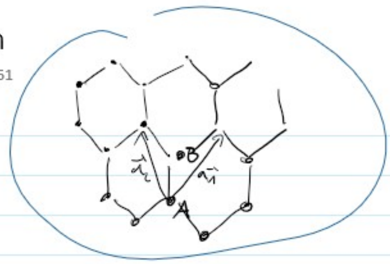
Nanofizika - grafen

ponedeljek, 23. marec 2020 14:51

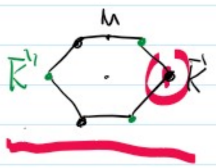


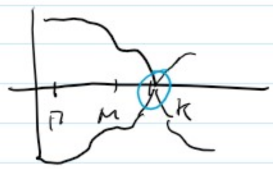
grafen: atomi C na mreži satovja (2d - kristal, kalupa upogajem v z-smeri)

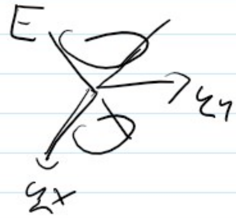
Geim, Novoselov (Nobel '10) '04

- zanimiv tehnološko (trden, lahek) 0,77 mg/m², 4kg obremenitve, ...

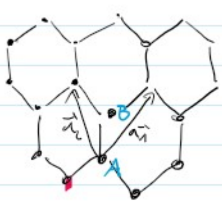
- lahko ga dopiramo z elektrodami
- nizkoenergijska stanja imajo Diracovo disperzijo $E = \pm|k|c\hbar$





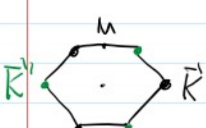


- relativistični Hallov pojav



$$\vec{a}_1 = a\left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)$$
$$\vec{a}_2 = a\left(-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)$$
$$\vec{b}_1 = \frac{2\pi}{a}\left(1, \tfrac{1}{\sqrt{3}}\right)$$
$$\vec{b}_2 = \frac{2\pi}{a}\left(-1, \tfrac{1}{\sqrt{3}}\right)$$

$\vec{a}_i \cdot \vec{b}_j = 2\pi \delta_{ij}$



$$\vec{K} = \tfrac{1}{3}(\vec{b}_1 - \vec{b}_2)$$
$$\vec{K}' = \tfrac{1}{3}(\vec{b}_2 - \vec{b}_1)$$

$$H = -t\sum_{\vec{R}}\Big(|\psi^B_{\vec{R}-\vec{a}_1}\rangle\langle\psi^A_{\vec{R}}| + |\psi^B_{\vec{R}-\vec{a}_2}\rangle\langle\psi^A_{\vec{R}}| + |\psi^B_{\vec{R}}\rangle\langle\psi^A_{\vec{R}}| + h.c.\Big)$$

$$|\psi^A_{\vec{k}}\rangle = \frac{1}{\sqrt{N}}\sum_{\vec{R}} e^{i\vec{k}\cdot\vec{R}} |\psi^A_{\vec{R}}\rangle$$, isto za B

$$H(\vec{k}) = \Big[\langle\psi^A_{\vec{k}}|H|\psi^A_{\vec{k}}\rangle \quad \langle\psi^A_{\vec{k}}|H|\psi^B_{\vec{k}}\rangle \Big]$$

$$H(\vec{k}) = \begin{bmatrix} \langle \psi_k^A | H | \psi_k^A \rangle & \langle \psi_k^A | H | \psi_k^B \rangle \\ \langle \psi_k^B | H | \psi_k^A \rangle & \langle \psi_k^B | H | \psi_k^B \rangle \end{bmatrix}$$

$$H(\vec{k}) = \begin{bmatrix} & e^*(\vec{k}) \\ e(\vec{k}) & \end{bmatrix}$$

$$e(\vec{k}) = -t\left(1 + e^{i\vec{k}\cdot\vec{a_1}} + e^{i\vec{k}\cdot\vec{a_2}}\right)$$

