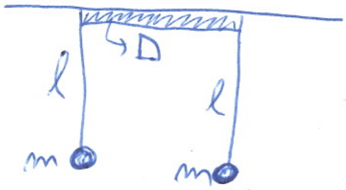
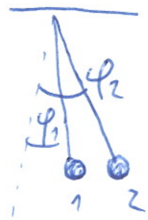
TORZIJSKO SKLOPLJENI TEŽNI NIHALI

Obravnavajmo dve težni nihali, ki se lahko prosto vrtita okrog skupne osi, ki deluje hkrati kot torzijska sklopitev z velikostjo D. Poišči stacionarne stabilne lege, in lastne frekvence in lastne nihajne načine okrog njih!





Sistem opišemo s koordinatama zasuka $\varphi = \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}$

NAMIGI

- $V = -mgl\left(\cos\varphi_1 + \cos\varphi_2 - \frac{\alpha}{2}(\varphi_1 - \varphi_2)^2\right)$; $\alpha = \frac{D}{mgl} > 0$
- Ravnovesna lega:

① $\frac{\partial V}{\partial \varphi_1}\Big|_{\varphi_0} = 0$

② $\frac{\partial V}{\partial \varphi_2}\Big|_{\varphi_0} = 0$

$\xrightarrow[\varphi_0]{\text{rešujemo}}$

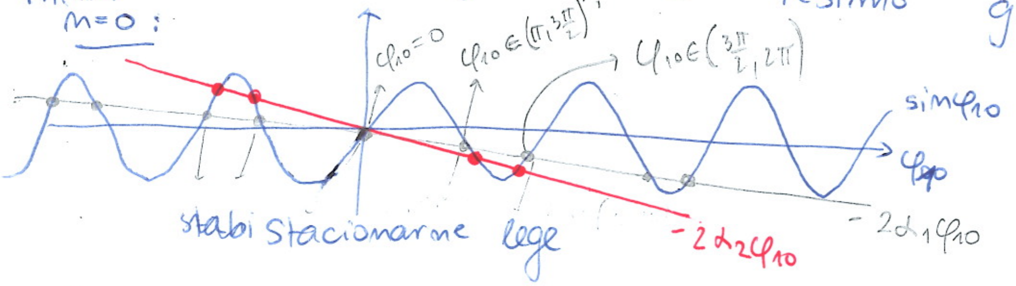
①+②: $\sin\varphi_{10} = -\sin\varphi_{20} \Rightarrow \varphi_{10} = \begin{cases} -\varphi_{20} + 2\pi m \\ \varphi_{20} + \pi + 2\pi m \end{cases}$

①−②: $\begin{cases} \cancel{2}\sin\varphi_{10} + 2\alpha\varphi_{10} - \cancel{\alpha}\pi m = 0 \\ \sin\varphi_{10} - 2\pi/2 - 2\pi m = 0 \end{cases}$

~~ni prave rešitve~~

Enačbo $\sin\varphi_{10} + 2\alpha\varphi_{10} - \pi m = 0$ rešimo grafično:

Primer $m=0$:



$\alpha_2 > \alpha_1$; za α_2 manj rešitev

- Stabilna lega, če $V(\varphi_0 + \delta\varphi) > V(\varphi_0)$ v bližini stacionarne φ_0

Taylor: $V(\varphi_0 + \delta\varphi) = V(\varphi_0) + \underbrace{\sum_i \frac{\partial V}{\partial \varphi_i}\Big|_{\varphi_0}\delta\varphi_i}_{0} + \frac{1}{2}\underbrace{\sum_{ij} \frac{\partial^2 V}{\partial\varphi_i \partial\varphi_j}\Big|_{\varphi_0}\delta\varphi_i\delta\varphi_j}_{\delta\varphi^T \tilde{\underline{V}}\,\delta\varphi} + \dots$

$\Rightarrow \varphi_0$ stabilna, če:

$\delta\varphi^T \tilde{\underline{V}}\,\delta\varphi > 0$ za $\forall\, \delta\varphi$ = ko je $\tilde{\underline{V}}$ pozitivno definitna (vse lastne vrednosti pozitivne)

- Zapiši matriko $\tilde{\underline{V}}$, kjer $\tilde{V}_{ij} = \frac{\partial^2 V}{\partial\varphi_i \partial\varphi_j}$ in poišči lastne vrednosti

$\tilde{\underline{V}} = mgl\begin{pmatrix} \cos\varphi_{10} + \alpha & -\alpha \\ -\alpha & \cos\varphi_{10} + \alpha \end{pmatrix}$; $\lambda_1 = \cos\varphi_{10} \cdot mgl$, $\lambda_2 = (\cos\varphi_{10} + 2\alpha) \cdot mgl$

- stabilne lege za $m=0$, prve tri:

a) $\varphi_{10} = 0$: $\lambda_1 > 0$, $\lambda_2 > 0$ — stabilna ✓

b) $\varphi_{10} \in (\pi, \frac{3\pi}{2})$: $\lambda_1 < 0$ — labilna

c) $\varphi_{10} \in (\frac{3\pi}{2}, 2\pi)$: $\lambda_1 > 0$, $\lambda_2 > 0$ — stabilna

Izračun lastnih nihanj: Zapiši T in V za majhne odmike iz φ_0

$T = \frac{1}{2} \delta\dot{\varphi}^T \, \underline{\underline{\tilde{T}}} \, \delta\dot{\varphi}$, $\underline{\underline{\tilde{T}}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} ml^2$, V imamo

- Lastna nihanja: z nastavkom $\underline{\delta\varphi} = \varphi_0 \, \underline{a} \, e^{-i\omega t}$ iz Euler-Lag. enačb dobimo:

$$\underline{\underline{\tilde{V}}} \, \underline{a} = \omega^2 \, \underline{\underline{\tilde{T}}} \, \underline{a} \quad \Rightarrow \quad \det\left(\underline{\underline{\tilde{V}}} - \underbrace{\omega^2 ml^2}_{\lambda} \mathbb{1}\right) = 0$$

λ_1, λ_2 že poznamo, torej

$$\lambda_1 = mgl\cos\varphi_{10} = \omega_1^2 ml^2$$
$$\Rightarrow \quad \omega_1^2 = \omega_0^2 \cos\varphi_{10}; \; \omega_0^2 = \frac{g}{l}$$
$$\omega_2^2 = (\cos\varphi_{10} + 2\alpha)\,\omega_0^2$$

- Lastni vektorji; izračunaj!

λ_1 pripada $\underline{a_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, λ_2: $\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \underline{a_2}$

Lastni nihajni načini so torej:

$\cancel{\underline{\delta\varphi_1} = \varphi_1} \quad \cancel{\underline{\delta\varphi_2} = \varphi_0}$

$$\underline{\delta\varphi_1} = (A_1 \cos\omega_1 t + B_1 \sin\omega_1 t)\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$\underline{\delta\varphi_2} = (A_2 \cos\omega_2 t + B_2 \sin\omega_2 t)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$