Kvantna računalništva

N

q o o o o o o

$0, 1$

stanje 2^N različnih stanj

$$|\psi\rangle = \sum_n c_n |n\rangle$$

$|0\rangle = |0000 \ldots 0\rangle$
$|1\rangle = |0 \ldots 1\rangle$
$|2\rangle = |000 \ldots 10\rangle$

c_n 10 biti

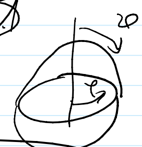
$10 \cdot 2^N$ bitov za zapis kvantnega stanja

Enota kv. informacije kubit
dvonivojski kv. sistem

$\uparrow, \downarrow$

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + \sin\frac{\theta}{2} e^{i\varphi}|\downarrow\rangle$$

$$\langle\psi|\vec{\sigma}|\psi\rangle = \begin{pmatrix} \cos\varphi \sin\theta \\ \sin\varphi \sin\theta \\ \cos\theta \end{pmatrix} = \begin{pmatrix} \langle\sigma_x\rangle \\ \langle\sigma_y\rangle \\ \langle\sigma_z\rangle \end{pmatrix}$$



(1) $$|\psi\rangle = \hat{R}_z(\varphi)\hat{R}_y(\theta)|\uparrow\rangle$$

$$\hat{R}_{\hat{n}}(\alpha) = e^{-i\frac{\hat{n}\cdot\vec{\sigma}}{2}\alpha} = \cos\frac{\alpha}{2}\,\hat{1} - i\hat{n}\cdot\vec{\sigma}\,\sin\frac{\alpha}{2}$$

D.n. pokaži zvezo (1).

Izrek o nekloniranju (NO-CLONING TH.)

$|\psi\rangle$ ..... $|\psi\rangle|\psi\rangle|\psi\rangle\ldots$

$|e\rangle$ .. znano zač. stanje

Ali $\exists\, U$?

$$U|\psi\rangle|e\rangle = |\psi\rangle|\psi\rangle$$

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$$

$$U(a|\uparrow\rangle + b|\downarrow\rangle)|e\rangle = (a|\uparrow\rangle + b|\downarrow\rangle)(a|\uparrow\rangle + b|\downarrow\rangle)$$

$\parallel$

$$Ua|\uparrow\rangle|e\rangle + Ub|\downarrow\rangle|e\rangle = a|\uparrow\rangle|\uparrow\rangle + b|\downarrow\rangle|\downarrow\rangle$$

$b = 0 \quad a = 1$

Operacije na kubitih

- enokubitne operacije

$U|\psi\rangle$ ... elek. potencial V

$U|\psi\rangle$ ... elek. potencial V

$$H = \hat{1}\, eV - \mu_B \vec{B}\cdot\vec{\sigma}$$

$$\boxed{U = e^{-iHt}}$$

$$\boxed{\begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \ \dots \ NOT = e^{i\pi/2} R_{\hat{x}}(\pi)}$$

σ_x matrika $\quad \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$R_{\hat{n}}(\varphi) = e^{-i\frac{\vec{n}\vec{\sigma}}{2}\varphi} = \cos\tfrac{\varphi}{2}\,\hat{1} - i\hat{n}\cdot\vec{\sigma}\,\sin\tfrac{\varphi}{2}$$

$$|\psi(t)\rangle = e^{-i\frac{Ht}{\hbar}}|\psi\rangle$$

$R_{\hat{x}}(\pi) = -i\sigma_x \qquad \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} = NOT = e^{i\pi/2} R_x(\pi)$

$\varphi/2 = -\frac{\mu_B B t}{\hbar} = \pi/2 \qquad \vec{B} = B\cdot\hat{x}$

Kvantna vezja

$2|0\rangle + 3|1\rangle$  $2|1\rangle + 3|0\rangle \qquad |\uparrow\rangle = |0\rangle \quad |\downarrow\rangle = |1\rangle$

