Oblika vodila, pri katerem h(t)=c t

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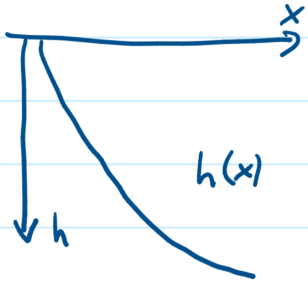

A

$$h(x) = ?$$

$$v = \sqrt{\dot{x}^2 + \dot{h}^2} = \sqrt{\left[\left(\frac{dx}{dh}\right)^2 + 1\right] \cdot \dot{h}^2}$$

c

$$h = c\,t \qquad dx/dh = x'$$

$$v = \sqrt{x'^2 + 1}\; c = v(h)$$

Iz ohranitve energije $\Rightarrow$ $\frac{mv^2}{2} = mg(h_0 + h)$

$\underline{m\,(1 + f')^2}$

$$v = \sqrt{2g(h + h_0)}$$

h_0 parametrizira začetni pogoj (v_0, pri $h=0$, $= \sqrt{2gh_0}$)

Združimo:

$$\sqrt{2g(h + h_0)} = \sqrt{(x'^2 + 1)} \cdot c$$

$$2g(h + h_0) - c^2 = x'^2 c^2$$

$$\frac{1}{c}\left[2g(h + h_0) - c^2\right]^{1/2} = dx/dh$$

$$\int dh \downarrow \; = \int dx$$

$$\Downarrow$$

$$\frac{2}{3 \cdot 2g}\left(2g(h + h_0) - c^2\right)^{3/2} = c\,x$$

$$x(h) = \frac{1}{3gc}\left[2g(h + h_0) - c^2\right]^{3/2}$$

(1/2)

Za vodila tovej velja

$$(1+x'^2) = \frac{2g}{c^2}(h+h_0)$$

Enačba gibanja se izpelje iz

$$L = \frac{1}{2} m \sqrt{1+x'^2}\, \dot{h}^2 + mgh$$

$$L = \frac{1}{2} m \frac{2g}{c^2}(h+h_0)\,\dot{h}^2 + mgh$$

$$\frac{d}{dt}\, m \frac{g}{c^2}(h+h_0)\,\dot{h} = \frac{mg}{c^2}\dot{h}^2 + mg$$

$$2\frac{g}{c^2}\dot{h}^2 + \frac{g}{c^2}(h+h_0)\ddot{h} = \frac{g}{c^2}\dot{h}^2 + g$$

enačba gibanja

$$\frac{g}{c^2}(h+h_0)\ddot{h} = -\frac{g}{c^2}\dot{h}^2 + g$$

če $\dot{h} = c$, $\ddot{h} = 0$

če je $\dot{h}$ večji, je $\ddot{h} < 0$

če je $\dot{h}$ manjši, je $\ddot{h} > 0$

asimptotsko torej $\dot{h} \to c$.

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