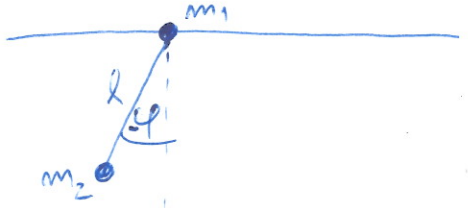
MATEMATIČNO NIHALO NA PREMIČNEM PRITRDIŠČU

Imamo matematično nihalo na vrvici dolžine l z maso m_2. Vrvica je pritrjena na maso m_1, ki je vpeta na ravno horizontalno palico, po kateri se prosto giblje. Kakšno je gibanje sistema, frekvenca nihanja nihala?



- <u>Koordinate</u>:

  x_1 - pozicija mase 1

  x_2, y_2 - koordinati mase 2

- <u>Vezi</u>:

  Masa 2 je na stalni razdalji (l - vrvica) od mase 1.

  $(x_1 - x_2)^2 + y_2^2 = l^2$

- <u>Generalizirane koordinate</u>:

  $3 - 1 = 2$

  Izberem x_1 in φ: kot med vrvico in navpičnico, ki gre ~~sk~~ čez maso m_1.

  $x_2 = x_1 + l\sin\varphi$

  $y_2 = -l\cos\varphi$

- <u>Kinetična energija</u>:

  $T = \frac{1}{2} m_1 (\dot{x}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \big(l^2 \cos^2\varphi\, \dot{\varphi}^2 + l^2 \sin^2\varphi\, \dot{\varphi}^2 + \dot{x}_1^2 + 2\dot{x}_1 l \cos\varphi\, \dot{\varphi} \big)$

  $T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_1^2 + \frac{1}{2} m_2 l^2 \dot{\varphi}^2 + m_2 l \dot{x}_1 \dot{\varphi} \cos\varphi$

- <u>Potencialna energija</u>:

  $V = -m_2 g l \cos\varphi$

- <u>Lagrangean</u>:

  $L = T - V = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_1^2 + \frac{1}{2} m_2 l^2 \dot{\varphi}^2 + m_2 l \dot{x}_1 \dot{\varphi} \cos\varphi + m_2 g l \cos\varphi$

Euler-Lagrangeove enačbe:

x_1: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = 0$

Ker L ne vsebuje x_1, je x_1 ciklična koordinata in $\frac{\partial L}{\partial \dot{x}_1} = p_x$ ohranjena količina.

① $p_x = m_1\dot{x}_1 + m_2\dot{x}_1 + m_2 l\dot{\varphi}\cos\varphi$

φ: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0$

$$m_2 l^2\ddot{\varphi} + m_2 l\ddot{x}_1\cos\varphi + m_2 l\dot{x}_1(-\sin\varphi)\dot{\varphi} - m_2 l\dot{x}_1\dot{\varphi}(-\sin\varphi)$$
$$- m_2 g l(-\sin\varphi) = 0$$

$$m_2 l^2\ddot{\varphi} + m_2 l\ddot{x}_1\cos\varphi \cancel{+ 2m_2 l\dot{x}_1\dot{\varphi}} + m_2 g l\sin\varphi = 0 \quad (*)$$

Izrazimo $\ddot{x}_1$ iz ~~pri~~ ①; odvajamo po času in dobimo:

$\frac{d}{dt}$ ①:

$$(m_1+m_2)\ddot{x}_1 + m_2 l\ddot{\varphi}\cos\varphi - m_2 l\dot{\varphi}^2\sin\varphi = 0$$

$$\ddot{x}_1 = \frac{m_2 l}{m_1+m_2}(\dot{\varphi}^2\sin\varphi - \ddot{\varphi}\cos\varphi)$$

Vstavimo v (*) in dobimo:

$$m_2 l^2\ddot{\varphi} + m_2 l\cos\varphi\frac{m_2 l}{m_1+m_2}(\dot{\varphi}^2\sin\varphi - \ddot{\varphi}\cos\varphi) + m_2 g l\sin\varphi = 0$$

Delimo z $m_2 l^2$:

$$\ddot{\varphi}\left(1 - \frac{m_2}{m_1+m_2}\cos^2\varphi\right) + \dot{\varphi}^2\frac{m_2}{m_1+m_2}\sin\varphi\cos\varphi + \frac{g}{l}\sin\varphi = 0$$

Za majhne odmike iz ravnovesne lege pri $\varphi = 0$:

$$\cos\varphi \approx 1, \quad \sin\varphi \approx \varphi$$

$$\ddot{\varphi}\frac{m_1}{m_1+m_2} + \dot{\varphi}^2\frac{m_2}{m_1+m_2}\varphi + \frac{g}{l}\varphi = 0$$

Pri majhnih odmikih iz ravnovesja so tudi hitrosti majhne, zato zanemarim člen $\dot{\varphi}^2\varphi$, saj je reda $O(\varphi^3)$. Dobimo

$$\boxed{\ddot{\varphi} + \frac{g}{l}\left(1 + \frac{m_2}{m_1}\right)\varphi = 0}$$

Nihanje s frekvenco $\omega_0^2 = \frac{g}{l}\left(1 + \frac{m_2}{m_1}\right)$. Če $m_1 \to \infty$, je $\omega_0^2 = g/l \Rightarrow$ frekvenca pri togega nihala.