Hadamardove operacije

$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad H\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

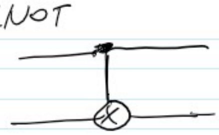
$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$H = \hat{R}_{1,0,1}(\pi)\, e^{i\pi/2} = \left(\cos\tfrac{\pi}{2}\,\hat{1} - i\hat{n}\vec{\sigma}\,\sin\tfrac{\pi}{2}\right) e^{i\pi/2}$

Fazna op $U_\varphi = e^{i\varphi}\hat{1} \qquad = \left(\frac{-i}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\cdot 1\right) e^{i\pi/2}$

večkubitne operacije

CNOT



CNOT: izvedi NOT na (2), če (1) = $|1\rangle$

$|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$$CNOT = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{bmatrix} =$$

$$CNOT|xy\rangle = |x\rangle|x \oplus y\rangle$$

$0 \oplus 0 = 0$

$1 \oplus 0 = 1$

$0 \oplus 1 = 1$

$1 \oplus 1 = 0$

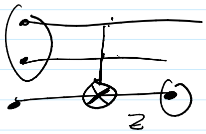
$$CNOT = R_y^{(2)}(\pi/2) \cdot R_z^{(1)}(\pi/2)\, R_z^{(2)}(\pi/2)\, U_{zz}(\pi/4)\, R_y(-\pi/2)\, U_y(\pi/4)$$

$H = J\, \sigma_{1z}\, \sigma_{2z} \qquad U_{zz}(\varphi)$

$\uparrow\uparrow, \downarrow\downarrow \ldots J \qquad \uparrow\downarrow, \downarrow\uparrow \ldots -J$

Toffolijeva vrata

C-CNOT



obrni $|3\rangle$, če $|1\rangle \,\&\, |2\rangle$ $|1\rangle$

$T|xyz\rangle = T|xy\,(z \oplus xy)\rangle$

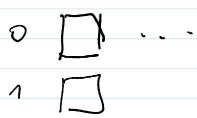
$z = |1\rangle \;\ldots\;$ NAND $\; xy$

Kvantni algoritmi

Deutschev algoritem

$\ldots \boxed{f} \ldots$

	črna	škatla		
f_1	$0,1$	$\to 0$		konst.
f_2	$0,1$	$\to 0,1$	$\hat{1}$	uravnotežene
f_3	$0,1$	$\to 1,0$	$\otimes$	uravnotežene
f_4	$0,1$	$\to 1$		konst.



? ali je f konst, ali je f uravnotežen?

Deutsch – Jozsa $\;\underbrace{01007}_{n} \to \begin{cases} 0 \\ 1 \end{cases}$

kv. vezje

$|xy\rangle \xrightarrow{f} |x\, f(x) \oplus y\rangle$

$\uparrow 0$

$|x0\rangle \xrightarrow{f} |x\, f(x)\rangle$

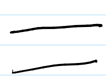
	črna	škatla
f_1	$0,1$	$\to 0$
f_2	$0,1$	$\to 0,1$
f_3	$0,1$	$\to 1,0$
f_4	$0,1$	$\to 1$

$f_1\; |00\rangle \to |00\rangle$

$f_1\; |01\rangle \to |01\rangle$

$f_1\; |10\rangle \to |10\rangle$

$f_1\; |11\rangle \to |11\rangle$

f_1  $= \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

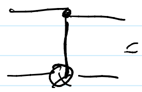
$f_2: |00\rangle \to |00\rangle$

$|01\rangle \to |01\rangle$

$|10\rangle \to |11\rangle$

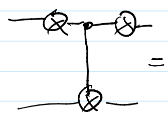
 $= \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \end{pmatrix}$

f_2 : $|00\rangle \to |00\rangle$
$|01\rangle \to |01\rangle$
$|10\rangle \to |11\rangle$
$|11\rangle \to |10\rangle$



