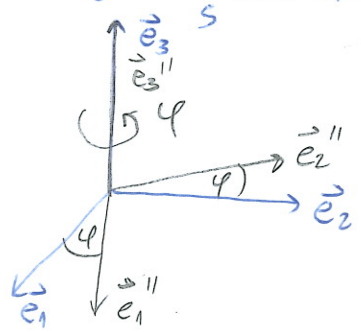
# EULERJEVI KOTI

Od prej: rotacijo telesa opišejo tri neodvisne koordinate. Eulerjevi koti so take možne koordinate, povejo kako je togo telo - lastni sistem togega telesa S' zarotiran glede na fiksen sistem S.

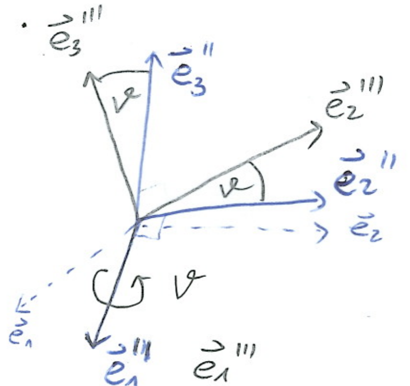
1. Eulerjev kot: $\varphi$ - kot precesije



Prehod med sistemoma opiše:

$$R_\varphi = \begin{pmatrix}\cos\varphi & \sin\varphi & 0\\ -\sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{pmatrix}$$

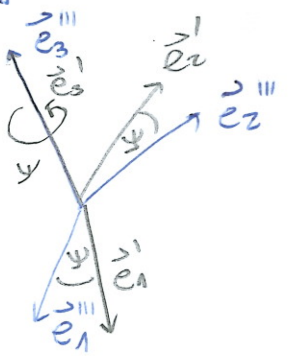
2. Eulerjev kot: $\vartheta$ - kot nutacije



Prehod med sistemoma:

$$R_\vartheta = \begin{pmatrix}1 & 0 & 0\\ 0 & \cos\vartheta & \sin\vartheta\\ 0 & -\sin\vartheta & \cos\vartheta\end{pmatrix}$$

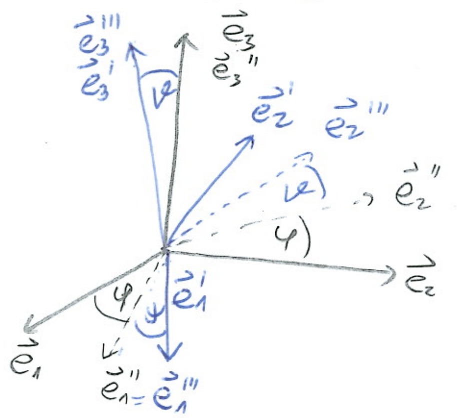
3. Eulerjev kot: $\psi$ - kot zasuka



Prehod med sistemoma:

$$R_\psi = \begin{pmatrix}\cos\psi & \sin\psi & 0\\ -\sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{pmatrix}$$

Celotno:



- Vektor kotne hitrosti:
  - V lastnem sistemu vrtavke S' je: $\vec{\omega}' = \omega_x'\vec{e}_1' + \omega_y'\vec{e}_2' + \omega_z'\vec{e}_3'$
  - z Eulerjevimi koti pa: $\vec{\omega} = \dot\varphi\,\vec{e}_3 + \dot\vartheta\,\vec{e}_1'' + \dot\psi\,\vec{e}_3'$

$\omega_x'$, $\omega_y'$ in $\omega_z'$ dobimo tako, da zapišemo to v S' sistemu:

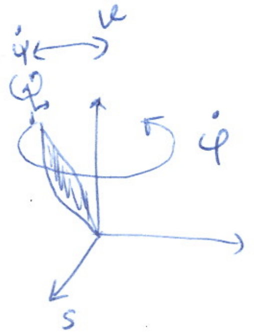
$$\vec{\omega} = R_\psi R_\vartheta R_\varphi \begin{pmatrix}0\\0\\\dot\varphi\end{pmatrix} + R_\psi R_\vartheta \begin{pmatrix}\dot\vartheta\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\\dot\psi\end{pmatrix}$$

$$\vec{\omega} = \vec{e}_1{}'(\dot{\varphi}\sin\vartheta\sin\psi + \dot{\vartheta}\cos\psi) + \vec{e}_2{}'(\dot{\varphi}\sin\vartheta\cos\psi - \dot{\vartheta}\sin\psi) + \vec{e}_3{}'(\dot{\varphi}\cos\vartheta + \dot{\psi})$$

- Kinetična energija pa je:

$$T = \frac{1}{2}(J_1\omega_1'^2 + J_2\omega_2'^2 + J_3\omega_3'^2) \rightarrow J_1 = J_2 = J \ ; \ J_3 = J'$$

$$= \frac{1}{2}\left[J(\dot{\varphi}^2\sin^2\vartheta + \dot{\vartheta}^2) + J'(\dot{\varphi}\cos\vartheta + \dot{\psi})^2\right]$$




$$\omega_x' = \dot{\varphi}\sin\vartheta\sin\psi + \dot{\vartheta}\cos\psi$$
$$\omega_y' = \dot{\varphi}\sin\vartheta\cos\psi - \dot{\vartheta}\sin\psi$$
$$\omega_z' = \dot{\varphi}\cos\vartheta + \dot{\psi}$$