$\det(H(\vec{k}) - E) = 0 \quad E^2 - |e|^2 = 0 \quad E = \pm|e|$

$\vec{k} = \vec{K} + \vec{q}$

$\vec{K}\cdot\vec{a_1} = 2\pi/3 \quad ; \vec{K}\cdot\vec{a_2} = -\frac{2\pi}{3}$

$$e(\vec{k}) = -t\left(1 + e^{i2\pi/3}(1 + i\vec{q}\cdot\vec{a_1}) + e^{-i2\pi/3}(1 + i\vec{q}\cdot\vec{a_2})\right)$$

$$= -t\left(\cos\frac{2\pi}{3}\,\vec{q}\cdot(\vec{a_1}+\vec{a_2})\,i - \sin\frac{2\pi}{3}\,\vec{q}(\vec{a_1}-\vec{a_2})\right)$$

$$= -t\left(i - \tfrac{1}{2}\sqrt{3}\,q_y a - \tfrac{1}{2}\sqrt{3}\,q_x a\right) = +t\frac{\sqrt{3}}{2}(q_x a + i q_y a)$$

$$\boxed{H(k) = t\frac{\sqrt{3}}{2}\begin{pmatrix} 0 & q_x - iq_y \\ q_x + iq_y & 0 \end{pmatrix} a} = \vec{q}\cdot\vec{\sigma}\,\frac{t\sqrt{3}}{2}a = c^*\hbar\,\vec{q}\cdot\vec{\sigma} \qquad \vec{\sigma} = (\sigma_x, \sigma_y)$$

$$c^* = \frac{t\sqrt{3}}{2}\frac{a}{\hbar} \approx 10^6\,m/s \qquad \begin{pmatrix} & q_x \\ q_x & \end{pmatrix} = q_x\sigma_x$$

$H(\vec{p}) = c^*\vec{\sigma}\cdot\vec{p}$

$\boxed{H(\vec{q}) = c^*\hbar(\vec{\sigma}\cdot\vec{q})}$ Diracova / Weylova

$\boxed{c^*\hbar\,\vec{\sigma}\cdot\vec{q}\,\psi = E\psi}$ enačba

drug K'

$-c^*\hbar\,\vec{\sigma}\cdot\tilde{\vec{q}}\,\psi = E\psi$

$\vec{\sigma}\cdot\vec{p}$ – projekcija (pseudo–) spina na moment : sučnost

$\vec{p} = \hbar\vec{q}$

Klein-Gordonova & Diracova eq.

$$\left(E = \frac{p^2}{2m}\right)\psi$$

$E \to i\hbar\partial_t \qquad \vec{p} \to -i\hbar\nabla$

$$E^2 = m^2c^4 + c^2p^2 \qquad E = \pm\sqrt{m^2c^4+c^2p^2}$$

$$\left(-\hbar^2\partial_t^2 = m^2c^4 - c^2\hbar^2\nabla^2\right)\psi$$

Klein-Gordon

$$\psi = e^{i\left(\vec{k}\vec{x} - \frac{Et}{\hbar}\right)}$$

$$E^2\psi = m^2c^4\psi + \hbar^2k^2c^2\psi$$

$E \qquad\qquad \rho \propto \psi^*\partial_t\psi - \psi\partial_t\psi^*$

Diracova enačba $\quad \nabla^2 = \partial_i\partial_i$

$$\hat{O}_{KG}\left(-\hbar^2\partial_t^2 + c^2\hbar^2\nabla^2 = m^2c^4\right)\psi$$

$$(\underbrace{A i\hbar\partial_t - i\hbar B_i\partial_i c}_{\hat{O}} = mc^2)\psi \qquad (\gamma_0 i\hbar\partial_t - i\hbar\gamma^i\partial_i c = mc^2)\psi$$

$i = 1,2,3 \quad x,y,z \qquad \partial_0 = \frac{1}{c}\partial_t$

$\mu = 0,1,2,3$

$$\hat{O}^2 = \hat{O}_{KG} \qquad B_iB_j = \frac{1}{2}(B_iB_j + B_jB_i)\partial_i\partial_j$$

$$(i\hbar\gamma^\mu\partial_\mu = mc)\psi$$

$$\hat{O}^2 = A^2 - \hbar^2\partial_t^2 - c^2\hbar^2\frac{B_iB_j + B_jB_i}{2}\partial_i\partial_j$$

$$\underbrace{+cA\hbar^2B_i\partial_t\partial_i + B_i\hbar^2A\partial_i\partial_t c}_{\underbrace{(AB_i + B_iA)}_{=0}\ldots}$$