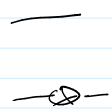
$= \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{pmatrix}$

f_3 : $|00\rangle \to |01\rangle$
$|01\rangle \to |00\rangle$
$|10\rangle \to |10\rangle$
$|11\rangle \to |11\rangle$

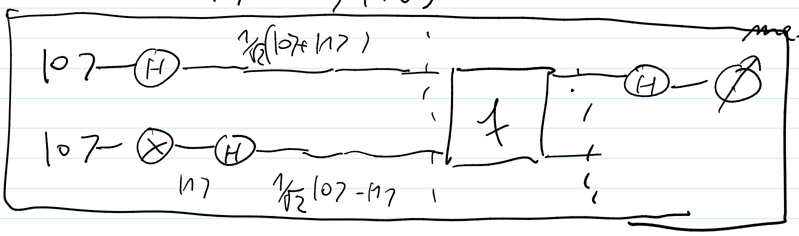


$= \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

f_4 : $|00\rangle \to |01\rangle$
$|01\rangle \to |10\rangle$
$|10\rangle \to |11\rangle$
$|11\rangle \to |10\rangle$



$= \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & 1 \\ & & 1 & \end{pmatrix}$



meritev $|0\rangle, |1\rangle$

$$|\psi\rangle = \frac{1}{2}\left(|00\rangle - |01\rangle + |10\rangle - |11\rangle\right)$$

$f: |\psi\rangle$

f_1 $\frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) = \psi_1$

f_2 $\frac{1}{2}(|00\rangle - |01\rangle + |11\rangle - |10\rangle) = \psi_2$

f_3 $\frac{1}{2}(|01\rangle - |00\rangle + |10\rangle - |11\rangle) = \psi_3$

f_4 $\frac{1}{2}(|01\rangle - |00\rangle + |11\rangle - |10\rangle) = \psi_4$

$$\psi_1 = (|0\rangle + |1\rangle)(|0\rangle - |1\rangle)\tfrac{1}{2} = -\psi_4$$

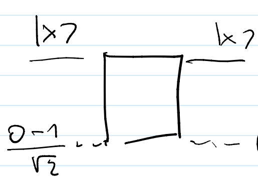
$$\psi_2 = (|0\rangle - |1\rangle)(|0\rangle - |1\rangle)\tfrac{1}{2} = -\psi_3$$

$$H(|0\rangle + |1\rangle)\tfrac{1}{\sqrt{2}} = |0\rangle$$

$$H(|0\rangle - |1\rangle)\tfrac{1}{\sqrt{2}} = |1\rangle$$

na (1) ... $\pm|0\rangle$ za f_1, f_4 ↑

$\pm|1\rangle$ za f_2, f_3 ↓



$(-1)^{f(x)} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$\frac{|0\rangle - |1\rangle}{\sqrt{2}} \otimes \frac{|1\rangle - |0\rangle}{\sqrt{2}} = (-1)\frac{|0\rangle - |1\rangle}{\sqrt{2}}$

f_1, f_4

$\underbrace{\frac{|0\rangle + |1\rangle}{\sqrt{2}} \; \frac{|0\rangle - |1\rangle}{\sqrt{2}}}^{\psi_I} \xrightarrow{f} \pm \psi_I$

f_2, f_3 $\xrightarrow{f} \pm \frac{|0\rangle - |1\rangle}{\sqrt{2}} \; \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

Računs ku [illegible]

τ_2, t_3 $\quad \xrightarrow{\pm} \frac{|0\rangle - |1\rangle}{\sqrt{2}} \quad \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

Računska kompleksnost

število. N $\log_2 N$ bitov

n .... bitov

101
010 $\quad n$ ...

