Nanofizika QD

nedelja, 05. april 2020 15:20

Kvantne pike in Coulombska blokada

$$\frac{e^2}{4\pi\varepsilon_0 r \varepsilon}$$

mali koščki imajo Coulombska energije

$l \sim 10\,nm$

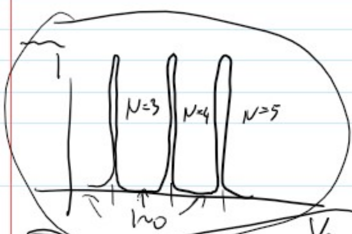
$$E_C = \frac{e^2}{4\pi\varepsilon_0 r} = \frac{e^2}{4\pi\varepsilon_0 \hbar c}\frac{\hbar c}{r} \qquad \frac{\hbar c}{r} = \frac{1}{137}\,\frac{1240\,eV\,nm}{2\pi r}$$

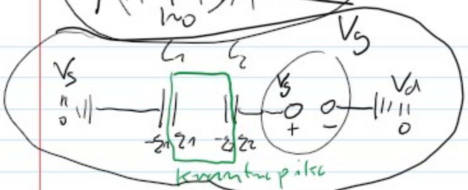
$$\frac{e^2}{4\pi\varepsilon_0\hbar c} = 1/137 \qquad \sim \frac{eV\,nm}{r}$$

diskretni nivoji

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

$$\frac{\hbar^2}{2m\,l^2} = \frac{\hbar^2 c^2}{2mc^2\,l^2} = \frac{1000^2\,eV^2\,nm^2}{10^6\,MeV\,l^2} = eV\,\frac{nm^2}{l^2}$$





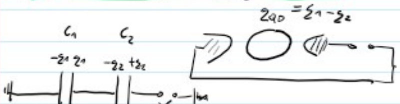
$q_1 - q_2 = eN \leftarrow$ cel

$V_1 = \frac{q_1}{C_1} \qquad V_2 = \frac{q_2}{C_2} \qquad \mathcal{H} = E + pV$

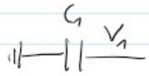
$V_g = V_1 + V_2$

$$\boxed{\mathcal{H} = \frac{q_1^2}{2C_1} + \frac{q_2^2}{2C_2} - q_2 V_g}$$

$q_{QD} = q_1 - q_2$



zunaj (v bakr. žici) je $\vec{E} = 0$



$E = \frac{C_1 V_1^2}{2}$

$q_1 - q_2 = eN$

$V_1 = \frac{q_1}{C_1} \qquad V_2 = \frac{-q_2}{C_2}$

$V_g = V_1 + V_2$

$V = \frac{q}{C}$

„Entalpija": $V\,dq = \frac{q_1^2}{2C_1}$

$$\mathcal{H} = q_1^2/2C_1 + q_2^2/2C_2 - q_2 V_g$$

$\mathcal{H} =$

$V_g = q_1/C_1 + q_2/C_2$

$V_g C_1 = q_1 + q_2\, C_1/C_2$

$q_1 = V_g C_1 - q_2 \frac{C_1}{C_2}$

$q_1 - q_2 = eN$

$V_g \quad /C_1 \quad /C_2$

$V_g C_1 = q_1 + q_2 C_1/C_2$ $\qquad q_1 = V_g C_1 - q_2 \frac{C_1}{C_2}$

$q_1 - q_2 = eN$

$q_2(-1 - C_1/C_2) + V_g C_1 = eN$

$- q_2 = \frac{eN - V_g C_1}{1 + C_1/C_2}$

$q_2 = \frac{V_g C_1 - eN}{1 + C_1/C_2}$

$q_1 = eN + q_2 = \frac{V_g C_1 \cancel{-eN} \cancel{+eN} + eN C_1/C_2}{1 + C_1/C_2}$

