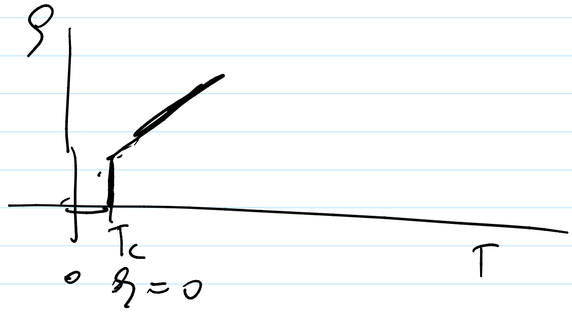
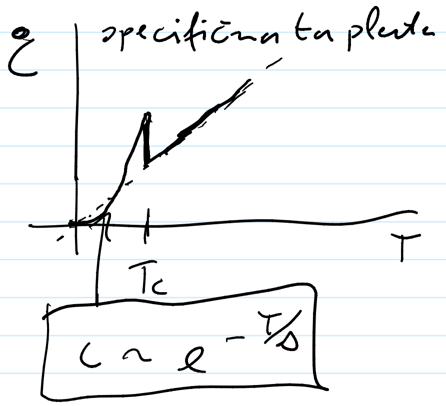
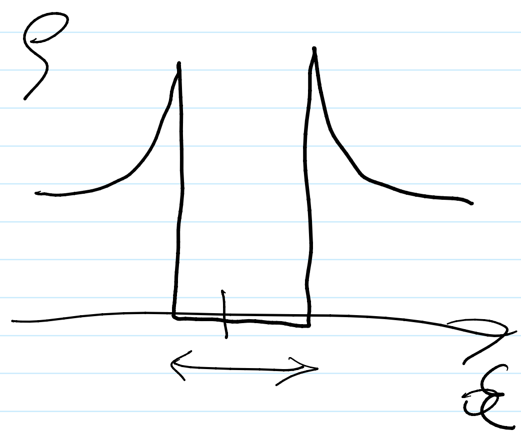
Coulombske blokade in superprevodnost







mikroskopski izvor

$$|\psi_0\rangle = \prod_{k\sigma} c^+_{k\sigma} |0\rangle$$

$$|\psi_{BCS}\rangle_\varphi = \prod_k \left(u_k + v_k e^{i\varphi} c^+_{k\uparrow} c^+_{-k\downarrow} \right) |0\rangle$$

$$H = \sum_k \varepsilon_k n_k + \sum_{kk'\sigma\sigma'} U_{kk'\ldots} \, c^+_{k\sigma} c^+ c\, c$$

$$n_k = c^+_{k\uparrow} c_{k\uparrow} + c^+_{k\downarrow} c_{k\downarrow}$$

$$c_i^+ c_i \qquad c_j^+ c_j$$

$$\underbrace{\langle c_i^+ c_i \rangle}_{n_i} c_j^+ c_j$$

$$\langle\Psi| c^+ c^+ |\Psi\rangle = \begin{cases} 0; & \text{ni superpozic.} \end{cases}$$

$$H_{BdG} = \sum_k \varepsilon_k n_k + \sum_k \Delta e^{i\varphi} c^+_{k\uparrow} c^+_{-k\downarrow} + h.c.$$

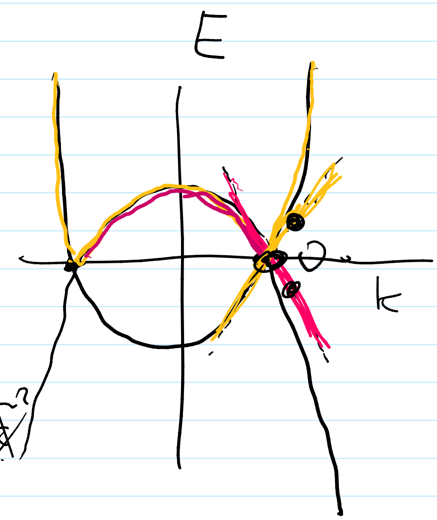
$$c^+_{-k\downarrow} = \tilde{d}_{k\uparrow} \qquad \underbrace{U\langle c^+_{k\uparrow} c^+_{-k\downarrow}\rangle}_{\Delta}$$

