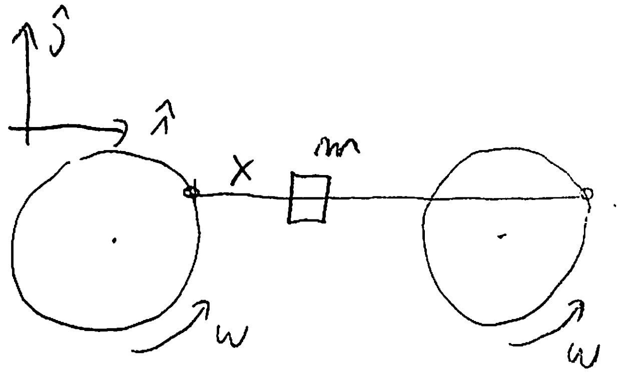
# Utež na gibajoči se prečki




a)

$$\vec{r} = \vec{r_0} + x\,\hat{\imath} = r_0 \cos\omega t\;\hat{\imath} + r_0 \sin\omega t\;\hat{j} + x\,\hat{\imath}$$

$$\vec{v} = \left(-r_0\omega \sin\omega t + \dot{x}\right)\hat{\imath} + \omega r_0 \cos\omega t\;\hat{j}$$

$$v^2 = \dot{x}^2 + \omega^2 r_0^2 + (-2 r_0 \omega \dot{x} \sin\omega t)$$

$$L = \frac{1}{2} m (\dot{x}^2 - 2 r_0 \omega \dot{x} \sin\omega t) - m g r_0 \sin\omega t$$

+ konst.

edina din. spremenljivka $x$

$$L = \frac{1}{2} m (\dot{x}^2 - 2 r_0 \omega \dot{x} \sin\omega t)$$

b)

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{d}{dt} \underbrace{m\,(\dot{x} - r_0 \omega \sin\omega t)}_{p_x} = \frac{\partial L}{\partial x} = 0$$

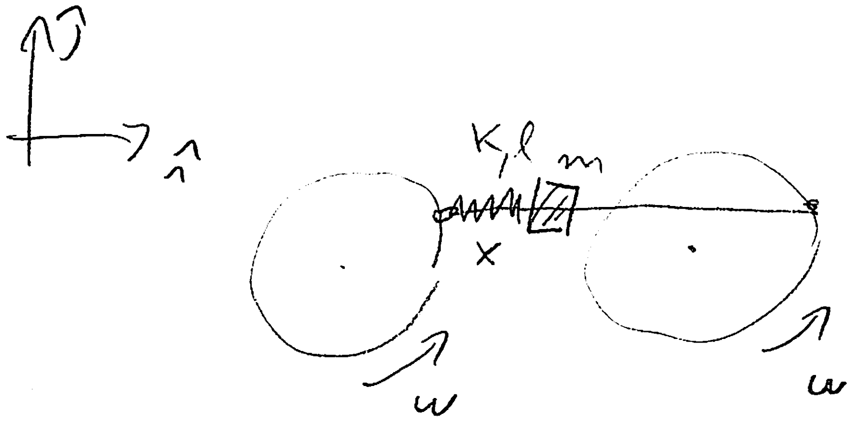
$p_x = \text{konst}$ (Edina ohranjena kol.!)

$$x = \frac{p_x}{m} t - r_0 \cos\omega t$$

Utež prosto drsi, hitrost v x smeri se ohranja.

$$\vec{v}\cdot\hat{\imath} = \frac{p_x t}{m}$$

Utež na gibajoči se prečki z vzmetjo




c)

$$L = \frac{1}{2} m (\dot{x}^2 - 2 r_0 \omega \dot{x} \sin \omega t) - \frac{1}{2} k (x - l)^2$$

$$\frac{d}{dt} m (\dot{x} - r_0 \omega \sin \omega t) = -k (x - l)$$

$$\tilde{x} = x - l$$

$$\ddot{\tilde{x}} - r_0 \omega^2 \cos \omega t = -\omega_0^2 \tilde{x}$$

$$\omega_0^2 = k/m$$

d) homogen del:

$$\tilde{x} = A \cos \omega_0 t + B \sin \omega_0 t$$

partikularni del:

$$\tilde{x} = C \cos \omega t$$

$$(-C\omega^2 - r_0 \omega^2 + \omega_0^2 C) \cos \omega t = 0$$

$$C = \frac{r_0 \omega^2}{\omega_0^2 - \omega^2}$$

Splošna rešitev:

$$\tilde{x} = \frac{r_0 \omega^2}{\omega_0^2 - \omega^2} \cos \omega t + A \cos \omega_0 t + B \sin \omega_0 t$$

$$x = \hat{x} + l$$

Ko $\omega \to \omega_0$ $C \to \infty$. V tej limiti je ugodneje splošno rešitev napisati v obliki

$$\hat{x} = \overbrace{\frac{r_0 \omega^2}{\omega_0^2 - \omega^2} \left(\cos \omega t - \cos \omega_0 t\right)}^{\tilde{x}_1} + A' \cos \omega_0 t + B \sin \omega_0 t$$

$$\omega = \omega_0 + \omega' \quad ; \omega' \to 0$$

$$\hat{x}_1 = \frac{r_0 \omega^2 \left(\cos \omega_0 t \cos \omega' t - \sin \omega_0 t \sin \omega' t - \cos \omega_0 t\right)}{\omega_0^2 - \omega_0^2 - 2\omega_0 \omega' - \omega'^2}$$

razvijemo za male $\omega'$, do lin reda

$$\tilde{x}_1 = \frac{r_0 \omega^2}{-2\omega_0 \cancel{\omega'}} \left(-\sin \omega_0 t \, \cancel{\omega'} t\right)$$

$$\tilde{x}_1 = \frac{r_0 \omega^2}{2 \omega_0} t \sin \omega_0 t = \frac{r_0 \omega_0}{2} t \sin \omega_0 t$$

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Amplituda lin. ~~narašča~~ narašča s časom.

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