$n^2 \qquad P \sim n^M$

faktorizacija $\qquad NP \sim e^n$

$1137 = \sqrt{N}$

NP: rezultat preizkusimo v P

Groverjev algoritem

iskanje v neurejeni bazi N

kvantno $\sqrt{N}$

Shor algoritem

'96 algoritem za faktorizacijo

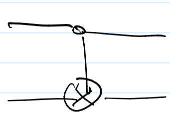
$15 = 5\,3$

2001

Nielsen – Chuang

John Preskill

Vaja: Realizacija CNOT z Isingovo int.



$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & & 1 \\ & & 1 & \end{pmatrix} \qquad |00\rangle\ |01\rangle\ |10\rangle\ |11\rangle$

$$CNOT = \left(\frac{1+\sigma_z^{(1)}}{2}\right) 1^{(2)} + \left(\frac{1-\sigma_z^{(1)}}{2}\right)\sigma_x^{(2)}$$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$$\sigma_x^{(2)} = R_y(\pi/2)\, \sigma_z^{(2)}\, R_y(-\pi/2)$$

$$\mathrm{CNOT} = R_y(\pi/2)\left[\left(\frac{1+\sigma_z^{(1)}}{2}\right) 1^{(2)} + \frac{1-\sigma_z^{(1)}}{2}\sigma_z^{(2)}\right] R_y(-\pi/2)$$

$$M = [\ ] = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix} = R_z^{(1)}(\varphi_1)\, R_z^{(2)}(\varphi_2)\, U_\varphi(\varphi_0)\, U_{zz}(\varphi_3)$$

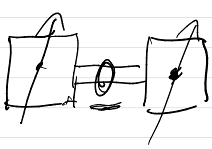
$$H = -J\sigma_z^{(1)}\sigma_z^{(2)}$$

$$U = e^{-\frac{iHt}{\hbar}} = e^{i\underbrace{\frac{Jt}{\hbar}}_{\varphi_3}\sigma_z^{(1)}\sigma_z^{(2)}} = \begin{bmatrix} e^{i\varphi_3} & & & \\ & e^{-i\varphi_3} & & \\ & & e^{-i\varphi_3} & \\ & & & e^{i\varphi_3} \end{bmatrix}$$

$$R_z(\varphi) = \cos\varphi/2\ 1 - i\sigma_z \sin\varphi/2$$
$$= \begin{bmatrix} \cos\varphi/2 - i\sin\varphi/2 & \\ & \cos\varphi/2 + i\sin\varphi/2 \end{bmatrix} = \begin{bmatrix} e^{-i\varphi/2} & \\ & e^{i\varphi/2} \end{bmatrix}$$

$$R_z^{(1)} = \begin{pmatrix} e^{-i\varphi/2} & & & \\ & e^{-i\varphi/2} & & \\ & & e^{i\varphi/2} & \\ & & & e^{i\varphi/2} \end{pmatrix} \qquad R_z^{(2)} = \begin{pmatrix} e^{-i\varphi/2} & & & \\ & e^{i\varphi/2} & & \\ & & e^{-i\varphi/2} & \\ & & & e^{i\varphi/2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} = R_z^{(1)}(\pi/2)\, R_z^{(2)}(\pi/2)\, U_\varphi(\pi/4)\, U_{zz}(\pi/4)$$



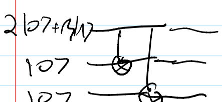
Kvantno odpravljanje napak

$\alpha|0\rangle + \beta|1\rangle$ logični kubit ~~$(\alpha|0\rangle+\beta|1\rangle)(\alpha|0\rangle+\beta|1\rangle)(\alpha|0\rangle+\beta|1\rangle)$~~

$\alpha|000\rangle + \beta|111\rangle$

$\Uparrow$

$\alpha|0\rangle + \beta|1\rangle$



$\alpha|000\rangle + \beta|111\rangle$

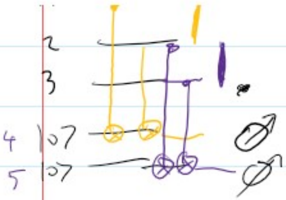
$|000\rangle \xrightarrow[\otimes]{\text{NAPAKA}} |010\rangle$

