3. Izpit KLM, 30. 8. 23

1.

a) $$L = \frac{1}{2} m \left(r^2 \sin^2\vartheta \, \dot{\varphi}^2 + r^2 \dot{\vartheta}^2 \right)$$

$r = \text{konst} = r_0$ (radij Zemlje)

φ: $$\frac{d}{dt} m r^2 \sin^2\vartheta \, \dot{\varphi} = 0 \qquad \sin^2\vartheta \, \dot{\varphi} := \ell = \text{konst}$$

ϑ: $$\frac{d}{dt} \cancel{m r^2} \dot{\vartheta} = \cancel{m r^2} \sin\vartheta \cos\vartheta \, \dot{\varphi}^2$$

$$\ddot{\vartheta} = \sin\vartheta \cos\vartheta \, \dot{\varphi}^2 = \frac{\ell^2}{\sin^3\vartheta} \cos\vartheta$$

b)

$$\ddot{\vartheta} = \frac{d}{dt} \frac{d}{dt} \vartheta = \dot{\varphi} \frac{d}{d\varphi} \dot{\varphi} \frac{d}{d\varphi} \vartheta =$$

$$= \frac{\cancel{\ell}}{\cancel{\sin^2\vartheta}} \frac{d}{d\varphi} \left(\frac{\cancel{\ell}}{\sin^2\vartheta} \frac{d}{d\varphi} \vartheta \right) = \frac{\cancel{\ell^2}}{\sin^{\cancel{3}}\vartheta} \cos\vartheta$$

$$\frac{d^2}{d\varphi^2} (-\mathrm{ctg}\,\vartheta) = \mathrm{ctg}\,\vartheta$$

$$\Downarrow$$

$$\mathrm{ctg}\,\vartheta = A \cos(\varphi - \varphi_0) \Rightarrow$$ gibanje po ravnini z $r = r_0 \Rightarrow$ kroženje okrog središča Zemlje

c) brez vrtenja:

$$\vec{r} = r \hat{e}_r$$

$$\vec{v} = \dot{\vec{r}} = \dot{r} \hat{e}_r + r \dot{\vartheta} \hat{e}_\vartheta + r \sin\vartheta \, \dot{\varphi} \hat{e}_\varphi$$

z vrtenjem:

$$\vec{v} = \vec{v}_{rel} + \vec{\omega} \times \vec{r} = \vec{v}_{rel} + r \sin\vartheta \, \omega \hat{e}_\varphi$$

$$\vec{v} = \dot{r} \hat{e}_r + r \dot{\vartheta} \hat{e}_\vartheta + r \sin\vartheta (\dot{\varphi} + \omega) \hat{e}_\varphi$$

d) V inercialnem sistemu ima utež začetne hitrost $\vec{v} = r\omega\,\hat{e}_{zah}$, $|\vec{v}| = r\omega \sin \frac{\pi}{6} = \frac{r\omega}{2}$.
v smeri vrtenja.

Utež gre v smeri jugo-zahoda in se po 2 dneh vrne na začetno lego.

V vrtečem sistemu utež na začetku miruje, potem pa se začne gibati v smeri jugo-zahoda in se po dveh dneh vrne v začetno lego.

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3. a)

$$H = \frac{p^2}{2} \cancel{+} \frac{q^{-2}}{2}$$

$$\frac{d}{dt} D = \frac{\partial}{\partial t} D + \{D, H\} = -H + \{D, H\}$$

$$\{D, H\} = \left\{\frac{pq}{2}, H\right\} = \frac{p}{2}\{q, H\} + \frac{q}{2}\{p, H\} =$$

$$= \frac{p}{2}\frac{1}{2}\{q, p^2\} \cancel{+} \frac{q}{2}\frac{1}{2}\{p, q^{-2}\} = \frac{p^2}{2} - \frac{q^{-2}}{2}$$

$$\frac{d}{dt} D = 0$$

b)

$$H = \left(\sum_i p_i^2\right)^{n/2} - a\left(\sum_i q_i^2\right)^{-n/2}$$

$$\{D, H\} = \frac{1}{n}\left\{\sum_j p_j q_j, H\right\} = \frac{1}{n}\left[\sum_j p_j \left\{q_j, \left(\sum_i p_i^2\right)^{n/2}\right\} + \sum_j q_j \left\{p_j, -a\left(\sum_i q_i^2\right)^{-n/2}\right\}\right]$$

$$\{D, H\} = \frac{n}{2} \cdot \frac{1}{n} \sum_j p_j 2p_j \left(\sum_i p_i^2\right)^{n/2 - 1}$$

$$- a \frac{n}{2} \frac{1}{n} \sum_j q_j 2q_j \left(\sum_i q_i^2\right)^{-n/2 - 1} =$$

$$= H$$

$$\frac{d}{dt} D = 0$$