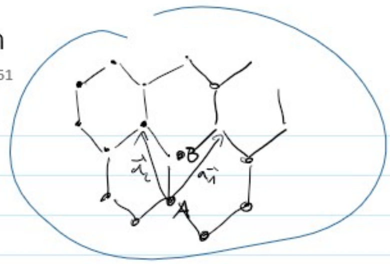
Nanofizika - grafen

ponedeljek, 23. marec 2020 14:51

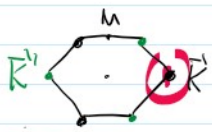


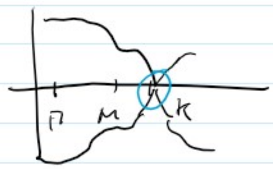
grafen: atomi C na mreži satovja (2d - kristal, blaga upogajem v z-smeri)

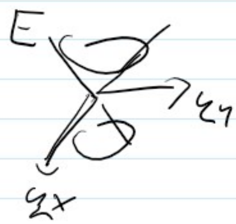
Geim, Novoselov ('04) (Nobel '10)

- zanimiv tehnološko (trden, lahek) 0,77 mg/m², 4kg obremenitve, ...

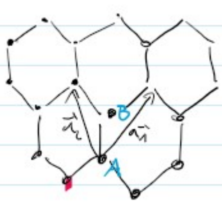
- lahko ga dopiramo z elektrodami
- nizkoenergijska stanja imajo Diracovo disperzijo $E = \pm|k|c\hbar$







- relativistični Hallov pojav



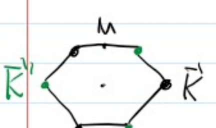
$$\vec{a}_1 = a\left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)$$

$$\vec{a}_2 = a\left(-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)$$

$$\vec{b}_1 = \frac{2\pi}{a}\left(1, \tfrac{1}{\sqrt{3}}\right)$$

$$\vec{b}_2 = \frac{2\pi}{a}\left(-1, \tfrac{1}{\sqrt{3}}\right)$$

$$\vec{a}_i \cdot \vec{b}_j = 2\pi\delta_{ij}$$



$$\vec{K} = \tfrac{1}{3}(\vec{b}_1 - \vec{b}_2)$$

$$\vec{K}' = \tfrac{1}{3}(\vec{b}_2 - \vec{b}_1)$$

$$H = -t\sum_{\vec{R}}\Big(|\psi^B_{\vec{R}-\vec{a}_1}\rangle\langle\psi^A_{\vec{R}}| + |\psi^B_{\vec{R}-\vec{a}_2}\rangle\langle\psi^A_{\vec{R}}| + |\psi^B_{\vec{R}}\rangle\langle\psi^A_{\vec{R}}| + h.c.\Big)$$

$$|\psi^A_{\vec{k}}\rangle = \frac{1}{\sqrt{N}}\sum_{\vec{R}} e^{i\vec{k}\cdot\vec{R}}\,|\psi^A_{\vec{R}}\rangle$$; isto za B

$$H(\vec{k}) = \big[\langle\psi^A_{\vec{k}}|H|\psi^A_{\vec{k}}\rangle \quad \langle\psi^A_{\vec{k}}|H|\psi^B_{\vec{k}}\rangle\big]$$

$$H(\vec{k}) = \begin{bmatrix} \langle \psi_{\vec{k}}^{A} | H | \psi_{\vec{k}}^{A} \rangle & \langle \psi_{\vec{k}}^{A} | H | \psi_{\vec{k}}^{B} \rangle \\ \langle \psi_{\vec{k}}^{B} | H | \psi_{\vec{k}}^{A} \rangle & \langle \psi_{\vec{k}}^{B} | H | \psi_{\vec{k}}^{B} \rangle \end{bmatrix}$$

$$H(\vec{k}) = \begin{bmatrix} & e^*(\vec{k}) \\ e(\vec{k}) & \end{bmatrix}$$

$$e(\vec{k}) = -t\left(1 + e^{i\vec{k}\cdot\vec{a_1}} + e^{i\vec{k}\cdot\vec{a_2}}\right)$$

