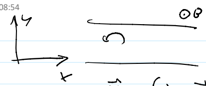
Nanofizika-Kvantni Hallov pojav

petek, 20. marec 2020 08:54

$$\begin{pmatrix}E_x\\E_y\end{pmatrix} = \underline{\underline{\rho}}\begin{pmatrix}j_x\\j_y\end{pmatrix}$$

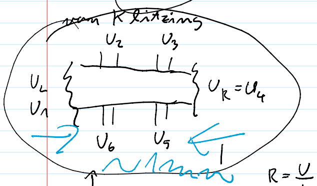


$$\vec F = e(\vec v\times\vec B + \vec E)$$

$$\underline{\underline{\rho}} = \begin{bmatrix}\rho_{xx}&\rho_{xy}\\\rho_{yx}&\rho_{yy}\end{bmatrix} =$$

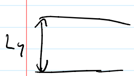
$$\rho_{xx} = \frac{m}{ne^2\tau} \qquad -\rho_{xy} = \rho_{yx} = \frac{B}{n|e|}$$

von Klitzing



$$R = \frac{U}{I}$$

$$R_H = \frac{(U_2 - U_6)}{I}$$

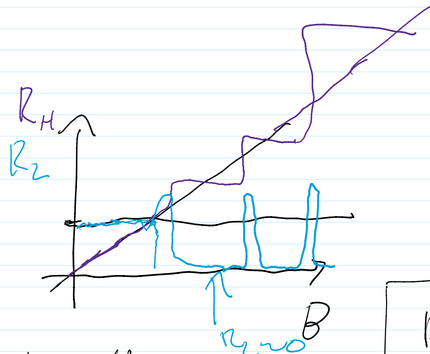


$$I = j_x L_y$$

$$U = -E_y L_y$$

$$R_H = U/I = \frac{-E_y \cancel{L_y}}{j_x \cancel{L_y}}$$

Klasični napoved $R_H = B/n|e|$



$$R_L = \frac{U_6 - U_5}{I}$$

$$\boxed{R_H = \frac{h}{2e^2\, n} \qquad n \in \mathbb{N}}$$

$$\frac{h}{e^2} = 25.8128075(5)$$

$$\frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{e^2 \mu_0 c}{4\pi\hbar} = \frac{e^2}{2h}\mu_0 c$$

$$c^2 = \frac{1}{\varepsilon_0\mu_0}$$

$$1/\varepsilon_0 = \mu_0 c^2$$

anomalni $(g-2)$ mag. moment

Kako pride do kvantizacije?

$$\frac{(\vec p - e\vec A)^2}{2m}\psi = E\psi$$

$$\vec A = B(-y,0,0) \qquad \vec B = \mathrm{rot}\,\vec A = (0,0,B)$$

$$\frac{1}{2m}\left[\left(-i\hbar\frac{\partial}{\partial x} + eBy\right)^2 - \hbar^2\left(\frac{\partial}{\partial y}\right)^2\right]\psi = E\psi$$

$$\psi(x,y) = e^{ikx}\chi(y)$$

$$\frac{1}{2m}\left[(\hbar k + eBy)^2 - \hbar^2\frac{\partial^2}{\partial y^2}\right]\chi(y) = E\chi$$

$$\left\{\frac{e^2B^2}{2m}\left[(y - y_k)^2\right] - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}\right\}\chi = E\chi$$

$$y_k = \frac{-\hbar k}{eB}$$

mag. dolžina $l_B^2 = \frac{\hbar}{|eB|}$

$$m v^2 = \hbar\omega_c$$

$$m\omega_c^2 r^2 = \hbar\omega_c \qquad \omega_c = \frac{|eB|}{m}$$

$$l_B^2 := r^2 = \frac{\hbar\omega_c}{m\omega_c^2} = \frac{\hbar\,\cancel{m}}{\cancel{m}\,|eB|}$$

