

② $V(r)$, ohranjene količine: $p_\varphi =$

$$L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\varphi}^2) - V(r)$$

$$1. p_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = \text{konst} = m r^2 \dot{\varphi}$$

$$2. H = \frac{1}{2} m \dot{r}^2 + \underbrace{\frac{p_\varphi^2}{2 m r^2}}_{V_{\text{ef}}} + V(r)$$

Krožna orbita: $\left. \frac{\partial V_{\text{ef}}}{\partial r} \right|_{r_0} = 0$ pogoj

obhodni čas:

$$-\frac{p_\varphi^2}{m r^3} \Big|_{r_0} + \left. \frac{\partial V}{\partial r} \right|_{r_0} = 0$$

$$\text{Obhodni čas: } \frac{m \dot{r}_0 \dot{\varphi}^2}{m r_0^2} = \frac{\partial V}{\partial r} \Rightarrow p_\varphi \dot{\varphi} = \sqrt{\frac{1}{r} \frac{\partial V}{\partial r}}$$

$$T = \frac{2\pi}{\dot{\varphi}} = 2\pi \left(\frac{1}{r} \frac{\partial V}{\partial r} \right)^{-1/2}$$

odmik iz krožne orbite: $r_0 \rightarrow r_0 + \varepsilon$; $|\varepsilon| \ll r_0$

$$H = \frac{1}{2} m \dot{\varepsilon}^2 + V_{\text{ef}}(r_0 + \varepsilon) \approx \frac{1}{2} m \dot{\varepsilon}^2 + V_{\text{ef}}(r_0) + \underbrace{\left. \frac{\partial V_{\text{ef}}}{\partial r} \right|_{r_0}}_0 \varepsilon + \frac{\partial^2 V_{\text{ef}}}{\partial r^2} \Big|_{r_0} \frac{\varepsilon^2}{2}$$

Krožna orbita $\leftarrow 0$

$$H - V_{\text{ef}}(r_0) = \frac{1}{2} m \dot{\varepsilon}^2 + \frac{\partial^2 V_{\text{ef}}}{\partial r^2} \Big|_{r_0} \frac{\varepsilon^2}{2}$$

Stabilna, ko $\frac{\partial^2 V_{\text{ef}}}{\partial r^2} \Big|_{r_0} > 0$, ko:

Harmonski oscilator: $H - V_{\text{ef}}(r_0) = \frac{1}{2} m \dot{\varepsilon}^2 + \frac{1}{2} m \omega^2 \varepsilon^2$

$$\omega^2 = \frac{1}{m} \frac{\partial^2 V_{\text{ef}}}{\partial r^2} \Big|_{r_0}$$

Rešitev: $\varepsilon(t) = A \cos(\omega t + \varphi)$

$$H - V_{\text{ef}}(r_0) = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \varphi) + \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \varphi)$$

$$\Rightarrow A = \sqrt{\frac{2(H - V_{\text{ef}}(r_0))}{\omega^2 m}}$$