass coordinate $\vec{P} = \vec{p}_1 + \vec{p}_2$ and relative coordinate $\vec{p} = (\vec{p}_1 - \vec{p}_2)/2$

$$H = \frac{P^2}{4m} + \frac{(\vec{p} - q\vec{A}_{\mathrm{rel}})^2}{m}$$

results in a decoupled motion of the centrum of mass (as expected due to the absence of an external potential), while the rest of the Hamiltonian has reduced to the system of a single charged particle of mass $m/2$ orbiting around a flux Φ. As the composite particles were constructed from the bosonic particles orbiting around a bosonic flux, the appropriate boundary conditions for the solution of the Hamiltonian proposed above are given by

$$\psi(r, \theta + \pi) = \psi(r, \theta), \tag{6}$$

meaning simply that the wavefunction is invariant on particle interchange (2b). Now we perform a (singular!) gauge transformation

$$\vec{A'} = \vec{A} - \nabla\Lambda$$

where $\Lambda = \phi\theta/2\pi$, which is not singly valued. In the primed gauge where $\vec{A'} = 0$, the Hamiltonian is transformed to

$$H = \frac{P^2}{4m} + \frac{p^2}{m}$$

which is a Hamiltonian of two free particles. During the transformation, however, the boundary conditions on the wavefunction have also changed because the wavefunction acquired the phase

$$\psi'(r,\theta) = \exp(-iq\Lambda)\psi(r,\theta) = \exp(-\frac{iq\Phi\theta}{2\pi})\psi(r,\theta)$$

what imposes nontrivial boundary condition. If we calculate $\psi(-\vec{r})$ we get

$$\psi'(r,\theta+\pi) = \exp(-iq\Phi/2)\psi'(r,\theta),$$

taking into account Eq.(6). The wavefunction describing two anyons is multiplied by a non-trivial phase factor upon particle interchange. The phase factor leads to the statistics

$$\alpha = \frac{q\Phi}{2} \tag{7}$$

As the flux is an independent parameter it the statistics is arbitrary . We have a physicaly relevant model for an anyon!

4 The Landau levels

Let us briefly review the quantum mechanics of electrons confined to the plane x-y in the presence of a uniform magnetic field in the z-direction [19]. We choose the symmetric gauge [2] $\vec{A} = (-y, x)/2 = \vec{e}_\phi r/2$. The Hamiltonian of the system[3]

$$H = \frac{1}{2m}(\vec{p} - e\vec{A})^2$$

is cylindrically symmetric, therefore the eigenstates

$$\psi_{nj}(r,\phi) = \mathcal{N}(r/l_B)^j e^{ij\phi} L_n^j(r/l_B)\exp(-\frac{(r/l_B)^2}{2}),$$

where L_n^j is a generalized Laguerre polynomial and $l_B = \sqrt{\hbar/m\omega_c}$ the magnetic length (a solution of $\hbar\omega_c = m\omega_c l_B^2$), are also the eigenfunctions of the angular momentum J, so the following holds:

$$H\psi_{nj} = \hbar\omega_c(n+1/2)\psi_{nj}$$

$$J\psi_{nj} = \hbar j\psi_{nj}.$$

It is convenient to introduce the dimensionless complex coordinates $z = (x+iy)/l_B$.

$$\psi_{nj}(z) = \mathcal{N} z^j L_n^j(|z|)\exp(-\frac{|z|^2}{2}) \tag{8}$$

For $j >> n$ the probability density $|\psi_{nj}|^2$ has the form of $n+1$ sharp concentric rings centered around the origin. The outermost ring has the radius $r_{\max} \sim l_B\sqrt{j}$, therefore the largest value of j that can occur on a disc with radius R (and area $S = \pi R^2$) is

$$j_{\max} = R^2/l_B^2 = BS/\Phi_0$$

[2] The reader might be more familiar with the calculation performed in the Landau gauge $\vec{A}(\vec{r}) = xB\hat{y}$, leading to eigenfunctions $\psi_k(x,y) = \exp(iky)\chi_k(x)$, where χ_k is a solution to the displaced harmonic oscillator equation

$$H_k = \frac{1}{2m}p_x^2 + \frac{1}{2}m\omega_c^2(x+X_k).$$

The central position $X_k = -kl^2$ is determined by the y momentum quantum number and magnetic length $l_B = \sqrt{\hbar/m\omega_c}$ and leads to the same degeneracy of the eigenstates as the calculation in a symmetric gauge [20].