$$p_\mu = i\hbar\partial_\mu$$

$$\boxed{(\gamma^\mu p_\mu = mc)\psi}$$

$\{A, B_i\} := AB_i + B_iA = 0$

$\{B_i, B_j\} = -2\delta_{ij}$

$A^2 = 1 \qquad A = \gamma^0 \qquad B^i = \gamma^i$

γ^μ

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \qquad \eta^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}$$

$$\boxed{(\gamma^\mu p_\mu = mc)\psi} \quad \text{Dirac}$$

Diracova upodobitev

$$\gamma^0 = \begin{pmatrix} 1_{2\times2} & \\ & -1_{2\times2} \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} & \sigma^i \\ -\sigma^i & \end{pmatrix}$$

$$\gamma^0\gamma^i = \begin{pmatrix} & \sigma^i \\ \sigma^i & \end{pmatrix}$$

$$A\psi = E\psi$$

Weylova upodobitev

$$\gamma^0 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \qquad \gamma_i = \begin{pmatrix} & \sigma^i \\ -\sigma^i & \end{pmatrix}$$

$$\gamma^0\gamma^i = \begin{pmatrix} \sigma^i & \\ & -\sigma^i \end{pmatrix}$$

primer: masivni delec, mirujoč

$$\gamma^0 = \begin{pmatrix} 1_{2\times2} & \\ & -1_{2\times2} \end{pmatrix} \qquad \gamma^\mu p_\mu \psi = mc\psi$$

$$\gamma^0\gamma^i = \begin{pmatrix} & \sigma^i \\ \sigma^i & \end{pmatrix} \qquad \psi = \underbrace{\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}}_{\psi_0} e^{i\vec{k}\cdot\vec{r} - i\frac{Et}{\hbar}}$$

$\vec{k} = 0$

$$\gamma^0 p_0 \psi = mc\psi$$

$$\gamma_0 E \psi_0 = mc\psi_0 \quad / \cdot \gamma_0$$

$$E/c\,\psi_0 = mc\,\gamma_0\psi_0$$

$$E\psi_0 = mc^2 \begin{pmatrix} 1_{2\times2} & \\ & -1_{2\times2} \end{pmatrix} \psi_0$$

$$\psi_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad E = mc^2$$

$$\psi_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad E = -mc^2$$

brezmasni delec

brezmasni delec

$$\gamma^\mu p_\mu \psi = \overset{0}{m} c \psi \quad / \gamma^0$$

$$\gamma^0 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} & \sigma^i \\ -\sigma^i & \end{pmatrix}$$

$$p_0 \psi = -\gamma^0\gamma^i p_i \psi$$

$$\gamma^0\gamma^i = \begin{pmatrix}\sigma^i & \\ & -\sigma^i\end{pmatrix}$$

$$i\hbar \frac{\partial_t}{c}\psi = \begin{pmatrix}\vec\sigma\cdot\vec p & 0\\ 0 & -\vec\sigma\cdot\vec p\end{pmatrix}\psi$$

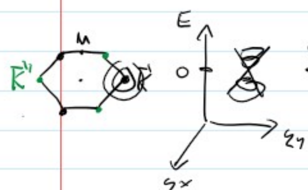
$$a^\mu b_\mu = a_0 b_0 - a_i b_i$$

$$\boxed{E\psi = c\begin{pmatrix}\vec\sigma\cdot\vec p & 0\\ 0 & -\vec\sigma\cdot\vec p\end{pmatrix}\psi}$$

$$\hbar \cancel{\omega} = c\hbar \begin{pmatrix}\vec\sigma\cdot\vec k & 0 \\ 0 & -\vec\sigma\cdot\vec k\end{pmatrix}\psi$$

(okrog K')

$$c\begin{bmatrix}\vec\sigma\cdot\vec p & \\ & -\vec\sigma\cdot\vec p\end{bmatrix}\begin{pmatrix}\psi_K\\ \psi_{K'}\end{pmatrix} = E\begin{pmatrix}\psi_K\\ \psi_{K'}\end{pmatrix}$$



Diracova relativistična disperzija za $m=0$.