$\alpha|000\rangle + \beta|111\rangle$

$\otimes \Downarrow$

$\alpha|010\rangle + \beta|101\rangle$

$|000\rangle \to |001\rangle$



$|000\rangle \to |001\rangle$

$\searrow |110\rangle$

0	0	1	1
0	1	0	1
	↓		
nap. X	3	1	2

VAJA: SWAP (IN CNOT) VRATA V DQD

$0 \dots 0 \qquad 2 \dots 2\varepsilon_1 + U$

$1 \dots \varepsilon_1$

1 2

$$H_0 = \boxed{\varepsilon_1 n_1 + U n_{1\uparrow} n_{1\downarrow}} + \varepsilon_2 n_2 + U n_{2\uparrow} n_{2\downarrow} \qquad -U < \varepsilon_1 < 0 \dots 1el.$$

t'

$\varepsilon_1 = \varepsilon_2 = \varepsilon$

$$H = H_0 + H'$$

$$H' = -t' \sum_{\sigma} \left(c_{1\sigma}^+ c_{2\sigma} + h.c. \right)$$

$t' = 0 \qquad\qquad c_{2\sigma}^+ c_{1\sigma}$

$|2 2' \rangle = c_{1\sigma}^+ c_{2\sigma'}^+ |0\rangle \quad \dots \quad E = 2\varepsilon$

$|\ell\rangle \dots |\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$ deg. osnovna stanja

$|h\rangle \dots |02\rangle = c_{2\uparrow}^+ c_{2\downarrow}^+ |0\rangle \qquad |20\rangle = c_{1\uparrow}^+ c_{1\downarrow}^+ |0\rangle$

vzbujena $E = 2\varepsilon + U$

$\langle \ell | H' | \ell' \rangle \dots 0$

2 red. $$H_{ij}^{eff} = \frac{1}{2} \sum_h \left[\frac{\langle i|H'|h\rangle \langle h|H'|j\rangle}{\underbrace{E_i - E_h}_{-U}} + \frac{\langle j|H'|h\rangle\langle h|H'|i\rangle}{\underbrace{E_j - E_h}_{-U}} \right]$$

$ij \in \ell$

$$H' = -t' \left(c_{1\uparrow}^+ c_{2\uparrow} + c_{1\downarrow}^+ c_{2\downarrow} + c_{2\uparrow}^+ c_{1\uparrow} + c_{2\downarrow}^+ c_{1\downarrow} \right)$$

$$H' |\uparrow\uparrow\rangle = \underbrace{H' c_{1\uparrow}^+ c_{2\uparrow}^+ |0\rangle}_{=0} = \qquad c_{2\uparrow}^+ c_{2\uparrow}^+ = 0$$

$$H' |\downarrow\downarrow\rangle = 0$$

$$H' |\uparrow\downarrow\rangle = H' c_{1\uparrow}^+ c_{2\downarrow}^+ |0\rangle = \left(-t' \sum_\sigma c_{1\sigma}^+ c_{2\sigma} + h.c. \right) c_{1\uparrow}^+ c_{2\downarrow}^+ |0\rangle$$

$$= -t' \left(|20\rangle + |02\rangle \right) \qquad -t' c_{1\downarrow}^+ c_{2\downarrow} c_{1\uparrow}^+ c_{2\downarrow}^+ |0\rangle$$

$$-t' |20\rangle = -t' c_{1\downarrow}^+ c_{1\uparrow}^+ (-1) |0\rangle = -t' c_{1\uparrow}^+ c_{1\downarrow}^+ |0\rangle$$

$$H' |\downarrow\uparrow\rangle = t' \left(|20\rangle + |02\rangle \right) \qquad \{ c_i, c_j^+ \} = \delta_{ij} \qquad c_{2\downarrow} c_{2\downarrow}^+ |0\rangle = 1 - c_{2\downarrow}^+ c_{2\downarrow} |0\rangle$$

