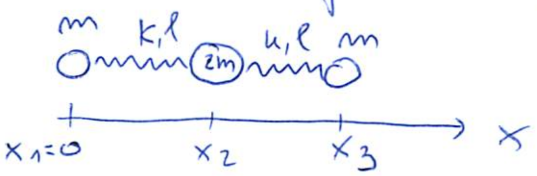
TRI UTEŽI POVEZANE V RAVNO ČRTO

Imamo tri uteži z masami $m_1 = m$, $m_2 = 2m$ in $m_3 = m$. Povezane so v ravni črti z dvema vzmetema s koeficientom k in neraztegnjeno dolžino l. Poišči lastne frekvence, lastne mihajne načine ~~...~~ in časovni razvoj sistema, če je ob $t=0$ utež 1 hitrost v_0 in $\underline{x}(t=0) = \begin{pmatrix} 0 \\ l \\ 2l \end{pmatrix}$



Koliko je energije v posameznih mihajnih načinih?

NAMIGI

- Koordinate: u_1, u_2, u_3, kjer: $\underline{x} = \begin{pmatrix} 0+u_1 \\ l+u_2 \\ 2l+u_3 \end{pmatrix}$: odmiki iz začetne lege kjer vzmetni raztegnjena.
- Kinetična: $T = \ldots\ldots = \frac{1}{2}\dot{\underline{u}}^T \underline{\underline{T}}\, \dot{\underline{u}}$; $\underline{\underline{T}} = m\tilde{\underline{\underline{T}}} = m\begin{pmatrix} 1 & & \\ & 2 & \\ & & 1 \end{pmatrix}$
- Potencialna:

$$V = \frac{1}{2}k\left(l-(x_2-x_1)\right)^2 + \frac{1}{2}k\left(l-(x_3-x_2)\right)^2 = \ldots = \frac{1}{2}\underline{u}^T \underline{\underline{V}}\,\underline{u}\ ;\ \underline{\underline{V}} = k\underbrace{\begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}}$$

- ($L = T - V$, Euler-Lagrangeove enačbe, nastavek $\underline{\eta}(t) = \underline{a}e^{-i\omega t}$) dobimo $\underline{\underline{V}}\,\underline{a} = \omega^2 \underline{\underline{T}}\,\underline{a}$. Poišči lastne frekvence ω^2! $\tilde{\underline{\underline{V}}}$

$\omega_1^2 = \omega_0^2$; $\omega_2^2 = 0$; $\omega_3^2 = 2\omega_0^2$

- Lastni vektorji / mihajni načini: $\underline{a}_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\frac{1}{\sqrt{2}}$; $\underline{a}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\frac{1}{\sqrt{3}}$; $\underline{a}_3 = \begin{pmatrix} 1 \\ -1 \\ +1 \end{pmatrix}\frac{1}{\sqrt{3}}$

$\underline{\eta_i}(t) = \underline{a_i}\,\alpha_i(t)$; $\alpha_1(t) = A_1\cos\omega_0 t + B_1\sin\omega_0 t$

$\alpha_3(t) = A_3\cos\sqrt{2}\omega_0 t + B_3\sin\sqrt{2}\omega_0 t$

$\alpha_2(t) = A_2 + B_2 t$

- Rešitev: $\underline{u}(t) = \sum_{i=1}^{3} \alpha_i(t)\,\underline{a_i}$, kjer A_i, B_i določi iz začetnih pogojev.

$$\underline{u}(t) = -\frac{1}{\sqrt{2}}\frac{v_0}{\omega_0}\sin(\omega_0 t)\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \frac{\sqrt{3}}{4}v_0 t\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \sqrt{\frac{3}{2}}\frac{1}{4}\frac{v_0}{\omega_0}\sin(\sqrt{2}\omega_0 t)\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

- $H = \sum_k H_k$; $H_k = \frac{1}{2}\dot{\alpha}_k^2 T_k' + \frac{1}{2}\alpha_k^2 V_k'$

$$\underline{\underline{T}}' = \underline{\underline{A}}^T\underline{\underline{T}}\,\underline{\underline{A}} = \begin{pmatrix} T_1' & & \\ & T_2' & \\ & & T_3' \end{pmatrix} ;\quad \underline{\underline{V}}' = \underline{\underline{A}}^T\underline{\underline{V}}\,\underline{\underline{A}} = \begin{pmatrix} V_1' & & \\ & V_2' & \\ & & V_3' \end{pmatrix}$$

$$\underline{\underline{A}} = (\underline{a_1}, \underline{a_2}, \underline{a_3})$$