[3] Here we do not include the spin term $\vec{\mu}\cdot\vec{B}$. In high magnetic fields the system is fully spin polarized. In our discussion we therefore neglect the spin degree of freedom.

As the energy of the system is not dependent on j we have a $j_{\max}$ fold degeneracy of each Landau level. In high magnetic fields it is sufficent to limit the discussion to the lowest Landau level, for which one-particle wave functions (all with energy $\hbar\omega_c/2$) can be put into form

$$\phi_m = \mathcal{N} z^m \exp(-\frac{|z|^2}{4})$$

which are the eigenstates of the angular momentum with a eigenvalue m. Because all the ϕ_m are degenerate any linear combination of them is also an acceptable eigenstate. This are all the solutions of the form $f(z)\exp(-|z|^2/4$, where $f(z)$ is *analytical*.

An important concept is also the *filling ratio* ν. It is equal to the number of filled Landau levels

$$\nu = N/Z, \tag{9}$$

where N is the number of electrons and $Z = j_{\max}$ the degeneracy of the Landau levels.

5 The fractional quantum Hall effect

The Hall effect was discovered in 1879 by Edwin Hall. The classical physics of the Hall effect can be understood in terms of the Lorentz force. As the current is flowing through the sample, which is penetrated by a magnetic field, the carriers are deflected towards its edge due to the action of the Lorentz force. The accumulated charge (Fig 3) is the source of electrical field. In the equilibrium the Lorentz and the electrical force exactly cancel

$$e\vec{E} = -e\vec{v} \times \vec{B}.$$

Introducing a coordinate system $\vec{v} = v\hat{x}$, $\vec{E} = E\hat{y}$, $\vec{B} = B\hat{z}$ and multiplying by the carrier density we derive the relation between the current density and the electrical field

$$neE = jB. \tag{10}$$

Experimentaly, the voltage $V = Ed$ and the current $I = jd$ are measured, so we multiply Eq.(10) by the width of the sample. The ratio between the two is known as the Hall resistance

$$R_H = \frac{B}{ne}.$$

It is dependent solely on the magnetic field and the density of the carriers. The result is not sensitive to the type, dimensions and the shape of the sample[4]. The Hall effect is routinely used

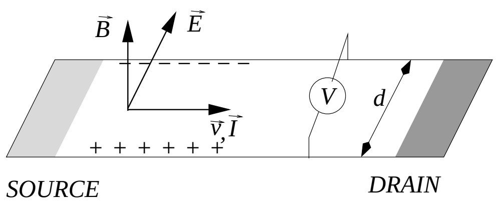

Figure 3: The experimental setup. The Hall resistance is the ratio between the voltage V and the current I. Forces on a particle in the classical picture are also sketched.

by solid state experimental phyisicists to determine the density of the carriers of the current. However, a surprise came in 1980 when Von Klitzing *et al.* [1] discovered that at high magnetic

[4] In 3D the relation between the current and the current density is $I = jdh$, where h is the height of the sample, so the Hall resistance is $R_H = B/(neh)$. Because this depends on the height, which can never be constructed very accurately, the apperance of Hall plateaus where the conductance is constant to the accuracy 10^{-10} in 3D is not possible.

