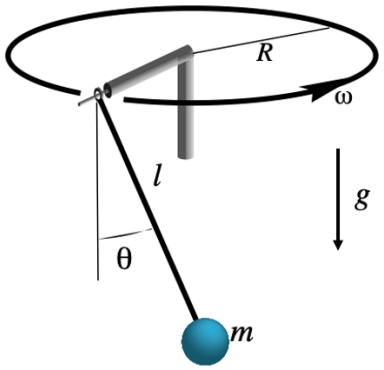
2. izpit iz Klasične mehanike, 19.6.2023

1. Consider a pendulum of length l and mass m, whose axis of rotation is moving in a circle with angular velocity ω, as shown in the figure. The plane in which the pendulum swings is perpendicular to the radial handle of length R. Neglect friction. Gravitational acceleration is denoted by g, and θ represents the displacement of the pendulum from the vertical line, as shown in the figure.

a) Using the labels in the figure, write down the position of the mass m! Then, express the kinetic and potential energy of the system!
b) Derive the equation of motion for θ!
c) Determine the critical value of ω and in different regimes, for different values of ω, identify stable and unstable equilibrium positions! Also, write down the frequencies of oscillation around the stable equilibrium positions.



2. A particle moves in the potential $V(r) = k/r^2$, where k is a real number (positive or negative). Sketch the form of the effective potential for different values of k! At which values of k are the orbits unbound? For such k derive the trajectory $r(\phi)$! Find the scattering angle and express it with the impact parameter b!

3. Consider a system of two bodies with masses M and m, connected by springs with spring constants k, as shown in the sketch. The surfaces are smooth, so friction can be neglected.

a) Write down the kinetic and potential energy of the system!
b) Calculate the natural frequencies and their corresponding normal modes. Sketch the modes and clearly indicate which mode has the highest frequency of oscillation!
c) Consider both limiting cases when $M \gg m$ and $m \gg M$. Calculate the natural frequencies of oscillation in each case!