GIBANJE NABITEGA DELCA V POLJU MAG. MONOPOLA

Za električno polje točkastega naboja velja: $\vec{E} = \frac{e\,\vec{r}}{4\pi\varepsilon_0 r^3}$

Analogno bi magnetno polje monopola izgledalo: $\vec{B} = \frac{mg}{e}\,\frac{\vec{r}}{r^3}$

Lorenzova sila: $m\ddot{\vec{r}} = e(\vec{v}\times\vec{B}) = e(\dot{\vec{r}}\times\vec{B})$

$$m\ddot{\vec{r}} = \frac{emg}{e\,r^3}(\dot{\vec{r}}\times\vec{r})$$

$$\circledast \qquad \ddot{\vec{r}} = \frac{g}{r^3}(\dot{\vec{r}}\times\vec{r})$$

Kaj sklepamo iz $\circledast$?

1. $\ddot{\vec{r}}\cdot\dot{\vec{r}} = \frac{g}{r^3}(\dot{\vec{r}}\times\vec{r})\cdot\dot{\vec{r}} = 0 \qquad \frac{d}{dt}\dot{\vec{r}}^2 = \frac{d}{dt}(\dot{\vec{r}}\cdot\dot{\vec{r}}) = \ddot{\vec{r}}\cdot\dot{\vec{r}} + \dot{\vec{r}}\cdot\ddot{\vec{r}} = 2\ddot{\vec{r}}\cdot\dot{\vec{r}}$

$$\ddot{\vec{r}}\cdot\dot{\vec{r}} = 0 = \frac{1}{2}\frac{d}{dt}\dot{\vec{r}}^2$$

$$\dot{\vec{r}}^2 = v^2 \qquad \Rightarrow v^2 = konst.$$

2. $\vec{\Gamma} = m(\vec{r}\times\vec{v})$

$$\vec{\gamma} = \frac{\vec{\Gamma}}{m} = \vec{r}\times\dot{\vec{r}} \qquad \dot{\vec{\gamma}} = \dot{\vec{r}}\times\dot{\vec{r}} + \vec{r}\times\ddot{\vec{r}} = \vec{r}\times\ddot{\vec{r}}$$

$$\vec{r}\times\ddot{\vec{r}} = \vec{r}\times\frac{g}{r^3}(\dot{\vec{r}}\times\vec{r}) = \frac{g}{r^3}\left(\dot{\vec{r}}(\vec{r}\cdot\vec{r}) - \vec{r}(\vec{r}\cdot\dot{\vec{r}})\right)$$

$$= g\left(\frac{\dot{\vec{r}}}{r} - \frac{\vec{r}(\vec{r}\cdot\dot{\vec{r}})}{r^3}\right) \qquad \frac{d}{dt}\frac{\vec{r}}{r} = \frac{d}{dt}\frac{\vec{r}}{\sqrt{\vec{r}\cdot\vec{r}}} = \frac{\dot{\vec{r}}r - \vec{r}\,\frac{1}{2}(\vec{r}\cdot\vec{r})^{-\frac{1}{2}}(\dot{\vec{r}}\cdot\vec{r} + \vec{r}\cdot\dot{\vec{r}})}{r^2}$$

$$= \frac{\dot{\vec{r}}}{r} - \frac{\vec{r}(\vec{r}\cdot\dot{\vec{r}})}{r^3}$$

$$\Rightarrow \dot{\vec{\gamma}} = g\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$$

$$\frac{d}{dt}\vec{\gamma} - \frac{d}{dt}\left(g\frac{\vec{r}}{r}\right) = 0$$

$$\frac{d}{dt}\left(\vec{\gamma} - g\frac{\vec{r}}{r}\right) = 0 \qquad \text{vpeljemo } \tilde{\vec{\gamma}} = \vec{\gamma} - g\frac{\vec{r}}{r} = konst.$$

$$\tilde{\vec{\gamma}}\cdot\vec{r} = \vec{\gamma}\cdot\vec{r} - g\frac{\vec{r}\cdot\vec{r}}{r} = (\vec{r}\times\dot{\vec{r}})\cdot\vec{r} - g\frac{r^2}{r} = -gr$$

$$\tilde{\vec{\gamma}}\cdot\vec{r} = |\tilde{\vec{\gamma}}||\vec{r}|\cos\vartheta' = |\tilde{\vec{\gamma}}|\,r\cos\vartheta'$$

$$\Rightarrow \cos\vartheta' = -\frac{g}{|\tilde{\vec{\gamma}}|} = konst. \Rightarrow \text{giblje se po stožcu}$$