$\det(H(\vec{k}) - E) = 0 \quad E^2 - |e|^2 = 0 \quad E = \pm|e|$

$\vec{k} = \vec{K} + \vec{q}$

$\vec{K}\cdot\vec{a_1} = 2\pi/3 \quad ; \vec{K}\cdot\vec{a_2} = -\frac{2\pi}{3}$

$$e(\vec{k}) = -t\left(1 + e^{i2\pi/3}(1 + i\vec{q}\cdot\vec{a_1}) + e^{-i2\pi/3}(1 + i\vec{q}\cdot\vec{a_2})\right)$$

$$= -t\left(\cos\tfrac{2\pi}{3}\ \vec{q}\cdot(\vec{a_1}+\vec{a_2})\, i \ - \sin\tfrac{2\pi}{3}\ \vec{q}(\vec{a_1}-\vec{a_2})\right)$$

$$= -t\left(i - \tfrac{1}{2}\sqrt{3}\, q_y a - \tfrac{1}{2}\sqrt{3}\, q_x a\right) = +t\frac{\sqrt{3}}{2}(q_x a + i q_y a)$$

$$\boxed{H(k) = t\frac{\sqrt{3}}{2}\begin{pmatrix} 0 & q_x - iq_y \\ q_x + iq_y & 0 \end{pmatrix} a} = \vec{q}\cdot\vec{\sigma}\ t\frac{\sqrt{3}}{2}a = c^*\hbar\,\vec{q}\cdot\vec{\sigma} \qquad \vec{\sigma} = (\sigma_x, \sigma_y)$$

$$c^* = \frac{t\sqrt{3}}{2}\frac{a}{\hbar} \approx 10^6\,m/s \qquad \begin{pmatrix} & q_x \\ q_x & \end{pmatrix} = q_x \sigma_x$$

$H(\vec{p}) = c^* \vec{\sigma}\cdot\vec{p}$

$$\boxed{H(\vec{q}) = c^*\hbar(\vec{\sigma}\cdot\vec{q})}$$ Diracova / Weylova

$$\boxed{c^*\hbar\,\vec{\sigma}\cdot\vec{q}\ \psi = E\psi}$$ enačba

okrog K'

$$-c^*\hbar\,\vec{\sigma}\cdot\tilde{\vec{q}}\ \psi = E\psi$$

$\vec{\sigma}\cdot\vec{p}$ – projekcija (pseudo–) spina na moment : sučnost

$\vec{p} = \hbar\vec{q}$

Klein - Gordonova & Diracova eq

$$\left(E = \frac{p^2}{2m}\right)\psi$$

$E \to i\hbar\partial_t \qquad \vec{p} \to -i\hbar\nabla$

$$E^2 = m^2c^4 + c^2p^2 \qquad E = \pm\sqrt{m^2c^4 + c^2p^2}$$

$$\left(-\hbar^2\partial_t^2 = m^2c^4 - c^2\hbar^2\nabla^2\right)\psi$$

Klein - Gordon

$$\psi = e^{i\left(\vec{k}\vec{x} - \frac{Et}{\hbar}\right)}$$

$$E^2\psi = m^2c^4\psi + \hbar^2k^2c^2\psi$$

$E \qquad\qquad \rho \propto \psi^*\partial_t\psi - \psi\partial_t\psi^*$

Diracova enačba $\qquad \nabla^2 = \partial_i\partial_i$

$$\hat{O}_{KG}\left(-\hbar^2\partial_t^2 + c^2\hbar^2\nabla^2 = m^2c^4\right)\psi$$

$$(\underbrace{A\, i\hbar\partial_t - i\hbar B_i\partial_i c}_{\hat{O}} = mc^2)\psi$$

$i = 1,2,3 \quad x,y,z$

$\mu = 0,1,2,3$

$$(\gamma_0\, i\hbar\partial_t - i\hbar\gamma^i\partial_i c = mc^2)\psi$$

$$\partial_0 = \frac{1}{c}\partial_t$$

$\hat{O}^2 = \hat{O}_{KG}$

$$B_iB_j = \frac{1}{2}(B_iB_j + B_jB_i)\partial_i\partial_j$$

$$\hat{O}^2 = A^2 - \hbar^2\partial_t^2 \quad -c^2\hbar^2\frac{B_iB_j + B_jB_i}{2}\partial_i\partial_j$$

$$\underbrace{+cA\hbar^2B_i\,\partial_t\partial_i + B_i\hbar^2A\,\partial_i\partial_t c}_{\underbrace{(AB_i + B_iA)\ldots}_{=0}}$$

$$(i\hbar\gamma^\mu\partial_\mu = mc)\psi$$

$$p_\mu = i\hbar\partial_\mu$$

$$\boxed{(\gamma^\mu p_\mu = mc)\psi}$$

$\{A, B_i\} := AB_i + B_iA = 0$

$\{B_i, B_j\} = -2\delta_{ij}$

$A^2 = 1 \qquad A = \gamma^0 \qquad B^i = \gamma^i$

γ^μ

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \qquad \eta^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\boxed{(\gamma^\mu p_\mu = mc)\psi} \qquad \text{Dirac}$$

