$\vec{B} = B\hat{e}_z$; B = konst. Reši s Hamiltonovim formalizmom!

$$\vec{A} = \begin{cases} (Bz, 0, 0) = A(z)\hat{e}_r & (1) \\ (0, 0, -Br) = A(r)\hat{e}_z & (2) \end{cases}$$

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2) + e\vec{v}\cdot\vec{A}$$

$$\vec{v} = \dot{r}\hat{e}_r + r\sin\vartheta\,\dot{\varphi}\,\hat{e}_\varphi + r\dot{\vartheta}\hat{e}_\vartheta$$

① $$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2) + eBz\dot{r}$$

$$H = \sum_\alpha p_\alpha \dot{q}_\alpha - L = m\dot{r}^2 + eBz\dot{r} + mr^2\dot{\varphi}^2 + m\dot{z}^2 - \frac{1}{2}m\dot{r}^2 - \frac{1}{2}mr^2\dot{\varphi}^2 - \frac{1}{2}m\dot{z}^2 - eBz\dot{r}$$

$$p_r = m\dot{r} + eBz$$
$$p_\varphi = mr^2\dot{\varphi}$$
$$p_z = m\dot{z}$$

$$= \frac{m\dot{r}^2}{2} + \frac{mr^2\dot{\varphi}^2}{2} + \frac{m\dot{z}^2}{2}$$

$$H = \frac{(p_r - eBz)^2}{2m} + \frac{p_\varphi^2}{2mr^2} + \frac{p_z^2}{2m}$$

② $$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2) - eB\dot{z}r$$

$$p_r = m\dot{r}$$
$$p_\varphi = mr^2\dot{\varphi}$$
$$p_z = m\dot{z} - eBr$$

$$H = \frac{m\dot{r}^2}{2} + \frac{mr^2\dot{\varphi}^2}{2} + \frac{m\dot{z}^2}{2}$$

$$H = \frac{p_r^2}{2m} + \frac{p_\varphi^2}{2mr^2} + \frac{(p_z + eBr)^2}{2m}$$

en. gibanja

(1)

$$\dot r = \frac{p_r - eBz}{m}$$

$$\dot\varphi = \frac{p_\varphi}{mr^2}$$

$$\dot z = \frac{p_z}{m}$$

$$\dot p_r = \frac{p_\varphi^2}{mr^3}$$

$$\dot p_\varphi = 0$$

$$\dot p_z = \frac{(p_r - eBz)eB}{m} = \omega_c (p_r - eBz)$$

(2)

$$\dot r = \frac{p_r}{m}$$

$$\dot\varphi = \frac{p_\varphi}{mr^2}$$

$$\dot z = \frac{p_z + eBr}{m}$$

$$\dot p_r = -\omega_c (p_z + eBr) + \frac{p_\varphi^2}{mr^3}$$

$$\dot p_\varphi = 0$$

$$\dot p_z = 0$$

združimo:

(1)

$$\ddot r = \frac{p_\varphi^2}{m^2 r^3} - \omega_c \dot z$$

$$\ddot z = \frac{\omega_c}{m}(p_r - eBz) = \omega_c \dot r$$

(2)

$$\ddot r = -\omega_c \frac{(p_z + eBr)}{m} + \frac{p_\varphi^2}{m^2 r^3}$$

$$\ddot r = -\omega_c \dot z + \frac{p_\varphi^2}{m^2 r^3}$$

$$\ddot z = \omega_c \dot r$$

Enake enačbe gibanja!

poseben primer

$p_\varphi = 0$, φ = konst.

$$\ddot r = -\omega_c \dot z$$

$$\ddot z = \omega_c \dot r$$

ciklotronsko kroženje v ravnini (r, z); φ = konst

$$r = r_0 \sin(\omega_c t + \delta) + r_1$$

$$z = r_0 \cos(\omega_c t + \delta) + z_1$$

Satelit kroži z $r = r_0$ okrog Zemlje.

V nekem trenutku se mu zaradi trka spremeni smer hitrosti, njena velikost v_0 pa ostane nespremenjena.

Najbližja razdalja med satelitom in Zemljo (trenutek v_0)

je $r_1 = \frac{2}{5} r_0$. Kolikšna hitrost ima satelit pri najbližji razdalji?

$$E_0 = \overbrace{\frac{1}{2} m \underset{\substack{\shortparallel \\ 0}}{\dot{r}^2} + \frac{p_e^2}{2 m r_0^2}}^{= \frac{1}{2} m v^2} - \frac{\alpha}{r_0}$$

$$V_{eff} = \frac{p_e^2}{2 m r^2} - \frac{\alpha}{r} \quad ; \quad \frac{\partial V_{eff}}{\partial r} = -\frac{p_e^2}{m r^3} + \frac{\alpha}{r^2}\bigg|_{r_0} = 0$$

$$r_0 = \frac{p_e^2}{m \alpha}$$

po trku

$$E_1 = E_0 = \frac{-\alpha}{2 r_0}$$

$$V_{eff} = \frac{p_{e1}^2}{2 m r^2} - \frac{\alpha}{r} = \frac{x^2}{2} \frac{\alpha r_0}{r^2} - \frac{\alpha}{r}$$

definiram: $p_{e1} = x\, p_{e0}$

v skrajnih legah

za $r = r_1$: $E_1 = \frac{p_{e1}^2}{2 m r^2} - \frac{\alpha}{r} = x^2 \frac{\alpha r_0}{2 r^2} - \frac{\alpha}{r} = \frac{-\alpha}{2 r_0}$

$$\Rightarrow \quad x^2 = \left(\frac{\alpha}{r_1} - \frac{\alpha}{2 r_0} \right) \Big/ \frac{\alpha r_0}{2 r_1^2} = 2 \left(\frac{r_1}{r_0} - \frac{r_1^2}{2 r_0^2} \right)$$

$$x = \frac{4}{5}$$

$$V_{eff} = \frac{x^2 \alpha r_0}{2 r^2} - \frac{\alpha}{r}$$

v skrajni točki

$$E_1 = \frac{1}{2} m \dot{r}^2 + V_{eff}(r_1)$$

tam velja

$$\frac{1}{2} m v^2 = \frac{p_{\varphi 1}^2}{2 m r_1^2} = x^2 \frac{\alpha r_0}{2 r_1^2}$$

$$v^2 = \frac{x^2 \alpha r_0}{m r_1^2}$$

pred trkom

$$v_0^2 = \frac{\alpha r_0}{m r_0^2} = \frac{\alpha}{m r_0}$$

po trku

$$v^2 / v_0^2 = x^2 \frac{r_0^2}{r_1^2}$$

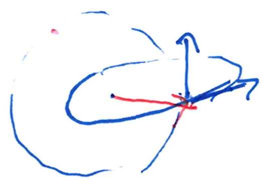
$$\underline{v/v_0 = x \frac{r_0}{r_1}} = \frac{4}{5} \cdot \frac{5}{2} = \underline{\underline{2}}$$

$$p_{\varphi 1} = m r_0 v_0 \sin\varphi \quad ; \quad p_{\varphi 0} = m r_0 v_0$$

$$x = \sin\varphi$$

φ kot med $\vec{r}$ in $\vec{v}$

$$\underline{\varphi = \arcsin x}$$



$$\varphi = \arcsin \frac{4}{5}$$

pri križanju $\varphi = \varphi_0 = \pi/2$.

kot med tirnicama $\underline{\underline{\varphi - \pi/2}}$.