$\mathcal{H} = q_1^2/2C_1 + q_2^2/2C_2 - q_2 V_g$

$$\mathcal{H} = \frac{(eN C_1/C_2 + V_g C_1)^2}{2C_1(1+C_1/C_2)^2} + \frac{(V_g C_1 - eN)^2}{2C_2(1+C_1/C_2)^2} + \frac{V_g C_1 + eN}{1 + C_1/C_2} V_g$$

$$\mathcal{H} = \frac{\frac{C_2}{C_1}(eN C_1/C_2 + V_g C_1)^2}{2C_2(1+C_1/C_2)^2} + \frac{(V_g C_1 - eN)^2}{2C_2(1+C_1/C_2)^2} + \frac{V_g C_1 + eN}{1 + C_1/C_2} V_g$$

$$= \frac{1}{2C_2(1+C_1/C_2)^2}\Big[e^2N^2(1 + C_1/C_2) + eN V_g C_1(2-2/ + V_g^2 C_1^2\left(\frac{C_2}{C_1} + 1\right)\Big] + \frac{V_g C_1 + eN}{1 + C_1/C_2} V_g$$

$$= V_g^2 C_2 C_1 \left(\frac{C_1}{C_2} + 1\right)\Big]$$

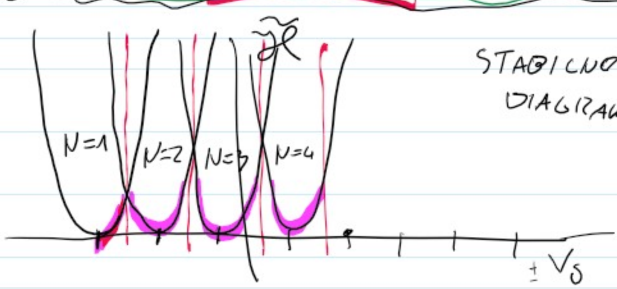
$$= \frac{1}{2C_2(1+C_1/C_2)}\left[e^2N^2 + V_g^2 C_2 C_1\right] + \frac{-2V_g^2 C_2 C_1 + eN V_g C_2 2}{2(C_1 + C_2)}$$

$$= \frac{1}{2(C_2 + C_1)}\left[e^2N^2 + 2eN V_g C_2 - V_g^2 C_1 C_2\right]$$

eN

$$= \frac{1}{2(C_2 + C_1)}\left[(eN + V_g C_2)^2 - V_g^2 C_2 (C_2 + C_1)\right]$$

$$\boxed{\mathcal{H} = \frac{1}{2(C_2 + C_1)}(eN + V_g C_2)^2 - \frac{V_g^2 C_2}{2}}$$



STABILNOSTNI DIAGRAM

$\tilde{\mathcal{H}} = \mathcal{H} + \frac{V_g^2 C_2}{2}$

$$V_g = -\frac{eN}{C_2}$$

$\mathcal{H}$ $\qquad q = -V_g C_2$

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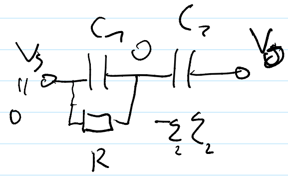


$$q = -q_2 = -V_g C_2$$

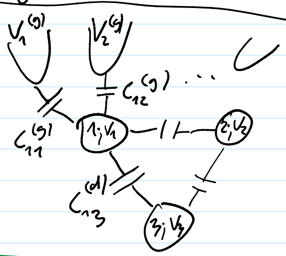
$$\frac{e^2}{C} \qquad T = RC$$

$$\Delta E\, T \gg \hbar$$

$$\frac{e^2}{\cancel{C}} R\cancel{C} \gg \hbar$$

$$\frac{e^2}{\hbar} \gg \frac{1}{R} = G \qquad G \ll \frac{e^2}{\hbar}$$

Stabilnostni diagram za N-kv. pik sklopljenih na M elektrod

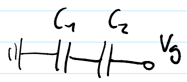


$$\boxed{\mathcal{H} = \frac{1}{2} \left(\underline{e} - \underline{q}\right)^T \underline{\underline{C}}^{-1} \left(\underline{e} - \underline{q}\right)}$$

$$\underline{e} = e\begin{pmatrix} N_1 \\ N_2 \\ \vdots \\ N_N \end{pmatrix} \qquad \underline{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix} \qquad q_i = -\sum C_{ik}^{(g)} V_k^{(g)}$$

$$\left(\underline{\underline{C}}\right)_{ij} = C_{ij} \qquad C_{ij} = \begin{cases} \sum_j C_{ij}^{(d)} + \sum_k C_{ik}^{(g)}; & i = j \\ -C_{ij}^{(d)} \end{cases}$$

zgled:



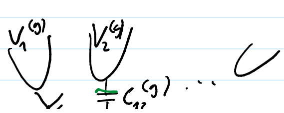
$$\underline{\underline{C}} = C_1 + C_2 \qquad \mathcal{H} = \frac{1}{2}\left(e + C_2 V_g\right) \frac{1}{C_1 + C_2} \left(e + C_2 V_g\right)$$

Dokaz:

$$\mathcal{H} = \frac{1}{2} \sum_{i<j} C_{ij}^{(d)} \left(V_i - V_j\right)^2 + \frac{1}{2} \sum_{ik} C_{ik}^{(g)} \left(V_i - V_k^{(g)}\right)^2$$

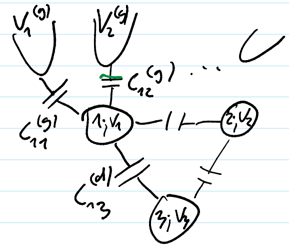
$$- \sum_{ik} q_{ik} V_k^{(g)}$$

$$q_{ik} = \left(V_k^{(g)} - V_i\right) C_{ik}^{(g)}$$



$$\sum_{ik} q_{ik} V_k$$

$$q_{ik} = \left(V_k^{(g)} - V_i\right) C_{ik}^{(g)}$$



$$\mathcal{H} = \frac{1}{2} \sum_{i<j} C_{ij}^{(d)} (V_i - V_j)^2 +$$

$$+ \frac{1}{2} \sum_{ik} C_{ik}^{(g)} \left(V_i^2 + V_k^{(g)2} \cancel{- 2 V_i V_k^{(g)}}\right)$$

$$- \sum_{ik} \left(V_k^{(g)} \cancel{- V_i}\right) C_{ik}^{(g)} V_k^{(g)}$$

$$\boxed{\mathcal{H} = \frac{1}{2} \sum_{i<j} C_{ij}^{(d)} (V_i - V_j)^2 + \underbrace{\frac{1}{2} \sum_{ik} C_{ik}^{(g)} V_i^2}_{A}} \underbrace{- \frac{1}{2} \sum_{ik} C_{ik}^{(g)} V_k^{(g)2}}_{\text{opuztimo}}$$

$$C_{ij}^{(d)} = C_{ji}^{(d)} \qquad C_{ii}^{(d)} = 0$$

$$\mathcal{H} = \frac{1}{4} \sum_{ij} C_{ij}^{(d)} (V_i - V_j)^2 + A$$

$$= \frac{1}{4} \sum_{ij} C_{ij}^{(d)} \left[\left(V_i^2 + V_j^2\right) - 2 V_i V_j \right] + A$$

$$C_{ij} V_j^2 = C_{ji} V_j^2$$

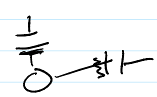
$$=$$

$$= \frac{1}{2} \sum_i V_i^2 \sum_j C_{ij}^{(d)} + \frac{1}{2} \sum_{ik} C_{ik}^{(g)} V_i^2 - \frac{1}{2} \sum_{ij} C_{ij}^{(d)} V_i V_j$$

$$= \frac{1}{2} \sum_{ij} V_i C_{ij} V_j \qquad C_{ij} = \begin{cases} \sum_{j'} C_{ij'}^{(d)} + \sum_{k} C_{ik}^{(g)} ; & i = j \\ - C_{ij}^{(d)} ; & i \neq j \end{cases}$$

$$\boxed{\mathcal{H} = \frac{1}{2} \underline{V}^T \underline{\underline{C}}\, \underline{V}}$$

naboj na i-ti piki



$$N_i e = \sum_j C_{ij}^{(d)} (V_i - V_j) + \sum_k C_{ik}^{(g)} \left(V_i - V_k^{(g)}\right)$$