$$c_{-k\downarrow} = d^+_{k\uparrow}$$

BdG

$$\begin{pmatrix} \hat{H} & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & -\hat{H}^* \end{pmatrix} \begin{pmatrix} \psi_e(\vec{r}) \\ \psi_h(\vec{r}) \end{pmatrix} = E \begin{pmatrix} \psi_e(\vec{r}) \\ \psi_h(\vec{r}) \end{pmatrix}$$

$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(r) - E_F$$



$\Delta = 0$

$$\psi_e(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$$
$$\psi_h(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$$

$$\hat{H}: \quad \tilde{E} = \frac{k^2\hbar^2}{2m} = \frac{\hbar^2 k_F^2}{2m} + \frac{\hbar^2 k_F \tilde{k}}{m} + \cancel{\tilde{k}^2}$$

$$k = k_F + \tilde{k}$$

$$v_F = \frac{\hbar k_F}{m}$$

$$= E_F + \hbar v_F \tilde{k}$$

$\Delta = 0$ $\qquad \boxed{E = \pm \hbar v_F \tilde{k}}$

$\Delta > 0$

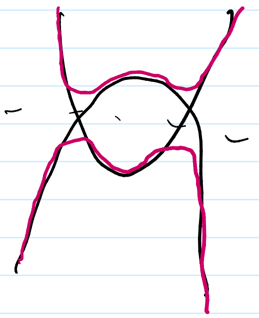
$$\begin{pmatrix} \tilde{E} & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & -\tilde{E} \end{pmatrix} \begin{pmatrix} \psi_e^0 \\ \psi_h^0 \end{pmatrix} = E \begin{pmatrix} \psi_e^0 \\ \psi_h^0 \end{pmatrix} \qquad \begin{aligned} \psi_e &= e^{i(k_F+\tilde{k})x} \psi_e^0 \\ \psi_h &= e^{i(k_F+\tilde{k})x} \psi_h^0 \end{aligned}$$

$$\tilde{E} = \hbar v_F \tilde{k}$$

$$\begin{vmatrix} \tilde{E} - E & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & -\tilde{E} - E \end{vmatrix} = 0 \qquad \tilde{E} = \hbar v_F \tilde{k}$$

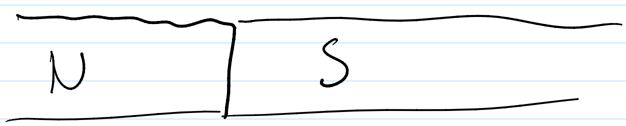
$$E^2 - \tilde{E}^2 - \Delta^2 = 0$$

$$E^2 = \tilde{E}^2 + \Delta^2 \qquad E = \pm\sqrt{\tilde{E}^2 + \Delta^2}$$

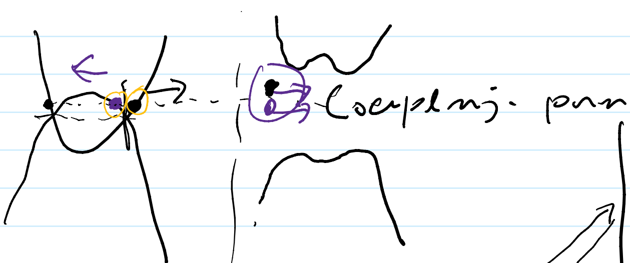


2Δ : REŽA, ENOEL. EKSITACIJ (PRI TEJ ENERGIJI.