Kvantni Hallov pojav v grafenu

$$H = c^*\vec\sigma\cdot\vec p = c\hbar\vec\sigma\cdot\vec q$$

$$\langle \vec r | \psi^{A,B}_{\vec R}\rangle = \phi_{2p_z}(\vec r - \vec R - \vec\delta_{AB})$$

$$|\psi^{A,B}_{k}\rangle = \sum_{\vec R} e^{i\vec k\cdot\vec R}\,|\psi^{A,B}_{\vec R}\rangle \quad / \langle \vec r|$$

$$\vec k = \vec K + \vec q$$

$$\psi_{\vec k}(\vec r) = \sum_{\vec R} \psi_A\, e^{i(\vec K+\vec q)\cdot\vec R}\,\phi(\vec r-\vec R-\vec\delta_A) + \psi_B\, e^{i(\vec K+\vec q)\cdot\vec R}\,\phi(\vec r - \vec R - \vec \delta_B)$$

hitro spr. z $\vec r$

ovojna funkcija $\psi_{B\vec q}(\vec r) := \psi_B\, e^{i\vec q\cdot\vec r}$

$$\hbar c^* \vec\sigma\cdot\vec q\;\psi_k = E\,\psi_k \quad / \cdot e^{i\vec q\cdot \vec r}$$

$$(-i)\hbar c\,\vec\sigma\cdot\nabla\, e^{i\vec q\cdot\vec r}\psi_k =$$

$$\psi_k = \begin{pmatrix}\psi_k^A\\ \psi_k^B\end{pmatrix}$$

$$\psi_k = \begin{pmatrix}\psi_k^A \\ \psi_k^B\end{pmatrix}$$

$$\hbar c\, \vec{\sigma} \cdot \nabla \begin{pmatrix} e^{i\vec{q}\cdot\vec{r}}\psi_k^A \\ e^{i\vec{q}\cdot\vec{r}}\psi_k^B \end{pmatrix}$$

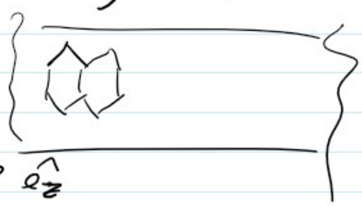
$$\hbar \qquad -i\nabla\hbar = \vec{p}$$

$$H = c^* \vec{p}\cdot\vec{\sigma}$$

OVOJNA FUNKCIJA

Landauovi nivoji v grafenu

$c^* \leftarrow c$



$$\boxed{c(\vec{p} - e\vec{A})\cdot\vec{\sigma}\,\psi = E\psi}$$

$$\vec{A} = -B(y,0,0) \qquad \vec{B} = B\,\hat{e}_z$$

$$\psi = e^{ikx}\chi(y) = e^{ikx}\begin{pmatrix}\chi_A(y) \\ \chi_B(y)\end{pmatrix}$$

$$c\left[(\hbar k + eBy)\sigma_x + (-i\hbar\partial_y)\sigma_y\right]\chi =$$

$e = -e_0$

$\omega_c = \frac{eB}{m}$

$$-ce_0B\left[\left(\frac{\hbar k}{-e_0B} + y\right)\sigma_x + \frac{i\hbar}{e_0B}\partial_y\sigma_y\right]\chi =$$

$ce_0Bl_B =$

$\hbar/e_0B = l_B^2$

$$= -ce_0B\left[(y - y_k)\sigma_x + i l_B^2\partial_y\sigma_y\right]\chi =$$

$y_k = k l_B^2$

$$= \underbrace{-ce_0Bl_B}_{\varepsilon_B}\left[\frac{y-y_k}{l_B}\sigma_x + i l_B\partial_y\sigma_y\right]\chi =$$

$ce_0Bl_B = \varepsilon_B$

$$\sigma_x = \sigma^+ + \sigma^- \qquad \sigma_y = \frac{\sigma^+ - \sigma^-}{i} \qquad \sigma^+ = \begin{pmatrix}0&1\\0&0\end{pmatrix} \quad \sigma^- = \begin{pmatrix}0&0\\1&0\end{pmatrix}$$