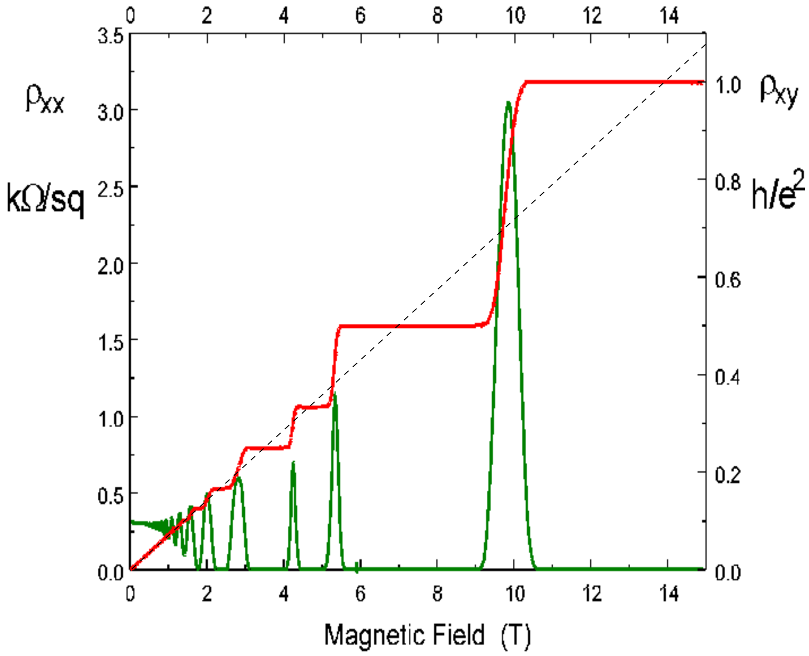

Figure 4: Plots of Hall (red) and longitudinal (green) resistivities as a function of magnetic field. The classical result is also plotted (dashed). The intersections of the classical result with plateaus correspond to integer fillings. After Ref. [1].

fields the Hall resistivity increases in steps (Fig 4) instead of being proportional to it as the classical equation suggests. This phenomenon became known as the integer quantum Hall effect (IQHE). The simplest way to understand it is by first examining the values where the classical result and the measurements agree. This happens to be exactly at integer fillings $\nu = n$. Each completely filled Landau level contributes a e^2/h to the Hall conductance $G_H = 1/R_H$ of the sample. When the filling is changed slightly the extra states do not contribute to the current because they are localized by the impurities and defects present in real samples [21]. Further insight can be obtained by studying the edge currents [22, 23]. The crucial issues in the understanding of the IQHE are therefore the gap between the Landau levels and the disorder which localizes the extra states.

The difference between Hall resistivities for different number of filled Landau levels which is exactly equal to nh^2/e_0 makes the IQHE useful as the standard of the resistivity. Von Klitzing constant equals $R_K = h/e^2 = 25812.807449(86)\Omega$. The same constant is also directly related to the fine structure constant $\alpha = e^2/(4\pi\epsilon\hbar c)$, so the IQHE is used as a from the particle physics independent measurement of α.

In 1982 the fractional quantum Hall effect [24, 25] was discovered [2]. With cleaner samples and at lower temperatures the plateaous occured not only at integer but also at fractional $\nu = p/q$ $(p, q \in \mathbb{I})$ fillings (Fig 5). FQHE at fillings of a type 1/(2j+1), j an integer occured was theoretically explained by Laughlin, who considered condensation of electrons into a special ground state, which he described by a trial wave function [26]. Its excitations were shown to carry fractional charge and obey fractional statistics [3]. The emergence of fractionally charged excitations was also confirmed experimentaly in Aharonov-Bohm type of experiments [28, 4, 5] and independly by shot-noise experiments [30, 31, 32]. Recently also the fractional statistics of quasi-particles of the FQHE was directly confirmed experimentally [29].

For fillings other than $1/(2n+1)$ various hierarchy schemes were proposed where the FQHE at some filling was explained as the FQHE of quasi-particles at another filling. Another approach was proposed by Jain [27] in which the FQHE as the IQHE of composite particles - the electrons with flux-tubes attached. This approach turned out to be very succesful at predicting the fractions at which the plateau appear, aswell as their stability to disorder or increase of temperature.

