DELEC V POTENCIALU $V(r) = -k/r^2$

Poišči vse možne orbite delca v $V(r) = -\frac{k}{r^2}$ potencialu. Najprej preko efektivnega potenciala poišči tipe orbit, potem pa iz celotne energije $r(\varphi)$!

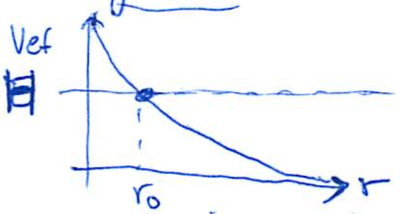
NAMIGI:

- centralni potencial $\Rightarrow$ ravninsko gibanje $\Rightarrow$ (r, φ) polarne koordinate.
- $p_\varphi = \frac{\partial L}{\partial \dot\varphi} = mr\dot\varphi^2 = $ konst saj $L \neq L(\varphi)$

① celotna energija: $H = \frac{m}{2}\dot r^2 + V_{ef}(r)$; $V_{ef}(r) = V(r) + \frac{p_\varphi^2}{2mr^2}$ efektivni potencial

- Tipi orbit: $V_{ef}(r) = \frac{1}{r^2}\left(\frac{p_\varphi^2}{2m} - k\right) = \frac{p_\varphi^2}{2m}(1-\varkappa)\frac{1}{r^2}$; $\varkappa = \frac{2mk}{p_\varphi^2}$

- $\varkappa < 1$:



Neomejeno gibanje delca, min. razdalja pri r_0

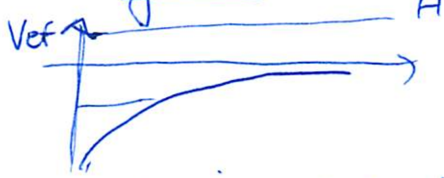
- $\varkappa = 1$

$H > 0$: padanje

$H = 0$: $\frac{m\dot r^2}{2} = 0$

kroženje

- $\varkappa > 1$



padanje v center

- Poišči $r(\varphi)$: $\dot r^2$ izrazi iz ①. ~~spremen~~ Uvedi novo spremenljivko $u = \frac{1}{r}$ in $\frac{d}{dt} \to \frac{d}{d\varphi}\dot\varphi$. Izrazi vse $\dot\varphi$ s p_φ, dobi se enačbo:

② $(u')^2 = \frac{2m}{p_\varphi^2}H - (1-\varkappa)u^2$; $u' = \frac{du}{d\varphi}$

- ② $\frac{d}{d\varphi}$ $\Rightarrow$ ③: $u'(u'' + (1-\varkappa)u) = 0$
- Rešitve enačbe ③:
  - $u' = 0 \Rightarrow u(\varphi) = u_0$ in $r(\varphi) = \frac{1}{u_0}$
  - $u'' + (1-\varkappa)u = 0$; $|1-\varkappa| = \alpha^2$ nova konstanta
    - $\varkappa < 1$: $u(\varphi) = A\cos[\alpha(\varphi - \varphi_0)]$, $\varphi \in \left(-\frac{\pi}{2\alpha}, \frac{\pi}{2\alpha}\right)$
    - $\varkappa = 1$: $u'' = 0 \Rightarrow u(\varphi) = A\varphi + B$
    - $\varkappa > 1$: $u(\varphi) = Ae^{-\alpha\varphi} + Be^{\alpha\varphi}$;