$y = \tilde{y} = y - y_k$

$\frac{\varepsilon_B}{\hbar\omega_c}$

$$= \varepsilon_B\left[y/l_B\,\sigma_x + i l_B\partial_y\sigma_y\right]\chi$$

$$\boxed{= \varepsilon_B\left[\sigma^+(y/l_B + l_B\partial_y) + \sigma^-(y/l_B - l_B\partial_y)\right]\chi}$$

$$\left\{\begin{aligned} a &= \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} + \frac{ipx_0}{\hbar}\right) = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} + \partial_x x_0\right) \\ a^+ &= \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} - \partial_x x_0\right)\end{aligned}\right.$$

$$\varepsilon_B\left[\sigma^+ \quad \sigma^- \quad\right]\chi \qquad \left(H = \hbar\omega_c\left(a^+a + \ldots\right)\right)$$

$$\left(a^+ = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} - \partial_x x_0\right)\right)$$

$$H\chi = \sqrt{2}\,\varepsilon_B\left[\sigma^+ a + \sigma^- a^+\right]\chi \qquad \left(H = \hbar\omega_c\left(a^+a + 1/2\right)\right)$$

$$H\begin{pmatrix}\chi_A\\ \chi_B\end{pmatrix} = \varepsilon_B\begin{bmatrix} & a\\ a^+ & \end{bmatrix}\begin{pmatrix}\chi_A\\ \chi_B\end{pmatrix} = E\begin{pmatrix}\chi_A\\ \chi_B\end{pmatrix}$$

$$\varepsilon_B\, a\,\chi_B = E\,\chi_A \Rightarrow \varepsilon_B^2\, a a^+ \chi_A = E^2\chi_A$$

$$\varepsilon_B\, a^+\chi_A = E\,\chi_B \Rightarrow \varepsilon_B^2\, a^+ a\,\chi_B = E^2\chi_B$$

$$E^2 = \varepsilon_B^2\, n$$

$$E = \pm|\varepsilon_B|\sqrt{n}$$

$$\chi_0 = \begin{pmatrix}0\\1\end{pmatrix}\mathcal{H}_0\left(\frac{y-y_k}{l_B}\right) \dots E=0 \qquad \mathcal{H}_N \dots N\text{-ta last. h.o.}$$

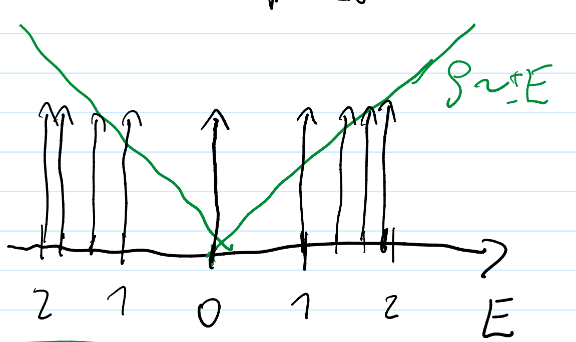
$$H_N\, e^{-\frac{(y-y_k)^2}{2l_B^2}}$$

$$\chi_n = \begin{pmatrix}0\\1\end{pmatrix}\mathcal{H}_N \pm \begin{pmatrix}1\\0\end{pmatrix}\mathcal{H}_{N-1} \qquad E = \pm\sqrt{n}\,\varepsilon_B$$

$$\varepsilon_B\begin{pmatrix} & a\\ a^+ & \end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\mathcal{H}_0 = \begin{pmatrix}a\,\mathcal{H}_0\\ 0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} = 0\begin{pmatrix}0\\1\end{pmatrix}\mathcal{H}_0$$

$$\varepsilon_B\begin{pmatrix} & a\\ a^+ & \end{pmatrix}\left[\begin{pmatrix}0\\ \mathcal{H}_N\end{pmatrix} \pm \begin{pmatrix}\mathcal{H}_{N-1}\\ 0\end{pmatrix}\right] = \varepsilon_B\left[\begin{pmatrix}\sqrt{N}\,\mathcal{H}_{N-1}\\ 0\end{pmatrix} \pm \begin{pmatrix}0\\ \sqrt{N}\,\mathcal{H}_N\end{pmatrix}\right]$$

