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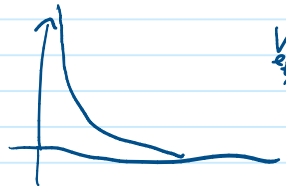
a) $$V_{eff}(r) = \frac{p_\varphi^2}{2mr^2} - \frac{k e^{-r/r_0}}{r}$$

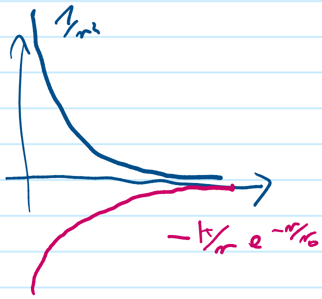
$k < 0$



$V_{eff}(r)$ te oblike za vse vrednosti p_φ.

Ni veznih orbit.

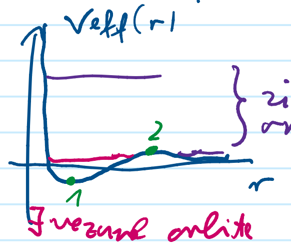
$k > 0$

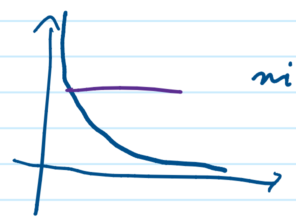


$V_{eff}(r) \sim 1/r^2$ za $r \to 0$, $r \to \infty$

(exponentno zadušen za velike r)

Skupaj: mali p_φ velik p_φ





ni veznih orbit

če $k <$ ni veznih orbit za poljuben p_φ

1 stabilna krožna orbita ($E =$ lok. $\min V_{eff}(r)$)

2 labilna krožna orbita ($E =$ lokalni $\max V_{eff}(r)$)

b) Ugodno prepisati v brezdimenzijsko obliko

$$V_{eff} = \frac{p_\varphi^2/r_0^2}{2mr^2/r_0^2} - \frac{k}{r_0}\,\frac{e^{-r/r_0}}{r/r_0}$$

$$V_{eff} = \frac{k}{r_0}\left(\frac{l^2}{2x^2} - \frac{e^{-x}}{x}\right) \quad ; \quad l^2 = \frac{p_\varphi^2}{k m r_0}$$

Vpeljemo $v = V_{eff}/(k/r_0)$

$$v(x) = \frac{l^2}{2x^2} - \frac{e^{-x/x_0}}{x}$$

da določimo obliko $v(x)$, odvajajmo:

$$\frac{dv}{dx} = -\frac{\ell^2}{x^3} + e^{-x}\left(\frac{1}{x} + \frac{1}{x^2}\right)$$

$$= \frac{1}{x^3}\left(-\ell^2 + e^{-x}(x + x^2)\right)$$

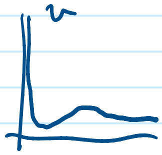
maksimalna vrednost $e^{-x}(x+x^2)$

$$\frac{d}{dx}\left(e^{-x}(x+x^2)\right) = e^{-x}(-x - x^2 + 2x + 1)$$

$$= e^{-x}(-x^2 + x + 1) \qquad x_{1,2} = \frac{1 \pm \sqrt{5}}{2} \;; (x > 0)$$

$$x^* = \frac{1+\sqrt{5}}{2}$$

Za $\ell^2 < \underbrace{e^{-x^*}(x^* + x^{*2})}_{:\ell_0^2}$ imamo dve ničli odvoda