$$\langle 20 | H' | \uparrow\downarrow \rangle = \langle 02 | H' | \uparrow\downarrow\rangle = -t'$$

$$\langle 20 | H' | \downarrow\uparrow \rangle = \langle 02 | H' | \downarrow\uparrow \rangle = t'$$

$$H_{ij}^{eff} = \frac{t'^2}{-U} \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = -2 \frac{t'^2}{U} \begin{array}{c} \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ & 1 & -1 & \\ & -1 & 1 & \\ & & & 0 \end{array}\right] \end{array} \begin{array}{c} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\uparrow \\ \downarrow\downarrow \end{array}$$

$\in |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle$

$$H^{eff} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right) = 0 \qquad H^{eff} |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle = 0 \quad \text{triplet}$$

$$H^{eff} \underbrace{\left(\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \right)}_{\text{SINGLET}} = -\frac{4t'^2}{U} \qquad -J \left(\vec{S}_i \cdot \vec{S}_j + const \right) = H^{eff}$$

$$|\Psi(t)\rangle = \underbrace{e^{-i\frac{H^{eff}}{\hbar}t}}_{\hat{U}} |\Psi\rangle(t=0)$$

$$|\uparrow\downarrow\rangle(t) = \hat{U}\, |\uparrow\downarrow\rangle(0) = \hat{U}\, \frac{|\uparrow\downarrow\rangle + |\uparrow\downarrow\rangle + |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{2}$$

$$= e^{-i0}\, \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{2} + e^{i\frac{J t}{\hbar}}\, \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{2}$$

$$|\uparrow\downarrow\rangle(t) = \frac{1+e^{i\frac{Jt}{\hbar}}}{2}|\uparrow\downarrow\rangle + \frac{1-e^{i\frac{Jt}{\hbar}}}{2}|\downarrow\uparrow\rangle$$

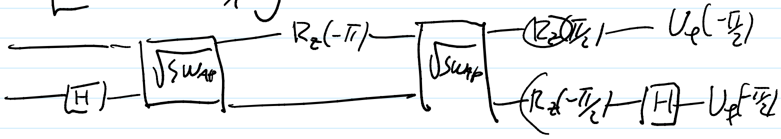
$$|\downarrow\uparrow\rangle(t) = \frac{1-e^{iJt}}{2}|\uparrow\downarrow\rangle + \frac{1+e^{iJt}}{2}|\downarrow\uparrow\rangle \qquad t = \frac{\pi\hbar}{J} = t_0$$

$$|\uparrow\downarrow\rangle \to |\downarrow\uparrow\rangle$$

$$U = \begin{array}{c} \begin{bmatrix} 1 & & & \\ & \frac{1+e^{i\frac{Jt}{\hbar}}}{2} & \frac{1-e^{i\frac{Jt}{\hbar}}}{2} & \\ & \frac{1-e^{i\frac{Jt}{\hbar}}}{2} & \frac{1+e^{i\frac{Jt}{\hbar}}}{2} & \\ & & & 1 \end{bmatrix} \end{array} \begin{array}{c} \uparrow\uparrow \\ \uparrow\downarrow \\ \downarrow\uparrow \\ \downarrow\downarrow \end{array} \qquad U\left(t = \frac{\pi\hbar}{J}\right) = \begin{bmatrix} 1 & & & \\ & 0 & 1 & \\ & 1 & 0 & \\ & & & 1 \end{bmatrix} := SWAP$$

$$t = t_0/2 \qquad \begin{bmatrix} 1 & & & \\ & \frac{1+i}{2} & \frac{1-i}{2} & \\ & \frac{1-i}{2} & \frac{1+i}{2} & \\ & & & 1 \end{bmatrix} = \sqrt{SWAP}$$

