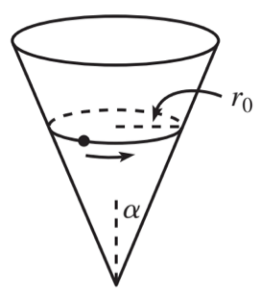
1A izpit iz Klasične mehanike, 6.4.2023

1. The shape of the Earth is similar to a slightly flattened ellipsoid, which is a consequence of its rotation with an angular velocity $\boldsymbol{\omega}$. A puck slides without friction on an icy surface with a velocity v. The surface is "flat" in the sense that it is perpendicular at all points to the effective gravitational acceleration $\boldsymbol{g}_{\text{eff}} = \boldsymbol{g} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$, where $\boldsymbol{g}$ is the gravitational acceleration on the surface of the Earth, and $\mathbf{r}$ denotes the location of the puck relative to the center of the Earth.

a) Draw a sketch of the Earth and the puck, and label $\boldsymbol{\omega}$, $\boldsymbol{g}$, $\boldsymbol{g}_{\text{eff}}$, and the force of the surface on the puck, $\mathbf{F}_s$.
b) Write down the equations of motion for the puck as viewed from a rotating coordinate system on the surface of the Earth. Assume that the motion is confined to a small region on the surface of the Earth ($\theta \approx$ const.), and that the puck is always in contact with the surface ($z' = 0$).
c) The puck is located in the Northern Hemisphere. Will the puck deflect in the clockwise or counterclockwise direction? Name the curve along which the puck moves!
d) Determine the time period of the puck's motion and the distance it travels during that.

2. A particle moves without friction inside a smooth cone. The cone is fixed with its tip downwards and its axis in a vertical position. Let α be the angle that the cone surface makes with its axis (see sketch). Let r denote the distance of the particle from the axis and let φ be the angle around the cone axis.

a) Write the kinetic and potential energy. Derive the equations of motion for the coordinates r and φ.
b) Assume that the particle moves in a circle with a fixed radius r_0. Compute the angular frequency ω!
c) The particle is slightly displaced in the transverse direction so that it begins to oscillate with frequency Ω around the circle with radius r_0. What is the frequency of the oscillation Ω? Express the result in terms of the gravitational acceleration g, the radius r_0, and the angle α.
d) When is $\omega = \Omega$? Describe or sketch the motion of the particle in this case.



3. Two rings with radius r_0 rotate uniformly with frequency ω around stationary axes, as shown on the figure. We attach a rod with length equal to the distance between the axes to the two disks through bearings. We place a weight with mass m, which slides on the rod without friction.

a) Write down the Lagrangian! Express the position of the weight using the coordinate x that measures the deviation from the left end of the rod.
b) Derive the equations of motion and solve them! Write down the constants of motion!
c) Let now the weight be attached to the left end of the rod with a spring of coefficient k and equlibrium length l. Write down the Lagrangian and the equations of motion!
d) Find the general solution and express the amplitude of the motion with frequency ω! What happens when $\omega \to \sqrt{k/m}$? Find the dependence of the oscillation amplitude on time in this case!