Diracova upodobitev

$$\gamma^0 = \begin{pmatrix} 1_{2\times2} & \\ & -1_{2\times2} \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} & \sigma^i \\ -\sigma^i & \end{pmatrix}$$

$$\gamma^0\gamma^i = \begin{pmatrix} & \sigma^i \\ \sigma^i & \end{pmatrix}$$

$$A\Psi = E\Psi$$

Weylova upodobitev

$$\gamma^0 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \qquad \gamma_i = \begin{pmatrix} & \sigma^i \\ -\sigma^i & \end{pmatrix}$$

$$\gamma^0\gamma^i = \begin{pmatrix} \sigma^i & \\ & -\sigma^i \end{pmatrix}$$

primer: masivni delec, mirujoč

$$\gamma^0 = \begin{pmatrix} 1_{2\times2} & \\ & -1_{2\times2} \end{pmatrix} \qquad \gamma^\mu p_\mu \Psi = mc\Psi$$

$$\gamma^0\gamma^i = \begin{pmatrix} & \sigma^i \\ \sigma^i & \end{pmatrix} \qquad \Psi = \underbrace{\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}}_{\psi_0} e^{i\vec{k}\cdot\vec{r} - i\frac{Et}{\hbar}}$$

$\vec{k} = 0$

$$\gamma^0 p_0 \Psi = mc\Psi$$

$$\gamma_0 E \psi_0 = mc\,\psi_0 \quad / \cdot \gamma_0$$

$$E/c\,\psi_0 = mc\,\gamma_0\psi_0$$

$$E\psi_0 = mc^2 \begin{pmatrix} 1_{2\times2} & \\ & -1_{2\times2} \end{pmatrix}\psi_0$$

$$\psi_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad E = mc^2$$

$$\psi_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad E = -mc^2$$

brezmasni delec

brezmasni delec

$$\gamma^\mu p_\mu \psi = \overset{0}{\overset{\|}{m}} c\,\psi \quad /\gamma^0$$

$$\gamma^0 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} & \sigma^i \\ -\sigma^i & \end{pmatrix}$$

$$p_0 \psi = -\gamma^0\gamma^i p_i \psi$$

$$\gamma^0\gamma^i = \begin{pmatrix} \sigma^i & \\ & -\sigma^i \end{pmatrix}$$

$$i\hbar \frac{\partial_t}{c} \psi = \begin{pmatrix} \vec\sigma\cdot\vec p & 0 \\ 0 & -\vec\sigma\cdot\vec p \end{pmatrix}\psi$$

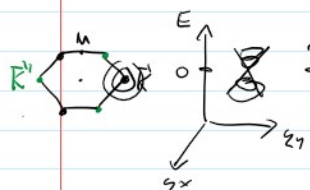
$$a^\mu b_\mu = a_0 b_0 - a_i b_i$$

$$\boxed{E\psi = c\begin{pmatrix} \vec\sigma\cdot\vec p & 0 \\ 0 & -\vec\sigma\cdot\vec p \end{pmatrix}\psi}$$

$$\hbar\,[illegible] = c\hbar\begin{pmatrix} \vec\sigma\cdot\vec k & 0 \\ 0 & -\vec\sigma\cdot\vec k \end{pmatrix}\psi$$

(okrog k')

$$c\left[\begin{matrix} \vec\sigma\cdot\vec p & \\ & -\vec\sigma\cdot\vec p \end{matrix}\right]\begin{pmatrix}\psi_K \\ \psi_{K'}\end{pmatrix} = E\begin{pmatrix}\psi_K \\ \psi_{K'}\end{pmatrix}$$



Diracova relativistična disperzija za $m=0$.

Kvantni Hallov pojav v grafenu

$$H = c^*\vec\sigma\cdot\vec p = c\hbar\vec\sigma\cdot\vec q$$

$$\langle \vec r | \psi^{A,B}_{\vec R}\rangle = \phi_{P_z}(\vec r - \vec R - \vec s_{AB})$$

$$|\psi_k^{A}\rangle = \sum_{\vec R} e^{i\vec k\cdot\vec R}\,|\psi^{A}_{\vec R}\rangle \quad / \langle\vec r|$$

$$\vec k = \vec K + \vec q$$

$$\psi_{\vec k}(\vec r) = \sum_{\vec R} \psi_A e^{i(\vec K+\vec q)\cdot\vec R}\,\phi(\vec r-\vec R-\vec s_A) + \psi_B e^{i(\vec K+\vec q)\cdot\vec R}\,\phi(\vec r - \vec R - \vec s_B)$$

hitra spr. z $\vec r$

ovojna funkcija $\psi_{B\vec q}(\vec r) := \psi_B e^{i\vec q\cdot\vec r}$

$$\hbar c^*\vec\sigma\cdot\vec q\,\psi_k = E\psi_k \quad /\cdot e^{i\vec q\cdot r}$$

