2. Homework Assignment from Nanophysics, 23.5.2024

1. *Quantum Mechanical Description of the LC Circuit.*

a) The LC circuit consists of an inductor with inductance L and a capacitor with capacitance C. Sketch the circuit, label the voltages on the elements, the charge on the capacitor q, and choose the direction of the current I. Find the differential equation that the charge on the capacitor satisfies!

b) The LC circuit can be treated within the framework of Lagrangian formalism, where the role of the dynamic variable x is taken by q. Find the Lagrangian function $\mathcal{L}(q, \dot{q})$ and verify that the Lagrangian equations give you the equation of motion from the previous point.

c) Write the Hamiltonian function $H = H(q, p)$, where $p = \partial\mathcal{L}/\dot{q}$. Write the Hamiltonian equations of motion.

d) Quantize the problem! Write the commutation relation between q and p.

e) Equivalently, the LC circuit can also be treated if we use I as the dynamic variable. In this case, the role of the potential term is taken by $LI^2/2$, and the role of the kinetic term by $CU_C^2/2$, where the voltage on the capacitor $U_C = \pm U_L = \pm L(\dot{I})$. (The signs depend on the choice of current direction and the choice of the sign of the charge on the capacitor). Repeat the previous steps for this choice!

2. *Quantum computers are freely available on the IBM Quantum Platform. Check the operation of quantum computers!*

a) Create a user account and review the usage examples. Quantum algorithms can be managed on the platform via the Python interface Qiskit or the graphical interface Composer.

b) Prepare a circuit that sequentially performs a certain number of NOT gates on an initial bit (10 or more). Run the circuit on a quantum computer! With what probability did you get the expected answer? Attach a picture of the circuit (or the algorithm printout) and the result!

c) Prepare a circuit that executes the Deutsch algorithm and run it on a quantum computer. With what probability did you get the expected answer? Attach a picture of the circuit (or the algorithm printout) and the result!