$$\boxed{\frac{\hbar\omega_c}{2}\left[\frac{(y-y_k)^2}{l_B^2} - l_B^2\frac{\partial^2}{\partial y^2}\right]\chi = E\chi}$$

har. oscilator

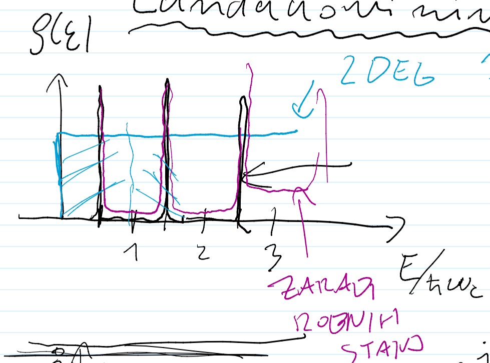
$$\boxed{\chi_{nk}(y) = \mathcal{H}_n\left(\frac{y-y_k}{l_B}\right) e^{-\frac{(y-y_k)^2}{2l_B^2}}} \qquad \boxed{E = \hbar\omega_c\left(n+\tfrac{1}{2}\right)}$$

$$\boxed{\psi = \chi_{nk}\, e^{ikx}}$$

$\neq f(k)$ $\qquad l_B^2 = \frac{\hbar}{|eB|}$ ~

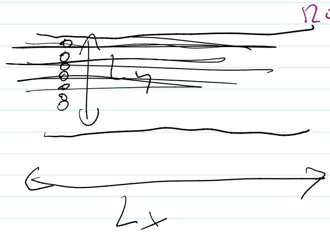
$l_B \sim 10\,\mathrm{nm}$ pri $1\,T$

Landauovi nivoji



2DEG $\frac{m}{\hbar^2\pi}$ $\quad g(E) = \frac{N}{S}$

$\frac{m}{\hbar^2\pi}(\hbar\omega_c)$



e^{ikx}

$\Delta k = \frac{2\pi}{L_x}$

$\Delta y = \frac{\hbar\Delta k}{|eB|}$

$$N = \frac{L_y}{\Delta y} = \frac{L_y|eB|}{\hbar\Delta k} = \frac{\overbrace{L_yL_x}^{S}|eB|}{2\pi\hbar} = \frac{m\omega_c}{2\pi\hbar}S = \frac{m(\omega_c\hbar)}{2\pi\hbar^2}S \quad \cdot 2$$

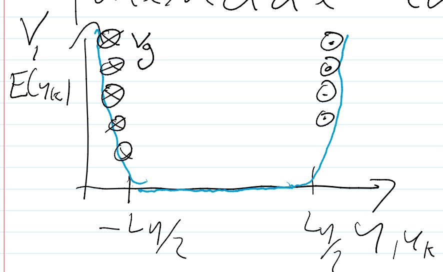
$$N = 2\frac{BSe}{h} = 2\frac{\Phi}{\Phi_0}$$

