

1.

a)  $L = \frac{1}{2} m (r^2 \sin^2 \vartheta \dot{\varphi}^2 + r^2 \dot{\vartheta}^2)$   
 $r = \text{konst} = r_0$  (radij Zemlje)

$\varphi$ :  $\frac{d}{dt} m r^2 \sin^2 \vartheta \dot{\varphi} = 0 \quad \sin^2 \vartheta \dot{\varphi} := l = \text{konst}$

$\vartheta$ :  $\frac{d}{dt} m r^2 \dot{\vartheta} = m r^2 \sin \vartheta \cos \vartheta \dot{\varphi}^2$

$\ddot{\vartheta} = \sin \vartheta \cos \vartheta \dot{\varphi}^2 = \frac{l^2}{r^2 \sin^3 \vartheta} \cos \vartheta$

b)

$\ddot{\vartheta} = \frac{d}{dt} \frac{d}{dt} \vartheta = \dot{\varphi} \frac{d}{d\varphi} \dot{\varphi} \frac{d}{d\varphi} \vartheta =$   
 $= \frac{l}{r^2 \sin^2 \vartheta} \frac{d}{d\varphi} \left( \frac{l}{\sin^2 \vartheta} \frac{d}{d\varphi} \vartheta \right) = \frac{l^2}{r^2 \sin^3 \vartheta} \cos \vartheta$

$\frac{d^2}{d\vartheta^2} (-\text{ctg} \vartheta) = \text{ctg} \vartheta$

$\text{ctg} \vartheta = A \cos(\vartheta - \vartheta_0) \Rightarrow$  gibanje po ravnini  
 z  $r = r_0 \Rightarrow$  kroženje  
 okrog središča Zemlje

c) brez vrtelja:

$\vec{r} = r \hat{e}_r$

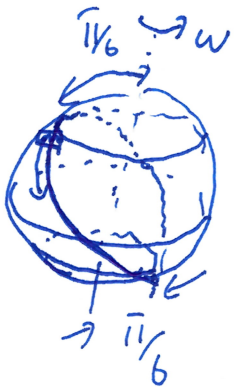
$\vec{v} = \dot{\vec{r}} = \dot{r} \hat{e}_r + r \dot{\vartheta} \hat{e}_\vartheta + r \sin \vartheta \dot{\varphi} \hat{e}_\varphi$

z vrteljem:

$\vec{v} = \vec{v}_{\text{rel}} + \vec{\omega} \times \vec{r} = \vec{v}_{\text{rel}} + r \sin \vartheta \omega \hat{e}_\varphi$

$\vec{v} = \dot{r} \hat{e}_r + r \dot{\vartheta} \hat{e}_\vartheta + r \sin \vartheta (\dot{\varphi} + \omega) \hat{e}_\varphi$

d) V inercialnem sistemu ima utež začetna hitrost  $\vec{v} = r\omega \hat{e}_{\text{in}}$ ,  $|\vec{v}| = r\omega \sin \frac{\pi}{6} = \frac{r\omega}{2}$  v smeri vzhoda.



Utež gane v smeri jugozahoda in se po 2 dneh vrne na začetno lega.

V vrtajočem sistemu utež na začetku miruje, potem pa se začne gibati v smeri jugozahoda in se po dveh dneh vrne na začetno lega.

3. Izpit KLM, 30.8.23

3. a)

$$H = \frac{p^2}{2} \neq \frac{z^2}{2}$$

$$\frac{d}{dt} D = \frac{\partial}{\partial t} D + \{D, H\} = -H + \{D, H\}$$

$$\{D, H\} = \left\{ \frac{pz}{2}, H \right\} = \frac{p}{2} \{z, H\} + \frac{z}{2} \{p, H\} =$$

$$= \frac{p}{2} \frac{1}{2} \{z, p^2\} \neq \frac{z}{2} \frac{1}{2} \{p, z^2\} = \frac{p^2}{2} - \frac{z^2}{2}$$

$$\underline{\underline{\frac{d}{dt} D = 0}}$$

$$e) H = \left( \sum_i p_i^2 \right)^{m/2} - a \left( \sum_i z_i^2 \right)^{-m/2}$$

$$\{D, H\} = \frac{1}{n} \left\{ \sum_j p_j z_j, H \right\} = \frac{1}{n} \sum_j p_j \left\{ z_j, \left( \sum_i p_i^2 \right)^{m/2} \right\} + \sum_j z_j \left\{ p_j, -a \left( \sum_i z_i^2 \right)^{-m/2} \right\}$$

$$\begin{aligned} \{D, H\} &= \frac{m}{2} \cdot \frac{1}{n} \sum_j p_j z_j \left( \sum_i p_i^2 \right)^{m/2 - 1} \\ &\quad - a \frac{m}{2} \frac{1}{n} \sum_j z_j z_j \left( \sum_i z_i^2 \right)^{-m/2 - 1} \\ &= H \end{aligned}$$

$$\underline{\underline{\frac{d}{dt} D = 0}}$$