$$(-i)\hbar c\,\vec\sigma\cdot\nabla\left(e^{i\vec q\cdot\vec r}\psi_k\right) =$$

$$\psi_k = \begin{pmatrix}\psi_k^A \\ \psi_k^B\end{pmatrix}$$

$$\psi_k = \begin{pmatrix}\psi_k^A\\ \psi_k^B\end{pmatrix}$$

$$\hbar c\, \vec\sigma\cdot\nabla \begin{pmatrix} e^{i\vec q\cdot\vec r}\,\psi_k^A \\ e^{i\vec q\cdot \vec r}\,\psi_k^B\end{pmatrix}$$

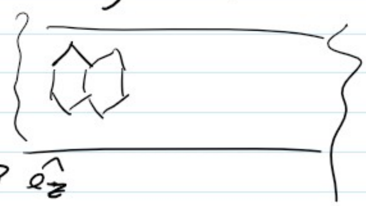
$$\hbar \qquad -i\nabla\hbar = \vec p$$

$$H = c^*\,\vec p\cdot\vec\sigma$$

OVOJNA FUNKCIJA

Landauovi nivoji v grafenu

$c^* \leftarrow c$



$$\boxed{c(\vec p - e\vec A)\cdot\vec\sigma\,\psi = E\psi}$$

$$\vec A = -B(y,0,0) \qquad \vec B = B\,\hat e_z$$

$$\psi = e^{ikx}\chi(y) = e^{ikx}\begin{pmatrix}\chi_A(y)\\ \chi_B(y)\end{pmatrix}$$

$$c\left[(\hbar k + eBy)\sigma_x + (-i\hbar\partial_y)\sigma_y\right]\chi =$$

$e = -e_0$

$$-ce_0B\left[\left(\frac{\hbar k}{-e_0B}+y\right)\sigma_x + \frac{i\hbar}{e_0B}\partial_y\sigma_y\right]\chi =$$

$\hbar/e_0B = l_B^2$

$$= -ce_0B\left[(y-y_k)\sigma_x + i\, l_B^2\,\partial_y\sigma_y\right]\chi =$$

$y_k = k\, l_B^2$

$$= \underbrace{-ce_0B\,l_B}_{\varepsilon_B}\left[\frac{y-y_k}{l_B}\sigma_x + i\, l_B\,\partial_y\sigma_y\right]\chi =$$

$$\sigma_x = \sigma^+ + \sigma^- \qquad \sigma_y = \frac{\sigma^+-\sigma^-}{i} \qquad \sigma^+ = \begin{pmatrix}0&1\\0&0\end{pmatrix} \quad \sigma^-=\begin{pmatrix}0&0\\1&0\end{pmatrix}$$

$y = \tilde y = y - y_k$

$$= \varepsilon_B\left[y/l_B\,\sigma_x + i\, l_B\,\partial_y\sigma_y\right]\chi$$

$$\boxed{= \varepsilon_B\left[\sigma^+(y/l_B + l_B\partial_y) + \sigma^-(y/l_B - l_B\partial_y)\right]\chi}$$

$$a = \frac{1}{\sqrt2}\left(x/x_0 + \frac{ipx_0}{\hbar}\right) = \frac{1}{\sqrt2}\left(\frac{x}{x_0}+\partial_x x_0\right)$$

$$a^+ = \frac{1}{\sqrt2}\left(\frac{x}{x_0} - \partial_x x_0\right)$$

$(H = \hbar\omega(a^+a + \ldots))$

$a^+ = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} - \partial_x x_0\right)$

$$H\chi = \varepsilon_B\left[\sigma^+ a + \sigma^- a^+\right]\chi \qquad \left(H = \hbar\omega_c\left(a^+a + \tfrac{1}{2}\right)\right)$$

$$H\begin{pmatrix}\chi_A \\ \chi_B\end{pmatrix} = \varepsilon_B\begin{bmatrix} & a \\ a^+ & \end{bmatrix}\begin{pmatrix}\chi_A \\ \chi_B\end{pmatrix} = E\begin{pmatrix}\chi_A \\ \chi_B\end{pmatrix}$$

$$\varepsilon_B\, a\chi_B = E\chi_A \Rightarrow \varepsilon_B^2\, a a^+ \chi_A = E^2\chi_A$$

$$\varepsilon_B\, a^+\chi_A = E\chi_B \Rightarrow \varepsilon_B^2\, a^+ a\, \chi_B = E^2\chi_B$$

$$E^2 = \varepsilon_B^2\, n$$

$$E = \pm|\varepsilon_B|\sqrt{n}$$

Landauovi nivoji