Prosti delec, nadaljevanje

1) $\dot{r}^2 + \frac{\lambda^2}{r^2} = a^2$

2) $\dot{\varphi} = \frac{\lambda}{r^2}$

Iščemo $r(t)$ in $\varphi(t)$.

$\boxed{1.}$ $\dot{r} = \pm\sqrt{a^2 - \frac{\lambda^2}{r^2}}$

$$\frac{dr}{\sqrt{a^2 - \frac{\lambda^2}{r^2}}} = \pm\, dt$$

$$\frac{r\,dr}{\sqrt{a^2 r^2 - \lambda^2}} = \pm\, dt \qquad u = a^2 r^2 - \lambda^2, \quad du = 2a^2 r\, dr$$

$$\frac{du}{2a^2\sqrt{u}} = \pm\, dt$$

$$\sqrt{u} = \pm\, a^2 (t - t_0)$$

$$u = a^2 r^2 - \lambda^2 = a^4 (t-t_0)^2$$

$$r^2 = a^2 (t-t_0)^2 + \left(\frac{\lambda}{a}\right)^2 = a^2 (t-t_0)^2 + r_0^2$$

$$r = \sqrt{a^2 (t-t_0)^2 + r_0^2}$$

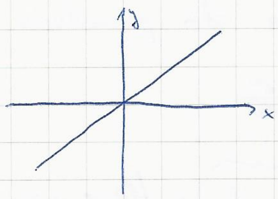
! Veljavna je samo $+\sqrt{\ }$ rešitev. r ne more biti manjši od 0.

$\dot{r} = \pm\sqrt{a^2 - \frac{\lambda^2}{r^2}}$ tu pa je pomemben tudi $-\sqrt{\ }$. Takrat se r manjša, približujemo se izhodišču.

Interpretacija:

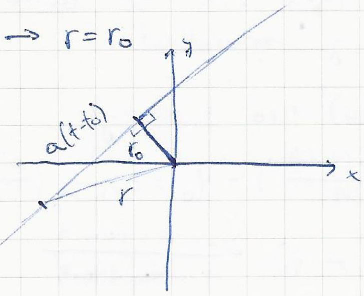
a) Če $r_0 = 0$

$r = a(t - t_0)$



b) $\lim_{t\to\infty} r$ tudi vidimo, da se r veča enakomerno z t

c) $t = t_0 \rightarrow r = r_0$



Sama oblika enačbe $\left(r^2 = a^2 (t-t_0)^2 + r_0^2\right)$ nas napeljuje na misel, da gre za pravokoten Δ. $h^2 = k_1^2 + k_2^2$

$\boxed{2.}$ $\dot{\varphi} = \frac{\lambda}{r^2} = \frac{\lambda}{a^2 (t-t_0)^2 + \left(\frac{\lambda}{a}\right)^2}$

$$d\varphi = \frac{\lambda\, dt\, \frac{a^2}{\lambda^2}}{\frac{a^4 (t-t_0)^2}{\lambda^2} + 1} \qquad \frac{a^2 (t-t_0)}{\lambda} = u, \quad du = \frac{a^2}{\lambda}\, dt$$

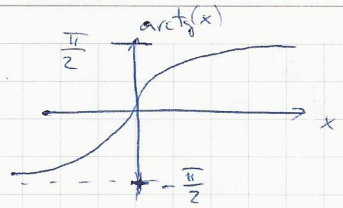
$$d\varphi = \frac{du}{u^2 + 1}$$

$$\varphi - \varphi_0 = \arctan(u) = \operatorname{arctg}\left(\frac{a^2 (t-t_0)}{\lambda}\right)$$

$$\varphi = \varphi_0 + \operatorname{arctg}\left(\frac{a^2 (t-t_0)}{\lambda}\right)$$

Interpretacija:

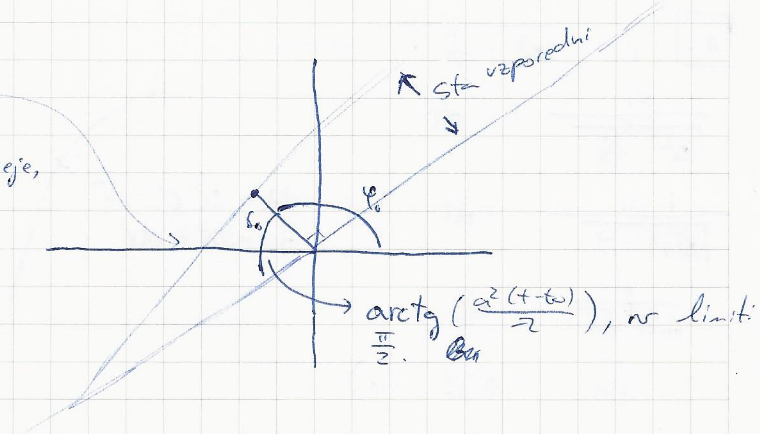
$$\varphi = \arctan\left(\frac{a^2(t-t_0)}{\lambda}\right) + \varphi_0$$



- $t = t_0 \rightarrow \varphi = \varphi_0$

- ko t raste, φ raste. Najprej hitro, potem vedno počasneje, pri $\lim\limits_{t\to\infty} \varphi = \frac{\pi}{2} + \varphi_0$

- ko t pada, φ pada. $\lim\limits_{t\to-\infty} \varphi = \varphi_0 - \frac{\pi}{2}$



- $$\vec{r} = r\cdot\vec{e}_r = \sqrt{a^2(t-t_0)^2 + r_0^2}\cdot\left(\cos\left(\operatorname{arctg}\left(\frac{a^2(t-t_0)}{\lambda}\right)+\varphi_0\right), \sin\left(\operatorname{arctg}\left(\frac{a^2(t-t_0)}{\lambda}\right)+\varphi_0\right)\right)$$

↳ imamo $\vec{r}$, krivuljo parametrizirano z t. V limitah smo pokazali da raste linearno za t, če bi hoteli povsod:

$$l = \int_{t_0}^{t} 1\cdot|\dot{\vec{r}}|\, d\tau \quad \text{, dolžina krivulje.}$$

- Da gre res za ravno črto se lahko prepričamo tudi tako, da prevedemo nazaj na x, y koordinate:

I) $r^2 = a^2(t-t_0)^2 + r_0^2 = x^2 + y^2$

II) $$\varphi = \operatorname{arctg}\left(\frac{a^2(t-t_0)}{\lambda}\right) + \varphi_0 = \operatorname{arctg}\left(\frac{a^2(t-t_0)}{\lambda}\right) + \operatorname{arctg}\left(\overbrace{\operatorname{tg}\varphi_0}^{\frac{y_0}{x_0}}\right) = \operatorname{arctg}\left(\frac{y}{x}\right)$$

$\boxed{\text{II}}$ $$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\left(\frac{a^2(t-t_0)}{\lambda} + \frac{y_0}{x_0}\right)x = y\left(1 - \frac{a^2(t-t_0)}{\lambda}\frac{y_0}{x_0}\right)$$

$$\frac{a^2x_0(t-t_0)+\lambda y_0}{\cancel{\lambda x_0}}\, x = y\,\frac{\lambda x_0 - a^2(t-t_0)y_0}{\cancel{\lambda x_0}}$$

: hmm.. Mogoče pa vseeno ni tako lahko :)

Jan Jenko