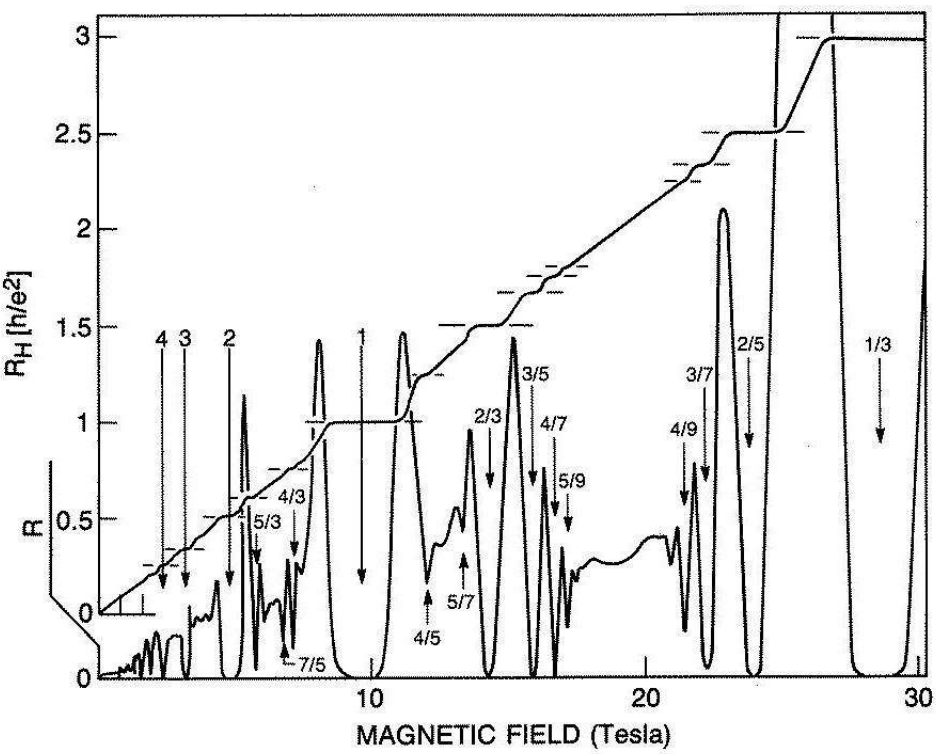

Figure 5: Plots of Hall and longitudinal resistivities as a function of magnetic field. Plateaus in Hall resistivity at several fillings are visible. After Ref. [28]

6 The Laughlin ground state and fractionally charged excitations

Many body wave function for fillings $\nu \leq 1$ can be written as the Slater determinant of one particle states Eq.(8), which we explicitely write for the case of two particles in the states of lowest angular momenta

$$\psi(z_1, z_2) = \mathcal{N}[(z_1)^0(z_2)^1 - (z_2)^0(z_1)^1] \exp[-\frac{|z_1|^2 + |z_2|^2}{4}] = \mathcal{N}(z_1 - z_2) \exp[-\frac{|z_1|^2 + |z_2|^2}{4}]$$

The Slater determinant of N particles in the states of lowest angular momenta can be simplified similary to give

$$\psi(z_1, z_2, ..., z_N) = \mathcal{N} \prod_{i<j}^{N} (z_i - z_j) \exp(-\frac{1}{4} \sum_j |z_j|^2).$$

Laughlin's suggestion was that the ground state wavefunction appropriate for the FQHE at filling $\nu = 1/m$ is

$$\psi_m = \prod_{i<j}^{N} (z_i - z_j)^m \exp(-\frac{1}{4} \sum_j |z_j|^2).$$

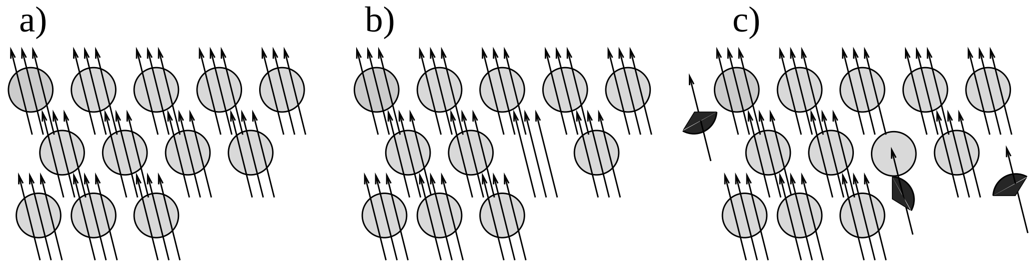

Figure 6: (a) The Laughlin ground state at $\nu = 1/3$. (b) If the density of electrons is slightly reduced there is a surplus of flux lines in multiples of 3. (c) The excitations of the Laughlin ground state develop around the flux tubes and are fractionally charged (in this case $+e_0/3$).