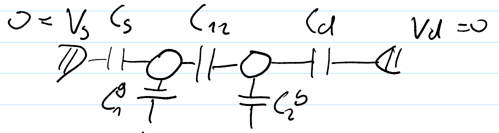
$$N_i e + \underbrace{\sum_k C_{ik}^{(g)} V_k^{(g)}}_{-q_i} = \left(\sum_j C_{ij}^{(d)} + \sum_k C_{ik}^{(g)}\right) V_i - \sum_j C_{ij}^{(d)} V_j$$

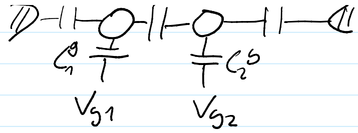
$$N_i e - q_i = \sum_j C_{ij} V_j$$

$$\underline{e} - \underline{q} = \underline{\underline{C}}\, \underline{V} \qquad \underline{V} = \underline{\underline{C}}^{-1} (\underline{e} - \underline{q})$$

$$\boxed{\mathcal{H} = \frac{1}{2} (\underline{e} - \underline{q})^T \underline{\underline{C}}^{-1} \cancel{\underline{\underline{C}}\, \underline{\underline{C}}^{-1}} (\underline{e} - \underline{q})}$$

Vaja: Dvojna kvantna pika





$$\underline{\underline{C}} = \begin{pmatrix} C_1 & -C_{12} \\ -C_{12} & C_2 \end{pmatrix} \qquad \begin{aligned} C_1 &= C_s + C_1^g + C_{12} = C_{12}(1+\alpha_1) \\ C_2 &= C_d + C_2^g + C_{12} = C_{12}(1+\alpha_2) \end{aligned}$$

$\alpha_1, \alpha_2 > 0$

$$\underline{\underline{C}}^{-1} = \frac{1}{C_1C_2 - C_{12}^2}\begin{pmatrix} C_2 & C_{12} \\ C_{12} & C_1 \end{pmatrix} = \frac{C_{12}}{C_1C_2 - C_{12}^2}\begin{pmatrix} 1+\alpha_2 & 1 \\ 1 & 1+\alpha_1 \end{pmatrix}$$

$1/C_0$

$$\mathcal{H} = (\underline{e} - \underline{q})^T \frac{\underline{\underline{C}}^{-1}}{2} (\underline{e} - \underline{q}) \qquad q_{1,2} = -C_{1,2}^g V_{g1,2} + C_s V_s, \quad V_s = 0$$

$$= \frac{1}{2}(eN_1 - q_1, eN_2 - q_2)\, \underline{\underline{C}}^{-1} \begin{pmatrix} eN_1 - q_1 \\ eN_2 - q_2 \end{pmatrix}$$

$$= \frac{1}{2C_0}\left[(1+\alpha_2)(eN_1 - q_1)^2 + (1+\alpha_1)(eN_2 - q_2)^2 + 2(eN_1 - q_1)(eN_2 - q_2)\right]$$

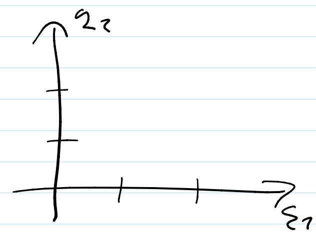
$$\boxed{\mathcal{H} = \alpha_2(eN_1 - q_1)^2 + \alpha_1(eN_2 - q_2)^2 + (eN_1 - q_1 + eN_2 - q_2)^2}$$

Če je N_1, N_2 osnovno stanje pri q_1, q_2 (V_1^g, V_2^g)

$q_1 \to q_1 + eM_1$, $q_2 \to q_2 + eM_2$

bo osn. stanje pri $N_1 + M_1$, $N_2 + M_2$

periodično tlakovanje



Kdaj je stabilna konf. z $(0,0)$ elektroni?