CNOT =



KVANTNI HARDWARE

di Vincenzojevi kriteriji
- inicializacija
- manipulacija
- read-out (meritev)
- koherenca
- skalabilnost

logični kubiti   fizični kubiti

$$H = H_{qc} + H_{okolica} + H'$$

$$\Psi \quad \psi \quad \chi$$

$$\Psi = \underbrace{|\psi\rangle}_{1}|\chi\rangle = (\alpha|0\rangle + \beta|1\rangle)|\chi\rangle$$

$$= \dots = \frac{\alpha|0\rangle|\chi_0\rangle}{2} + \beta|1\rangle|\chi_1\rangle$$

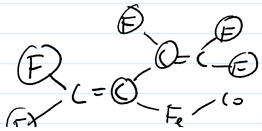
čisto stanje

$$\rho_{qc} = |\psi\rangle\langle\psi|$$

$$\rho_c = |\Psi\rangle\langle\Psi| \Rightarrow \mathrm{Tr}_{|\chi\rangle}\, \rho_c \begin{cases} 1; & (\alpha|0\rangle + \beta|1\rangle)(\alpha\langle 0| + \beta\langle 1|) \\ 0; & \text{mešano, nekoherentno st.} \end{cases}$$

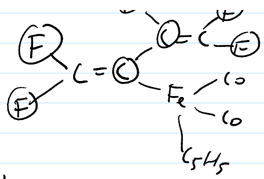
Nekaj arhitektur

- spini v kv. pikah
  t_c ... koherenčni čas
  t_m ... manipulacij.
- NMR



- NMR

$Cl-{}^{13}C(Cl)(Cl)-H^{S=1/2}$ Deutsch



$|00\rangle\langle 00|$

$$\rho_1 = \begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & d \end{bmatrix} \qquad \rho = \begin{bmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$$

$$\rho_2 = \begin{bmatrix} a & & & \\ & c & & \\ & & d & \\ & & & b \end{bmatrix} \qquad \rho_3 = \begin{bmatrix} a & & & \\ & d & & \\ & & b & \\ & & & c \end{bmatrix}$$

$$\frac{\rho_1+\rho_2+\rho_3}{3} = \begin{bmatrix} 3a & & & \\ & b+c+d & & \\ & & b+c+d & \\ & & & b+c+d \end{bmatrix} \frac{1}{3}$$

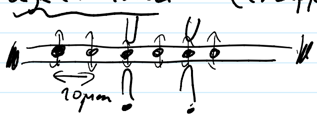
$$= \begin{bmatrix} 3a & & & \\ & 1-a & & \\ & & 1-a & \\ & & & 1-a \end{bmatrix} \frac{1}{3}$$

$$= \frac{1}{3}\left(\begin{bmatrix} 4a-1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix} + \begin{bmatrix} 1-a & & & \\ & 1-a & & \\ & & 1-a & \\ & & & 1-a \end{bmatrix} \right)$$

$$= \underbrace{\frac{4a-1}{3}|00\rangle\langle 00|}_{\rho_1} + \underbrace{\frac{1-a}{3}\mathbb{1}}_{\rho_2}$$

$$\langle Z_z \rangle = \mathrm{Tr}(\rho\, \sigma_z) = \mathrm{Tr}(\rho_1 \sigma_z)$$

ujeti ioni (trapped ion)

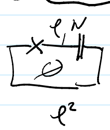


$|0\rangle \to |1\rangle$ za oba hkrati

$|00\rangle$

$$\begin{bmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{bmatrix}$$

Josephsonovi kubiti

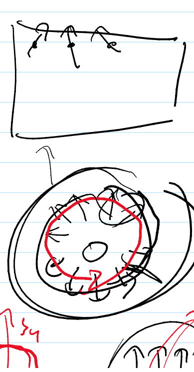


$$H = \frac{E_c}{2}\left(-\frac{\partial^2}{\partial\varphi^2}\right) + E_J\underbrace{(1-\cos\varphi)}_{\varphi^2/2}$$