spin $\qquad$ kvant mag. pretoka

grupna hitrost

$v = \frac{\partial E}{\partial k} = 0$

potencial zaradi robov

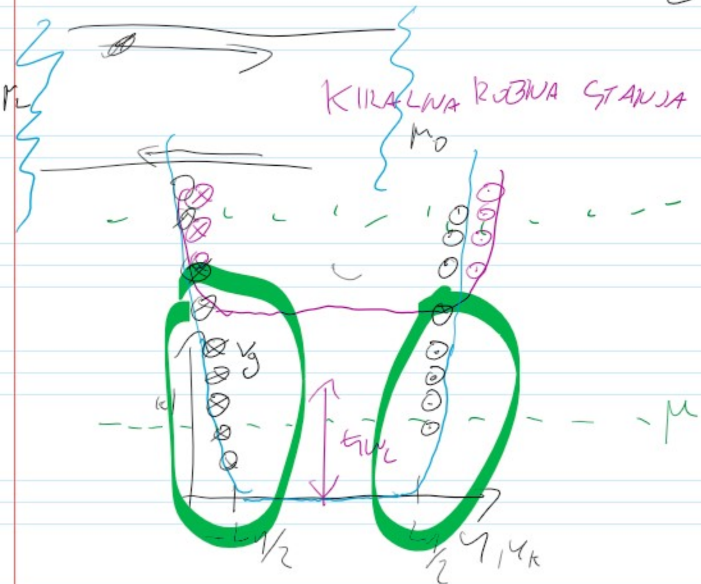


$$E \doteq \hbar\omega_c\left(n+\tfrac{1}{2}\right) + \langle\psi_{nk}|V(y)|\psi_{nk}\rangle$$

$$\boxed{E \doteq \hbar\omega_c\left(n+\tfrac{1}{2}\right) + V(y_k)}$$

$$v_g = \frac{1}{\hbar}\frac{\partial E}{\partial k} = \frac{1}{\hbar}\frac{\partial V}{\partial y_k}\frac{\partial y_k}{\partial k} = \boxed{\frac{1}{\hbar}\frac{\partial V}{\partial y_k}\cdot\left(-\frac{\hbar}{eB}\right)}$$

Nielsen – Ninomiya



Sipanje ni mogoče, ker sta $+v_g$, $-v_g$ 1mm narazen

bulk stanja pa so pri drugi energiji

$$I = 2e\int \frac{dk}{2\pi} v_g (f_L - f_0) = 2e\int \frac{dk}{2\pi}\frac{1}{\hbar}\frac{dE}{dk}(f_L - f_0)$$

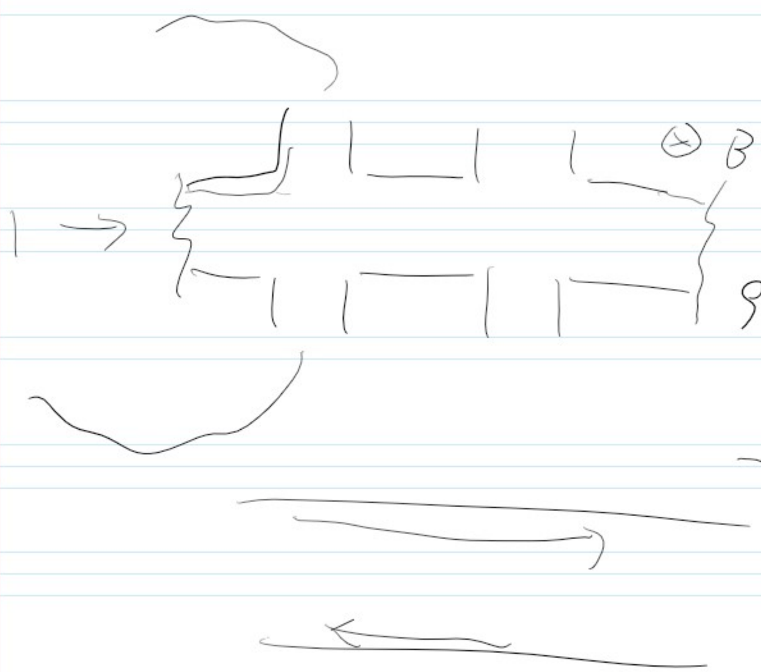
$$= 2\frac{e}{h}\int dE\,(f_L - f_0)$$

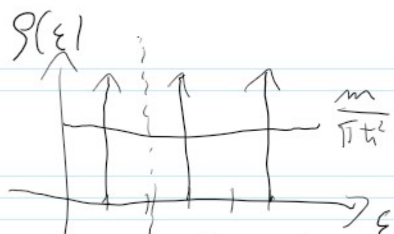
$T = 1$

$$\boxed{I = \frac{2e^2}{h}(U_L - U_0)M}$$

Samo $n = 0$ prispeva

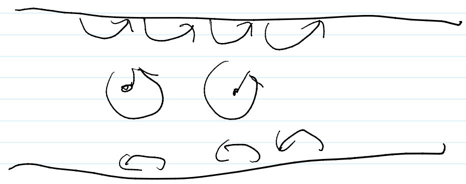
M – število zasedenih Landauovih nivojev





