POISSONOVI OKLEPAJI

Izračunaj Poissonove oklepaje:

① $\{L^2, \vec{L}\cdot\vec{m}\}$, $\vec{m} = m_x\hat{i} + m_y\hat{j} + m_z\hat{k}$ poljubna smer, m_i = konst

② $\{\vec{p}, \vec{L}\cdot\vec{m}\}$, $\vec{L}$ – vrtilna količina $= L_x\hat{i} + L_y\hat{j} + L_z\hat{k} = \vec{r}\times\vec{p}$

③ $\{f(A), A\}$, $\vec{p}$ – ~~impulz~~ gibalna količina

$A = A(\underline{q}, \underline{p})$ funkcija generaliziranih koordinat in impulzov, f ~~zvezno odve~~ gladka

①

$$L^2 = L_x^2 + L_y^2 + L_z^2 = \sum_i L_i^2$$

$$\vec{L}\cdot\vec{m} = \sum_i L_i m_i$$

$$\{L^2, \vec{L}\cdot\vec{m}\} = \{\sum_i L_i^2, \sum_j L_j m_j\} \underset{\text{linearnost}}{=} \sum_i\sum_j m_j\{L_i^2, L_j\} =$$

$$\underset{\text{produkt}}{=} \sum_{i,j} m_j\, L_i\{L_i, L_j\} + \{L_i, L_j\}L_i = 2\sum_{i,j} m_j L_i\{L_i, L_j\} = 2\sum_{i,j} m_j L_i \varepsilon_{ijk} L_k$$

$\{L_i, L_j\} = \varepsilon_{ijk}L_k$ (**)

$$= 2L_x\sum_{i,j} m_j L_i \varepsilon_{ijx} + 2L_y\sum_{i,j}\varepsilon_{ijy} m_j L_i + 2L_z\sum\varepsilon_{ijz} m_j L_i = (*)$$

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{če } (i,j,k) = (1,2,3), (2,3,1), (3,1,2) \\ -1 & \text{če } (i,j,k) = (3,2,1), (2,1,3), (1,3,2) \\ 0 & \text{če } i=k \text{ ali } k=j \text{ ali } j=i \end{cases}$$

$1,2,3 = x,y,z$

$$(*) = 2L_x\Big(m_x L_x \underbrace{\varepsilon_{xxx}}_{0} + m_y L_y \underbrace{\varepsilon_{yyx}}_{0} + \ldots m_y L_z \underbrace{\varepsilon_{zyx}}_{-1} + m_z L_y \underbrace{\varepsilon_{yzx}}_{1}\Big) +$$

$$2L_y\Big(0 + \ldots 0 + \ldots m_x L_z \underbrace{\varepsilon_{zxy}}_{1} + m_z L_x \underbrace{\varepsilon_{xzy}}_{-1}\Big) +$$

$$2L_z\Big(0 + 0 + \ldots m_x L_y \underbrace{\varepsilon_{yxz}}_{-1} + m_y L_x \underbrace{\varepsilon_{xyz}}_{1}\Big) = 0$$

$$\{L^2, \vec{L}\cdot\vec{m}\} = 0$$

② $\{\vec{p}, \vec{L}\cdot\vec{m}\} = ?$

$$\{\sum_i \vec{e}_i p_i, (\vec{r}\times\vec{p})\cdot\vec{m}\} = \{\sum_i \vec{e}_i p_i, \sum_{j,k,l}\varepsilon_{jkl} m_j q_k p_l\} \underset{\text{linearnost}}{=} \sum_{i,j,k,l}\vec{e}_i m_j\{p_i, q_k p_l\}\varepsilon_{jkl} =$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = \sum_j q_j \vec{e}_j$$

$$\vec{m} = \sum_k m_k \vec{e}_k$$

$$(\vec{r}\times\vec{p})\cdot\vec{m} = \det\begin{pmatrix} m_x & m_y & m_z \\ x & y & z \\ p_x & p_y & p_z \end{pmatrix} = \sum_{j,k,l}\varepsilon_{jkl} m_j q_k p_l$$

$$= \sum_{i,j,k,l} \vec{e}_i m_j \varepsilon_{jkl} \{p_i, q_k p_l\} \underset{\downarrow}{=} \sum_{i,j,k,l} \vec{e}_i m_j \varepsilon_{jkl} \left(\underbrace{\{p_i, q_k\}}_{-\delta_{ik}} p_l + q_k \underbrace{\{p_i, p_l\}}_{0} \right) =$$

ker produkt

$$= - \sum_{j,k,l} \vec{e}_k m_j p_l \varepsilon_{jkl} \underset{\downarrow}{=} \vec{m} \times \vec{p}$$

Vektorski produkt

$$\vec{a} \times \vec{b} = \det \begin{pmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix} = \sum_{ijk} \varepsilon_{ijk} \vec{e}_i a_j b_k$$

③ $\{f(A), A\}$; $A = A(\underline{q}, p)$

$$= \sum_i \left(\frac{\partial f(A)}{\partial q_i} \frac{\partial A}{\partial p_i} - \frac{\partial f(A)}{\partial p_i} \frac{\partial A}{\partial q_i} \right) = \rightarrow \text{razvijemo } f(A) \text{ po Taylorju}$$

$$= \sum_i \frac{\partial A}{\partial p_i} \frac{\partial}{\partial q_i} \left(\sum_m \frac{f^{(m)}(0)}{m!} A^m \right) - \frac{\partial A}{\partial q_i} \frac{\partial}{\partial p_i} \left(\sum_m \frac{f^{(m)}(0)}{m!} A^m \right) \qquad f(A) = \sum_m \frac{f^{(m)}(0)}{m!} A^m$$

$$= \sum_i \sum_m \frac{f^{(m)}(0)}{m!} \left(\frac{\partial A}{\partial p_i} A^{m-1} m \frac{\partial A}{\partial q_i} - \frac{\partial A}{\partial q_i} A^{m-1} m \frac{\partial A}{\partial p_i} \right)$$

$$= 0$$

$\{f(A), A\} = 0$!

Uporabi Einsteinovo konvencijo naslednjič, že znajo iz matije!!