$\mathcal{H}(0,0) < \mathcal{H}(1,0)$

$\mathcal{H}(0,0) = \alpha_2 q_1^2 + \alpha_1 q_2^2 + (-q_1 - q_2)^2$

$\mathcal{H}(1,0) = \alpha_2(e - q_1)^2 + \alpha_1 q_2^2 + (e - q_1 - q_2)^2$

$= \alpha_2 e^2 - 2\alpha_2 e q_1 + \alpha_2 q_1^2 + \alpha_1 q_2^2 + e^2 - 2e(q_1 + q_2) + (q_1 + q_2)^2$

$0 < (1+\alpha_2)e^2 - 2e(1+\alpha_2)q_1 - 2eq_2 \quad : (1+\alpha_2)$

$0 < e^2 - 2eq_1 - \frac{2eq_2}{(1+\alpha_2)}$

$$\boxed{-\frac{e}{2} > q_1 + \frac{q_2}{(1+\alpha_2)} > \frac{e}{2}}$$

$e = -|e|$

$\mathcal{H}(0,0) < \mathcal{H}(-1,0)$

$q_1 + \frac{q_2}{1+\alpha_2} = -\frac{e}{2}$

$\mathcal{H}(0,0) < \mathcal{H}(0,\pm 1)$

$\vec{n} \cdot \vec{a} = -e/_2$

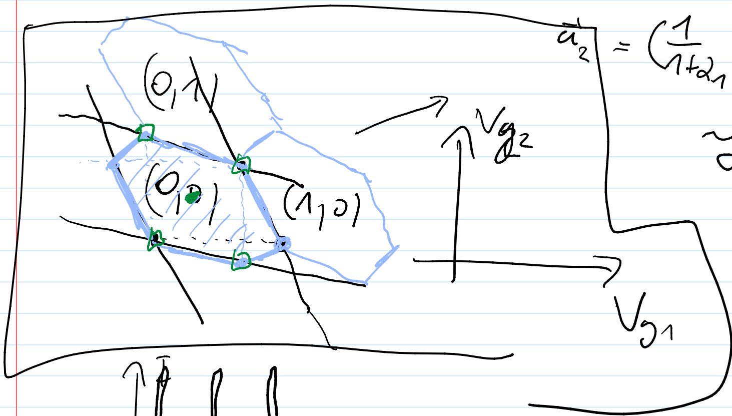
$$\mathcal{H}(0,0) < \mathcal{H}(0,\pm 1)$$

$$\boxed{-\frac{e}{2} > \frac{q_1}{1+\alpha_1} + q_2 > \frac{e}{2}}$$

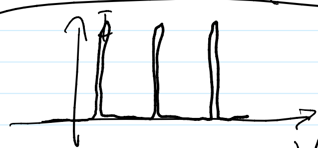
$$\vec{q} \cdot \vec{a} = -e/2$$

$$\vec{a}_1 = \left(1, \frac{1}{1+\alpha_2}\right)$$

$$\vec{a}_2 = \left(\frac{1}{1+\alpha_1}, 1\right)$$

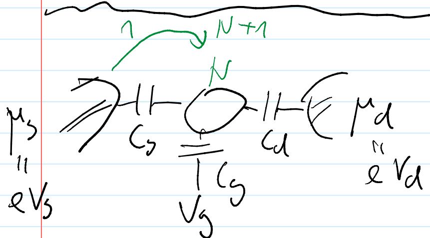


$$\mathcal{H}(0,0) < \mathcal{H}(1,-1) < \mathcal{H}(-1,1)$$



V_{g1} (pri malem $V_s - V_d$)

Končni bias (Coulombski diamanti)



$$\mu_{QD} = \mathcal{H}(N+1) - \mathcal{H}(N)$$

$$\mu_s > \mu_{QD} > \mu_d$$

$$\mathcal{H} = \frac{(eN - q)^2}{2\underbrace{(C_s + C_d + C_g)}_{C_\Sigma}}$$

$$q = -C_g V_g - C_s V_s - C_d V_d$$

$$\cancel{-C\frac{U_{sd}}{2} - C\left(-\frac{U_{sd}}{2}\right)}$$

$$C_s = C_d = C \qquad V_s = U_{sd}/2 \qquad V_d = -U_{sd}/2$$

$$q = -C_g V_g$$

$$\mathcal{H} = \frac{(eN + C_g V_g)^2}{2C_\Sigma}$$

$$\mu_{QD} = \frac{e^2}{2C_\Sigma}\left[\left((N+1) + \frac{C_g V_g}{e}\right)^2 - \left(N + \frac{C_g V_g}{e}\right)^2\right]$$

$$\mu_{QD} = \frac{e^2}{2C_\Sigma}\left[\cancel{N^2} + 2N + 1 + 2(\cancel{N}+1)\frac{C_g V_g}{e} + \cancel{\left(\frac{C_g V_g}{e}\right)^2} - \cancel{N^2} - \cancel{2N\frac{C_g V_g}{e}} - \cancel{\left(\frac{C_g V_g}{e}\right)^2}\right]$$

$$\cancel{- N^2} \pm \cancel{2N \frac{C_g V_g}{e}} - \cancel{\left(\frac{C_g V_g}{e}\right)^2} \Big]$$