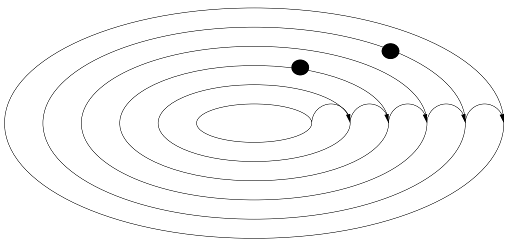

Figure 7: The determination of the charge of the excitations by the Schrieffer counting argument at filling $\nu = N/Z$. If we insert Z quantums of magnetic field flux (here $Z = 6$) all the electrons (black circles) will be removed from the sample. In reality the electrons are uniformly spread through the sample, therefore the addition of one quantum of magnetic flux results in the e_0/m charged quasi-hole.

He arrived at this wavefunction from certain general principles, a) the wavefunction should be formed of one particle states b) because the one particle states are polynomial in z the manyparticle state should also be such c) the wavefunction should be an eigenstate of the total angular momentum. Because the wavefunction should be antisymmetric the m is restricted to odd integers what is also consistent with the absence of plateaus at fillings $\nu = p/m$, m even.[5]

Laughlin further calculated numerically the ground-state of a few-particle system and determined that his trial-wavefunction exhibits excellent overlap with the numerical results.

The excitations correspond to slight deviation of the magnetic field from the value of precise integer filling. We penetrate the ground-state with a flux tube containg one quantum of magnetic flux. As the result of the addition of Φ_0 is the increase in angular momentum according to Eq.(5) for $\hbar$ its effect on single particle wavefunctions is

$$\phi_m \to \phi_{m+1}. \tag{11}$$

Laughlin accordingly proposed also the ansatz for the excitations from the ground state located at z_0

$$\psi'(z_0) = \prod_i (z_i - z_0)\psi_m.$$

Also in this case the overlap with numerically calculated few-body excited states turned out to be good.

We now determine the charge of these excitations at some filling factor $\nu = 1/m$. Inserting N quantums of magnetic flux (see Eq.(11) and Fig. 7) expells all the electrons, which carry total charge of $(N/m)(-e_0)$ from the system. As the excitation results from the insertion of

[5] There is a plateau at $\nu = 5/2$. For explanation of it one has to consider also the spin degree of freedom.[19]

one quantum of magnetic flux, its charge is equal to $(N/m)(e_0)/N = e_0/m$. Such excitation is readily identified as a fractionaly charged quasi-hole. The statistics of these quasiholes can be explicitly found by a Berry phase calculation [3], however, we avoid the lengthy calculation and recognize that we are dealing with objects with charge e_0/m connected to a flux equal to Φ_0. The statistics of these excitations is then determined according to Eq.(7) as

$$\alpha = \frac{(e/m)2\pi/e}{2} = \frac{\pi}{m}$$

7 The explanation of the FQHE

The explanation of FQHE in view of Laughlin's ideas is that there exist fillings (or, equivalently, magnetic field densities) at which the electron gas is particulary stable and condenses into a special ground state with high correlation. The slight deviations from these fillings result in quasi-particle excitations which carry fractional charge. As these excitations get pinned by the impurities and disorder present in the real samples, they do not contribute to the current. The conductance therefore does not change with the filling until the value of the filling for next stabile ground state is reached.

8 Conclusion

We have shown that the possibilities for the symmetry character of the particles are related to the dimensionality of the configurational space. As some physical systems are effectively two dimensional the low-energy excitations of such systems are anyonic. Since the discovery of the FQHE in 1982 the physical community pays a constant interest to the subject of anyons. Furthermore, the interest in anyons has grown recently once again, as the anyons were proposed as a means to perform a fault-tolerant quantum computation [35, 36]. The phase, which the anyonic wave-function develops as the anyons are transfered around each other is of topological nature. It is therefore expected to be more rigid against decoherence, which is one of the major obstacles in constructing an efficent quantum computer.

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