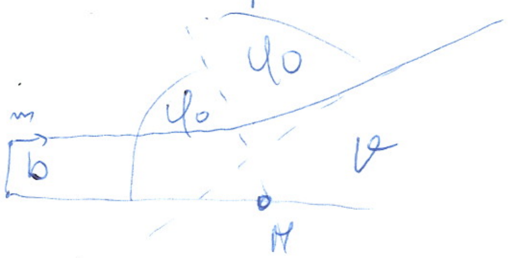
# SIPALNI PRESEK ZA SIPANJE DVEH + NABOJEV

Obravnavajmo sipanje dveh nabitih delcev, ki se čutita s potencialom $V(r) = \frac{\tilde{K}}{r}$. En delec ima $M \gg m$, takoden je $M$ pri miru, drugi pa leti proti njemu iz neskončnosti s hitrostjo $v_0$ in na oddaljenosti $b$. Kakšen je sipalni presek $\sigma(\vartheta)$?

Uporabimo izpeljano formulo $\sigma(\vartheta) = \frac{b}{\sin\vartheta}\frac{db}{d\vartheta}$.
Ker je to $V = \frac{\tilde{K}}{r}$, se bo lažji delec gibal po hiperboli:

$$\frac{1}{r} = \frac{1}{p}\left(1 - \varepsilon\cos(\varphi - \varphi_0)\right) = \frac{1}{\tilde{p}}\left(\varepsilon\cos(\varphi-\varphi_0) - 1\right),$$




$$p = \frac{p_\varphi^2}{mK} = -\frac{p_\varphi^2}{m\tilde{K}} = -\tilde{p} \quad ; \tilde{p} > 0$$

$$\varepsilon = \sqrt{1 + \frac{2E p_\varphi^2}{m\tilde{K}^2}} > 0$$

Sedaj moramo le iz enačbe orbite izrazit $b$ kot funkcijo $\vartheta$!
$(p_\varphi = m v_0 b)$

- Postavimo $\varphi$ tako, da je pri $\varphi = 0$ delec še daleč stran: $r \to \infty$:
  $\frac{1}{r} = 0 = \frac{1}{\tilde{p}}(\varepsilon\cos\varphi_0 - 1) \Rightarrow \cos\varphi_0 = \frac{1}{\varepsilon}$
- $r_{min}$, ko je: $\frac{1}{r_{min}} = max = \frac{1}{\tilde{p}}\left(\varepsilon\cos(\varphi_m - \varphi_0) - 1\right) \Rightarrow \varphi_m = \varphi_0$

Torej je $\vartheta = \pi - 2\varphi_0$:

$$\frac{1}{\varepsilon} = \cos\varphi_0 = \cos\frac{\pi - \vartheta}{2} = \sin\frac{\vartheta}{2}$$

$$\varepsilon^2 - 1 = \sin^{-2}\frac{\vartheta}{2} - 1 = \cot^2\frac{\vartheta}{2} = \frac{2E p_\varphi^2}{m\tilde{K}^2} = \left(\frac{2\,\cancel{m} v_0^2\, m^2 v_0^2 b^2}{2\,\cancel{m}\,\tilde{K}^2}\right) = \left(\frac{b}{\tilde{K}}2E\right)^2$$

$$\cot^2\frac{\vartheta}{2} = \frac{2Eb}{\tilde{k}} \quad \Big| \text{ Totalni diferencial}$$

$$db\,\frac{2E}{\tilde{k}} = -\frac{d\vartheta/2}{\sin^2\vartheta/2}$$

$$\Rightarrow db = -\frac{\tilde{K}}{4E}\frac{d\vartheta}{\sin^2\frac{\vartheta}{2}}$$

Torej je sipalni presek: $\sigma(\vartheta) = \frac{\tilde{K}}{2E}\frac{\cot^2\frac{\vartheta}{2}}{\sin\vartheta}\left|\frac{\tilde{K}}{4E}\frac{1}{\sin^2\frac{\vartheta}{2}}\right|$

$$\sigma(\vartheta) = \frac{\tilde{K}^2}{E^2 8}\frac{\cos\frac{\vartheta}{2}}{\sin\vartheta/2\; 2\sin\frac{\vartheta}{2}\cos\frac{\vartheta}{2}}\frac{1}{\sin^2\frac{\vartheta}{2}} = \left(\frac{\tilde{K}}{4E}\right)^2\frac{1}{\sin^4\frac{\vartheta}{2}}$$

To je znana formula za Rutherfordovo sipanje.

Totalni sipalni presek:

$$\sigma_{TOT} = \int_0^{\pi}\sigma(\vartheta)\,2\pi\sin\vartheta\,d\vartheta = \left(\frac{\tilde{K}}{4E}\right)^2\int_0^{\pi}4\pi\cos\frac{\vartheta}{2}\sin\frac{\vartheta}{2}\frac{1}{\sin^4\frac{\vartheta}{2}}\,d\vartheta$$

$$= \infty \quad \text{saj} \quad \frac{1}{\sin\frac{\vartheta}{2}} = \text{divergira}$$

To je posledica počasnega padanja potenciala, tako da se tudi neskončno oddaljeni delci odklonijo in je zato efektivna površina tarče neskončna.