1.

a) $S = \frac{1}{3}\begin{pmatrix} 1+2e^{i\varphi} & 1-e^{i\varphi} & 1-e^{i\varphi} \\ 1-e^{i\varphi} & 1+2e^{i\varphi} & 1-e^{i\varphi} \\ 1-e^{i\varphi} & 1-e^{i\varphi} & 1+2e^{i\varphi} \end{pmatrix}$

$S^{+}S = \hat{1}$

b) $\varphi = 0$ $\quad S = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$; se odbije

$\varphi = \pi$ $\quad S = \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$; z verjetnostjo $1/9$ se odbije, z verjetnostjo $4/9$ se prepusti (v vsakega od vodnikov)

c) $G_{23} = \frac{2e^2}{h}\left(-\delta_{23} + |S_{23}|^2\right) = \begin{pmatrix} -8/9 & 4/9 & 4/9 \\ 4/9 & -8/9 & 4/9 \\ 4/9 & 4/9 & -8/9 \end{pmatrix}$

d) $1: V$, $2: 0$; $3: V_x$

$I_3 = G_0\left(-\frac{8}{9}V_x + \frac{4}{9}V\right) = 0 \qquad V_x = V/2.$

$I_1 = G_0\left(-\frac{8}{9}V + \frac{4}{9}\frac{V}{2}\right) \Rightarrow \underline{I = \frac{6}{9} G_0 V}$

2.

a) $H^2 = c^2 \sum_{ij} \sigma_i q_i \sigma_j q_j = c^2 \frac{1}{2} \sum \underbrace{\{\sigma_i , \sigma_j\}}_{2\delta_{ij}} q_i q_j$

$= c^2 \sum_i q_i^2$

$H^2 |\psi_{\vec{q}}\rangle = (\varepsilon_{\vec{q}})^2 |\psi_{\vec{q}}\rangle$

$\varepsilon_q = \pm c \sqrt{\sum_i q_i^2} = \pm c |\vec{q}|$

b) $H = \tilde{c} \begin{pmatrix} & a^+ \\ a & \end{pmatrix} + c \sigma_z q_z$

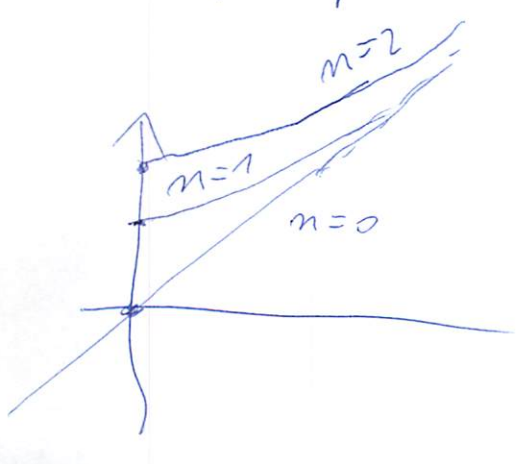
$H = \underbrace{\tilde{c}\, a (\sigma_x + i \sigma_y) + \tilde{c}\, a^+ (\sigma_x - i \sigma_y)}_{H_0} + \underbrace{c \sigma_z q_z}_{H_1}$

$H^2 = H_0^2 + H_1^2 + \underbrace{H_0 H_1 + H_1 H_0}_{= 0}$ (ker npr. $\sigma_x \sigma_z + \sigma_z \sigma_x = 0$

$H^2 = H_0^2 + c^2 q_z^2$

c)

$\varepsilon_n = \pm \sqrt{\tilde{c}^2 n^2 + c^2 q_z^2}$



3. $H = J \vec{S}_1 \cdot \vec{S}_2$; vzamemo $\hbar = 1$.

a)

lastna stanja:

singlet:

$\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

$E = -\frac{3}{4}J$

triplet

$|11\rangle$

$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle$

$|01\rangle$

$E = \frac{1}{4}J$

$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

če H deluje na lastna stanja, dobimo kvečjemu faze.

$|01\rangle = \frac{1}{2}(|01\rangle - |10\rangle + |01\rangle + |10\rangle)$

$|01\rangle(t) = \frac{1}{2}\left((|01\rangle - |10\rangle)e^{+i\frac{3}{4}Jt} + (|01\rangle + |10\rangle)e^{-i\frac{Jt}{4}}\right)$

$= e^{-iJt/4}\left[|01\rangle\frac{(1+e^{iJt})}{2} + |10\rangle\frac{1-e^{iJt}}{2}\right]$

$|10\rangle(t) = e^{-iJt/4}\left[|01\rangle\frac{1-e^{iJt}}{2} + |10\rangle\frac{1+e^{iJt}}{2}\right]$

za $t = t_0 = \frac{\pi}{J}$ je $|01\rangle \longleftrightarrow |10\rangle$ (do faze)

$|00\rangle(t) = e^{-i\frac{Jt}{4}}|00\rangle$ $\quad |11\rangle(t) = e^{-i\frac{Jt}{4}}|11\rangle$

b)

$H = J\vec{S}_1\vec{S}_2 + J/4 \cdot \hat{1}$ (v potencial, da se znebimo faze)

J le prižgemo za t_0. $\Rightarrow$ SWAP.

c) $H = B(t)\, S_z^{(2)}$. z $(S_z^{(1)} + S_z^{(2)})$ in $S_z^{(1)} - S_z^{(2)}$ odpravimo faze