Andreeva tuneliranje



$x<0$ $\quad$ $x>0$



$$\psi_e = e^{ik_F x}\tilde{\psi}_e(x)$$

$$\left(\frac{-\hbar^2\nabla^2}{2m}\right)\psi_e(x) = e^{ik_F x}\left(\frac{\hbar^2 k_F^2}{2m} + \frac{-\hbar^2 i k_F \nabla}{2m} + \frac{-\hbar^2}{2m}\nabla^2\right)\tilde{\psi}_e(x)$$

$$\begin{pmatrix} -i\hbar v_F\nabla & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & i\hbar v_F \nabla \end{pmatrix}\begin{pmatrix} \tilde{\psi}_e(x) \\ \tilde{\psi}_h(x) \end{pmatrix} = E\begin{pmatrix} \tilde{\psi}_e(x) \\ \tilde{\psi}_h(x) \end{pmatrix}$$

$\tilde{\psi} = C\,(e^{i\tilde{k}x} \ldots)$

$\Delta = 0$ N | S N

$$\tilde{\psi} = C \begin{pmatrix} e^{i\tilde{k}x} \\ r e^{-i\tilde{k}x} \end{pmatrix}$$

$$\begin{pmatrix} +i\hbar v_F K - E & \Delta e^{i\varphi} \\ \Delta e^{-i\varphi} & -i\hbar v_F K - E \end{pmatrix} \begin{pmatrix} \tilde{\psi}_e^0 \\ \tilde{\psi}_h^0 \end{pmatrix} = 0$$

$$E = \tilde{E} = \hbar v_F \tilde{k}$$

na S:

$$\tilde{\psi} = C \begin{pmatrix} \tilde{\psi}_e^0 \\ \tilde{\psi}_h^0 \end{pmatrix} e^{-Kx}$$

$$K = \frac{\sqrt{\Delta^2 - E^2}}{\hbar v_F}$$

$$\left(K^2 \hbar^2 v_F^2 - \Delta^2\right) = -E^2$$

$$E^2 = \Delta^2 - \hbar^2 v_F^2 K^2$$

$$(i\hbar v_F K - E)\tilde{\psi}_e^0 + \Delta \tilde{\psi}_h^0 e^{i\varphi} = 0$$

$$\tilde{\psi}_h^0 = \frac{\left(E/\Delta - i\sqrt{1 - (E/\Delta)^2}\right)\tilde{\psi}_e^0}{e^{i\varphi}} = \frac{(\cos\alpha - i\sin\alpha)\tilde{\psi}_e^0}{e^{i\varphi}}$$

$$E/\Delta = \cos\alpha$$

$$\frac{\tilde{\psi}_h^0}{\tilde{\psi}_e^0} = e^{-i\varphi} e^{-i\alpha} = e^{i\chi_e}$$

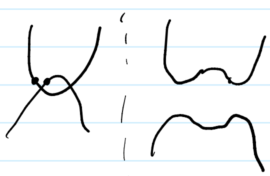
$$\psi(0^-) = \psi(0^+)$$

$$\begin{pmatrix} \tilde{\psi}_e \\ \tilde{\psi}_h \end{pmatrix} = \begin{pmatrix} e^{i\tilde{k}x} \\ r e^{-i\tilde{k}x} \end{pmatrix}\Bigg|_{0^-} = \begin{pmatrix} 1 \\ r \end{pmatrix} =$$

$$\tilde{\psi}_h / \tilde{\psi}_e = r = e^{i\chi_e} \qquad \chi_e = -\varphi - \alpha$$

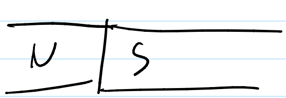
$$r_e = e^{i\chi_e} \qquad R_A = |r|^2 = 1$$

D.N 1:



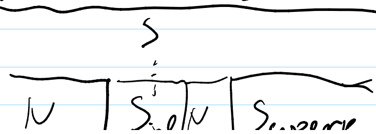
$$r_h = e^{i\varphi} e^{-i\alpha} = e^{i\chi_h}$$