arXiv : 1905.13641

Topologija v trdni snovi

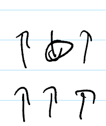
topološka klasifikacija defektov

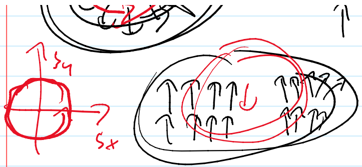


A ... $\vec{s}(\vec{r}) \in S$

$f\ A \to S$

$f(\vec{r}) = \vec{s}$

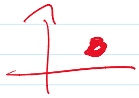




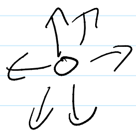
$$\Pi_1(S_1) = \mathbb{Z}$$

$\vec{S}$ ... pri tej poti

$\vec{S} = (S_x, S_y, 0)$

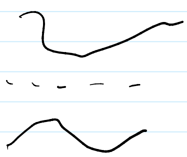


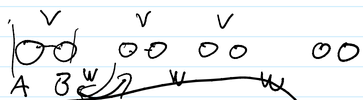
$$\Pi_1(S_2) = \text{identitet}$$



$$\Pi_2(S_2) = \mathbb{Z}$$

topološki izolatorji





$$H = v\sum_i \left(|A_i\rangle\langle B_i| + |B_i\rangle\langle A_i|\right)$$

$$w\sum_i \left(|A_{i+1}\rangle\langle B_i| + \sum_i |B_i\rangle\langle A_{i+1}|\right)$$

$$\left(|A\rangle, |B\rangle\right) |i\rangle$$

$$|k\rangle = \sum_i e^{iki} |i\rangle$$

$$H = \sum_k |k\rangle\langle k| \left[v\left(|A\rangle\langle B| + |B\rangle\langle A|\right) + w\, e^{ik} |A\rangle\langle B| + w\, e^{-ik} |B\rangle\langle A| \right]$$

$$|A i\rangle = \sum_k e^{-iki} |A k\rangle$$

$$e^{ik} = \cos k + i \sin k$$

$$\boxed{H = \sum_k \left(v\sigma_x + w\cos k\,\sigma_x + w \sin k\,\sigma_y\right)} |k\rangle\langle k|$$

$$i\begin{pmatrix} & 1 \\ -1 & \end{pmatrix} = \sigma_y$$

$$H(k) = (v + w\cos k)\sigma_x + w\sin k\,\sigma_y$$

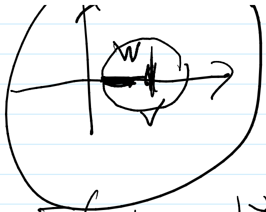
$$= \vec{d}(k)\cdot\vec{\sigma}$$

Topo.

TRIV

$|\vec{d}|$

TOPO. TRIV.

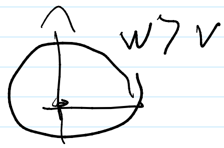


$d_z = 0$

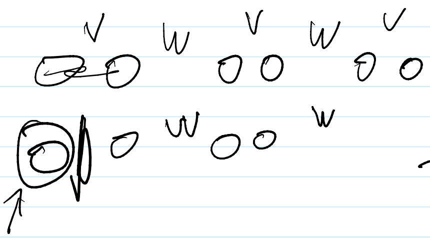
$E(k) = \pm|\vec{d}|$

$\vec{d}(k) = 0$ pri nekem k končna

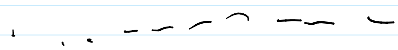
TOPOLOŠK

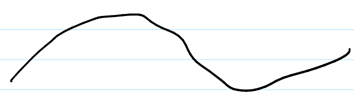


ovojno število $\mathbb{Z}$



ROBNO STANJE pri $E=0$





$$|\psi\rangle = |\psi_{VAL\,PAS}\rangle\, |\psi_{ROBNA}\rangle$$

Majorana

$$\gamma = c + c^{\dagger}$$