$$E_n = \hbar\omega_c(n + 1/2)$$

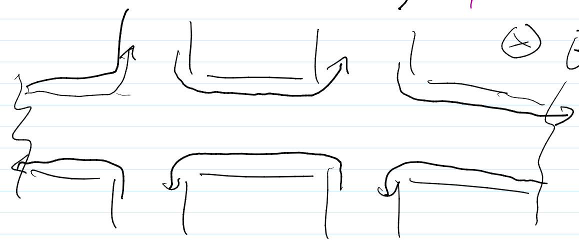
$$\omega_c = \frac{eB}{m}$$



$U_2 = U_1$ $\quad$ $U_3 = U_1$

$\otimes B$

IZVIR U_1



U_4 ponor

$I_1 = -I$

$I_4 = I$

$U_6 = U_4$ $\quad$ $U_5 = U_4$

$I_2 = I_3 = I_5 = I_6 = 0$

$$I_2 = \sum_3 G_{23} U_3$$

$$S_{23} = \begin{vmatrix} 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&1&0 \end{vmatrix}$$

$$G_{23} = G_0 \left(|S_{23}|^2 - S_{23} \right)$$

$$G_{23} = \begin{vmatrix} -1&0&0&0&0&1 \\ 1&-1&0&0&0&0 \\ 0&1&-1&0&0&0 \\ 0&0&1&-1&0&0 \\ 0&0&0&1&-1&0 \\ 0&0&0&0&1&-1 \end{vmatrix} G_0$$

$$I_2 = G_0 (U_1 - U_2) = 0$$

$$U_1 = U_2$$

$$I_3 = G_0 (U_2 - U_3) = 0$$

$$U_2 = U_3$$

$$U_5 = U_4 \quad , \quad U_6 = U_5$$

$$I_4 = I = G_0 (U_1 - U_4)$$

$$I_H = G_0 (U_1 - U_4)$$

$$U_H = U_2 - U_6 = U_1 - U_4$$

$$R_H = U_H / I_H = 1/G_0$$

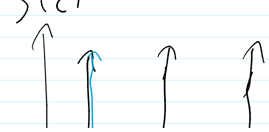
N-kanalov $\quad \dfrac{1}{N G_0}$

Spin

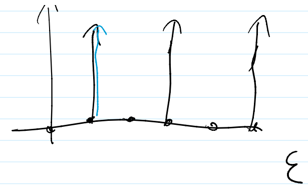
$$\Delta E_z = B g \mu_B = B g \frac{e\hbar}{2m_e}$$

prost el. $g=2$ $\quad = \underbrace{\frac{eB}{m_e}}_{\omega_c} \hbar$

$g(E)$



prost el. $g=2$ $\quad = \underbrace{\left(\frac{eB}{m_e}\right)}_{\omega_c} \hbar$



$$\Delta E_Z = \hbar\omega_c$$

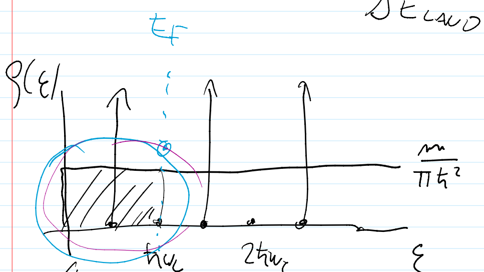
$$\omega_c = \frac{eB}{m^*}$$

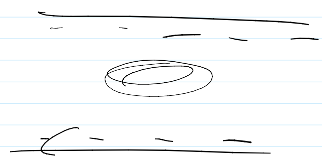
$$m^* = 0.067\, m_e$$

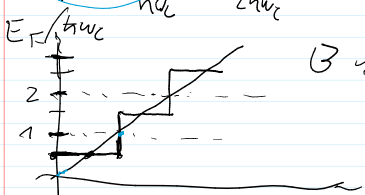
$$g = -0.44$$

$$\Delta E_Z = \frac{eB}{m_e}\, 0.44 \qquad \Delta E_{LANDAU} = \hbar\omega_c = \frac{eB}{0.067\, m_e}$$