; za $\ell < \ell_0$ imamo stabilne krožne orbite

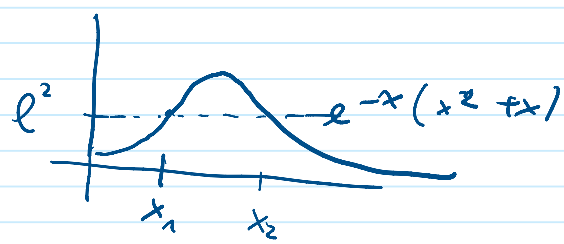
za $\ell = \ell_0$ imamo sedelna točka

za $\ell > \ell_0$ ni ničel (in ni vezanih orbit)

radij kroženja:

$$\frac{dv}{dx} = 0 \quad \Rightarrow \quad \ell^2 = e^{-x_1}(x_1 + x_1^2)$$

$r_1 = r_0 x_1$ (x_1 implicitno podan z manjšo od rešitev zgornje enačbe)



c) $E = \frac{1}{2} m\dot{r}^2 + V_{eff}(r)$ (razvoj blizu ekstrema

$$E = \frac{1}{2} m\dot{r}^2 + \frac{1}{2}\frac{d^2V_{eff}}{dr^2}\bigg|_{r=r_1}(r-r_1)^2$$

$$\frac{d^2V_{eff}}{dr^2} = \frac{1}{r_0^2}\,\frac{k}{r_0}\,\frac{d^2v}{dx^2} \qquad E = \frac{1}{2} m\dot{r}^2 + \frac{\tilde{k}(r-r_1)^2}{2}$$

$$\Rightarrow \omega^2 = \frac{\tilde{k}}{m} \quad \text{nihanje}$$

$$\frac{dv}{dx} = -\frac{\ell^2}{x^3} + e^{-x}\left(\frac{1}{x^2} + \frac{1}{x}\right)$$

$\rightarrow \quad \omega = \frac{k}{m}$ nihanje

$$\frac{dv}{dx} = -\frac{l^2}{x^3} + e^{-x}\left(\frac{1}{x^2} + \frac{1}{x}\right)$$

$$\frac{d^2v}{dx^2} = \frac{3l^2}{x^4} - e^{-x}\left(\frac{2}{x^3} + \frac{1}{x^2} + \frac{1}{x} + \frac{1}{x^2}\right)$$

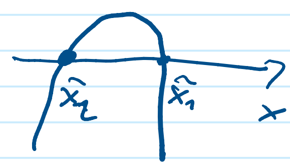
$$\frac{d^2v}{dx^2} = \frac{1}{x^4}\left(3l^2 - e^{-x}(2x + 2x^2 + x^3)\right)$$

upoštevam, da $l^2 = e^{-x}(x + x^2)$

$$\frac{d^2v}{dx^2} = \frac{1}{x^4} e^{-x} \underbrace{(x + x^2 - x^3)}_{}$$

> 0, ko $\quad 1 + x - x^2 > 0$

$$\hat{x}_{1,2} = \frac{1 \pm \sqrt{5}}{2}$$



za $x < \frac{1 + \sqrt{5}}{2}$

$$\tilde{k} = \frac{k}{r_0^3} \frac{1}{x_1^4}\left(e^{-x_1}(x_1 + x_1^2 - x_1^3)\right)$$

frekvenca oscilacij

$$\omega^2 = \tilde{k}/m$$

frekvenca vrtenja

$$m r^2 \dot{\varphi} = p_\varphi$$

$$\omega_r = \dot{\varphi} = \frac{p_\varphi}{m r^2}$$

$$\omega_r^2 = \frac{p_\varphi^2}{m r^4} \cdot \frac{1}{m} = \frac{p_\varphi^2}{k m r_0} \frac{k r_0}{m r^4}\Big|_{r = r_1}$$

$$= l^2 \frac{k}{r_0^3 m} \frac{1}{x^4}\Big|_{x = x_1}$$

$$= \frac{e^{-x}(x_1 + x_1^2)}{x_1^4} \frac{k}{m r_0^3}$$

$$\frac{\omega_r^2}{\omega^2} = \frac{x_1 + x_1^2}{(x_1 + x_1^2 - x_1^3)}$$

v splošnem ni racionalna številka

racionalna števila

$\Downarrow$

orbite niso sklenjene!