$$E = \pm\sqrt{N}\,\varepsilon_B$$

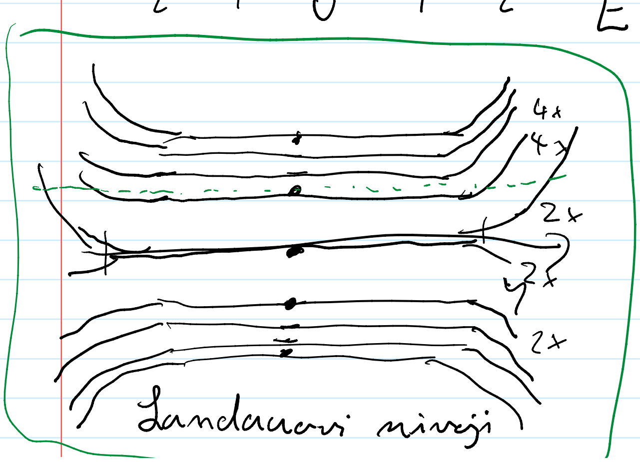


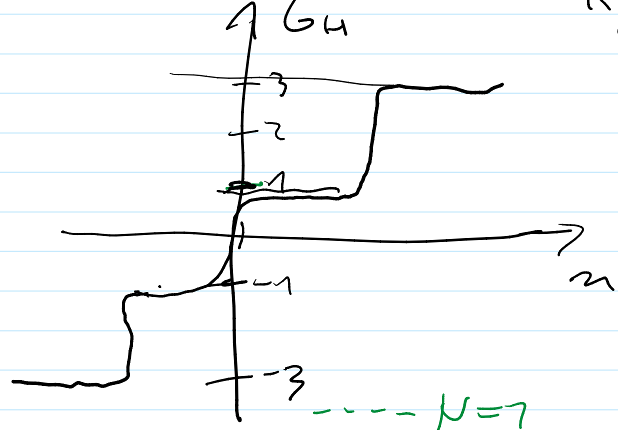
$$N = \frac{BS}{\Phi_0}\cdot 2\cdot 2$$

spin

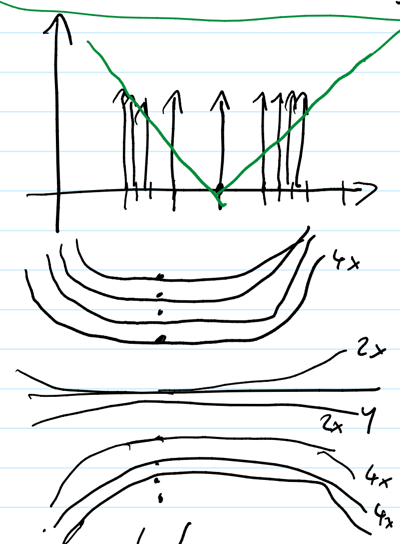
valley

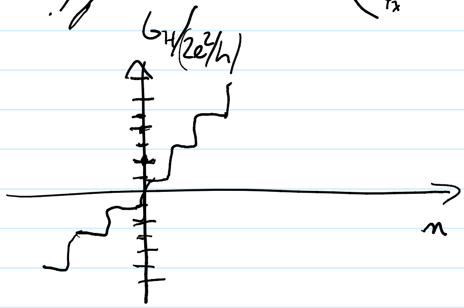
K, K'





Landauovi nivoji





-3 ---- $N=1$

$$\sigma_H = \frac{e^2}{h}(4N+2) = \frac{2e^2}{h}(2N+1)$$

$$\mathcal{E}_e =$$

$$c\, e_0 B l_B = \mathcal{E}_B$$

$$\frac{\mathcal{E}_B}{\hbar\omega_c} = \frac{c\,\cancel{e_0} B l_B\, m}{\cancel{e_0} B\, \hbar} = c\sqrt{\frac{\hbar}{e_0 B}}\ \frac{m}{\hbar}$$

$$l_B = \sqrt{\frac{\hbar}{e_0 B}}$$

$$= \sqrt{\frac{\cancel{\hbar}\, m c^2 m}{\hbar \cancel{\hbar}\, e_0 B}}$$

$$= \sqrt{\frac{mc^2}{\hbar\omega_c}}$$

$$= \sqrt{\frac{0.5\,\mathrm{MeV}\left(\frac{1}{300}\right)^2}{\mathrm{meV}\cdot 0.1}}$$

$$\frac{\mathcal{E}_B}{\hbar\omega_c} \approx 1000$$