$$\Delta E_{LAND} \approx 70 \cdot \Delta E_Z$$



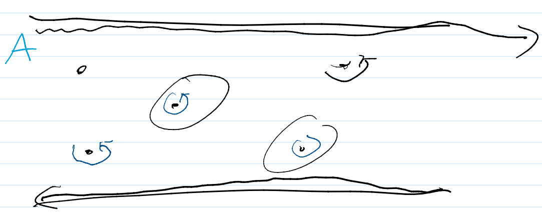




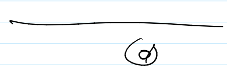
B fiksen

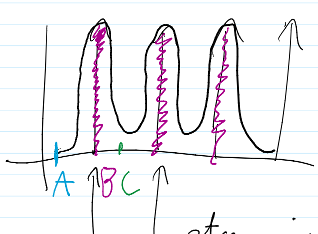
$$N = \frac{2BS}{\Phi_0}$$

$$\frac{eB}{m} = \hbar\omega_c\left(n+\tfrac{1}{2}\right)$$

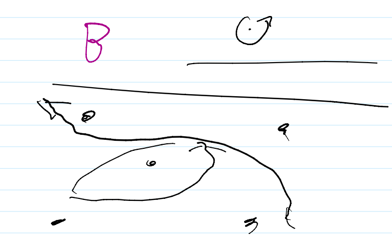


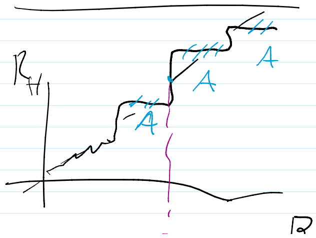
$$\langle \psi | V | \psi \rangle$$

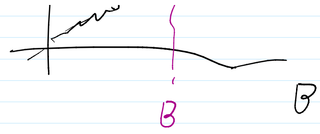




stanja B
ki segaja čez
cel vzorec







ULOMLJENI QHE (FRACTIONAL QHE)

$$\boxed{\frac{1}{R_H} = \frac{e^2}{h}\frac{N}{M}}, \quad N, M \in \mathbb{N}$$

Coulombske interakcije

'82 Tsui, Störmer (eksp.)

Laughlin (teorija) $\frac{1}{R_H} = \frac{e^2}{h}\frac{1}{\nu}$ $\nu = 3, 5, 7, \dots$

'98 Nobel

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + \frac{e^2}{4\pi\varepsilon_0 |r - r'|}$$

$N \dots (x_1, y_1), \dots, (x_N, y_N)$

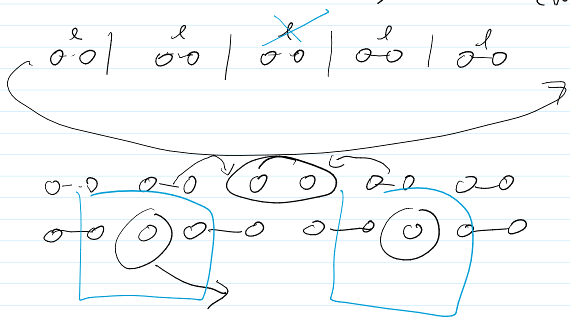
$$\boxed{\psi(x_i, y_i) = \mathcal{N} \prod_{i<j} (z_i - z_j)^m \, e^{-\sum_i |z_i|^2 / 4 l_0^2}}$$

$z_i = x_i + i y_i$

$m = \nu$

- frakcionalizacija naboja

naboj eksitacij e_0/m $m \in \mathbb{N}$



- ANYONSKA STATISTIKA

$$\psi(\vec{r}_1, \vec{r}_2)$$