$\vartheta' = \pi - \vartheta$

3. $\frac{d}{dt}\vec{r}\cdot\dot{\vec{r}} = \dot{\vec{r}}\cdot\dot{\vec{r}} + \vec{r}\cdot\ddot{\vec{r}} \qquad \vec{r}\cdot\ddot{\vec{r}} = \vec{r}\cdot\frac{g}{r^3}(\dot{\vec{r}}\times\vec{r}) = 0$

$$= v^2 = konst.$$

$$\vec{r}\cdot\dot{\vec{r}} = rv\cos\beta$$

$rv\cos\beta = v^2 t + konst.$ (višjih potenc t ni, ker $\frac{d}{dt}\vec{r}\cdot\dot{\vec{r}} = konst.$)

$rv\cos\beta = v^2(t - t_0) = v^2 t$ (izberemo ustrezen t_0)

$$|\tilde{\vec{\gamma}}|^2 = \left(\vec{\gamma} - g\frac{\vec{r}}{r}\right)^2 = \vec{\gamma}\cdot\vec{\gamma} - 2g\,\frac{\vec{\gamma}\cdot\vec{r}}{r} + g^2\,\frac{\vec{r}\cdot\vec{r}}{r^2}$$

$$\vec{\gamma} = \vec{r}\times\vec{v} \qquad \vec{\gamma}\cdot\vec{r} = (\vec{r}\times\vec{v})\cdot\vec{r} = 0$$

$$|\vec{\gamma}| = rv\sin\beta$$

$$|\tilde{\vec{\gamma}}|^2 = r^2v^2\sin^2\beta + g^2$$

$$r^2 v^2 \cos^2\beta = v^4 t^2$$

$$r^2 v^2 \sin^2\beta = \gamma^2$$

$$r^2 v^2 = v^4 t^2 + \gamma^2$$

$$r^2 = v^2 t^2 + \frac{\gamma^2}{v^2} = \frac{\gamma^2}{v^2}\left(1 + \frac{v^4 t^2}{\gamma^2}\right) = r_{min}^2\left(1 + \frac{t^2}{\tau^2}\right)$$

$$r = \frac{\gamma}{v}\sqrt{1 + \frac{v^4}{\gamma^2}t^2}$$

Za opis gibanja po stožcu so primerne sferične koordinate:

$$\vec{v} = \underset{\dot r}{\underset{\shortparallel}{v_r}}\,\hat{e}_r + \underset{r\sin\vartheta\,\dot\varphi}{\underset{\shortparallel}{v_\varphi}}\,\hat{e}_\varphi + \underset{r\dot\vartheta = 0}{\underset{\shortparallel}{v_\vartheta}}\,\hat{e}_\vartheta$$

$$r\sin\vartheta\,\dot\varphi = v\sin\beta = \frac{\gamma}{r}$$

$$\dot\varphi = \frac{\gamma}{r^2\sin\vartheta} = \frac{\gamma}{\sin\vartheta}\cdot\frac{v^2}{\gamma^2\left(1 + \frac{v^4 t^2}{\gamma^2}\right)} = \frac{v^2}{\sin\vartheta\,\gamma}\,\frac{1}{1 + \left(\frac{t}{\frac{\gamma}{v^2}}\right)^2}$$

$$\int d\varphi = \frac{v^2}{\sin\vartheta\,\gamma}\int\frac{1}{1 + \left(\frac{t}{\frac{\gamma}{v^2}}\right)^2}dt = \frac{v^2}{\sin\vartheta\,\gamma}\,\frac{\gamma}{v^2}\int\frac{1}{1+u^2}du$$

$$u = \frac{t v^2}{\gamma} \qquad du = \frac{v^2}{\gamma}dt$$

$$\varphi(t) + \varphi_0 = \frac{1}{\sin\vartheta}\,\mathrm{arctg}\,\frac{t v^2}{\gamma}$$

Izberemo $\varphi_0 = 0$.

Končna rešitev:

$$r(t) = \frac{\gamma}{v}\sqrt{1 + \frac{v^4}{\gamma^2}t^2}$$

$$\varphi(t) = \frac{1}{\sin\vartheta}\,\mathrm{arctg}\,\frac{t v^2}{\gamma}$$

$$\vartheta(t) = \vartheta = \text{konst.}$$