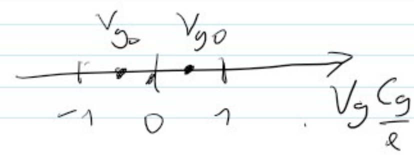
$$\mu_{g0} = \frac{e^2}{C_\Sigma}\left[N + \frac{1}{2} + \frac{C_g V_g}{e}\right]$$

$$\frac{e U_{sd}}{2} > \frac{e^2}{C_\Sigma}\left[N + 1/2 + \frac{C_g V_g}{e}\right] > -\frac{e U_{sd}}{2} \quad / : \frac{e}{2}$$

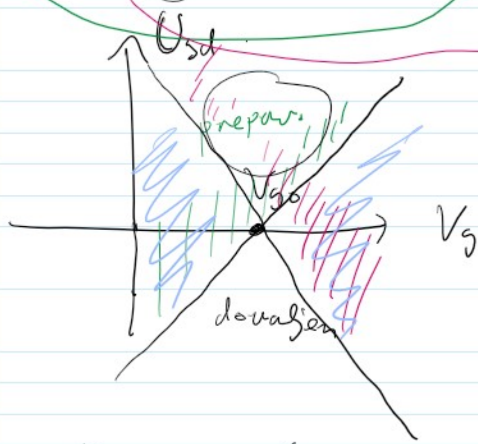
$$U_{sd} < \frac{e}{C_\Sigma}\left[2N + 1 + \frac{2 C_g V_g}{e}\right] < -U_{sd}$$

pogoj, da tok teče $|U_{sd}| \to 0$

$$V_g = \left(-\frac{2N+1}{2C_g}\right) \cdot e = V_{g0} = +\frac{e}{C_g}\left(-N - 1/2\right)$$

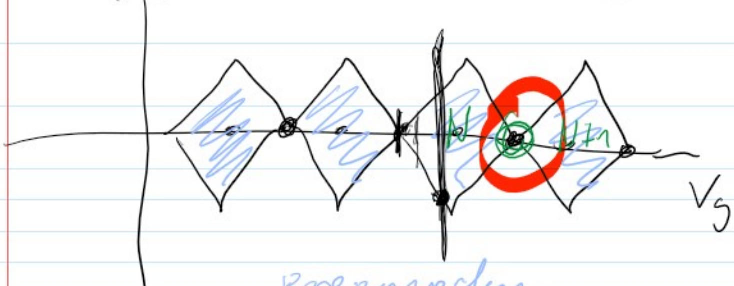


$$U_{sd} < \frac{2C_g}{C_\Sigma}\left[V_g - V_{g0}\right] < -U_{sd}$$



$$U_{sd} < \frac{2C_g}{C_\Sigma}(-1)(V_g - V_{g0})$$

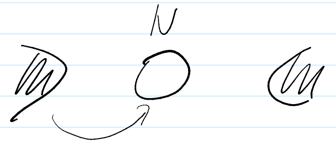
Coulombski diamant



Coulombska blokada

Obravnava transporta z master enačbo

tok kanien



tuk kancen

p_N ... verj., da je na piki N elektronov

p_{N+1} ... različen od 0

$\sum_n p_n = 1$

iz kjentali

$$\frac{dp_N}{dt} = \Gamma_{N-1}^{\to gd} p_{N-1} + \Gamma_{N+1}^{gd \to} p_{N+1} - \Gamma_N^{gd\to} p_N - \Gamma_N^{\to gd} p_N$$

gd → kontakt

št. procesov na enoto časa

$+ p_{N-2} \Gamma_{N-2}^{\dots}$

$+ \dots$

p_{N-2}, ... majhni