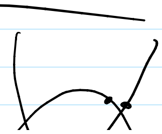
DN. 2:

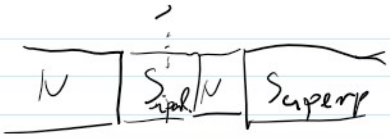


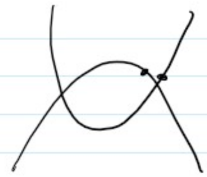
$$E > \Delta \qquad r_A = e^{-i\varphi}\left(E/\Delta - \left(\sqrt{E^2 - \Delta^2}\right)/\Delta\right)$$

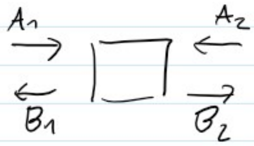
Przewodnictwo Andreeva









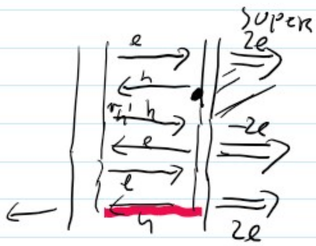


$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = S \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \qquad S_e = \begin{pmatrix} r_e & t_e' \\ t_e & r_e' \end{pmatrix} = \begin{bmatrix} r & t \\ t & r' \end{bmatrix}$$

$$S_e = S_e(E) = S(E_F + E) \doteq S(E_F)$$

$$S_h = S^*(E_F - E) \doteq S^*(E_F) \qquad S_h = \begin{pmatrix} r_h & t_h' \\ t_h & r_h' \end{pmatrix} = \begin{bmatrix} r^* & t^* \\ t^* & r'^* \end{bmatrix}$$

$$r_A = t_e e^{i\chi_e} t_h' + t_e \left(e^{i\chi_e} r_h' e^{i\chi_h} r_e' \right) e^{i\chi_e} t_h'$$
$$+ t_e \left(\qquad \right)^2 e^{i\chi_e} t_h'$$
$$+ \ \dots$$



$$r_A = \frac{t_e e^{i\chi_e} t_h}{1 - e^{i(\chi_e + \chi_h)} r_h' r_e'} = \frac{|t|^2 e^{i\chi_e}}{1 - (-1)|r'|^2}$$

$$e^{i\chi_e} = e^{-i\varphi} e^{-i\alpha} \qquad \alpha = \arccos E/\Delta$$
$$e^{i\chi_h} = e^{i\varphi} e^{-i\alpha} \qquad E \ll \Delta$$
$$e^{i(\chi_e + \chi_h)} = e^{-2i\alpha} \qquad \alpha \doteq \pi/2$$
$$\doteq e^{-i\pi} = -1$$

$$|t|^2 = T$$
$$|r'|^2 = |r|^2 = 1 - T$$

$$r_A = e^{i\chi_e} \frac{T}{2 - T}$$

$$R_A = |r_A|^2 = \frac{T^2}{(2-T)^2}$$

$$\boxed{G = \frac{2e^2}{h} \cdot 2 R_A = 2 G_0 R_A}$$

$T \sim 1 \qquad G = 2G_0$

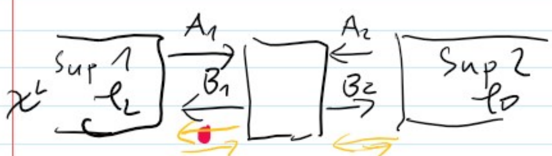
T mali $\qquad G = 2G_0 \dfrac{T^2}{4}$

navadna

$G = G_0$

$G = G_0 T$

Andreeva vezana stanja



$$\underline{B}^e = \begin{pmatrix} B_1^e \\ B_2^e \end{pmatrix} = S \begin{pmatrix} A_1^e \\ A_2^e \end{pmatrix} = S \underline{A}^e \qquad \begin{pmatrix} B_1^h \\ B_2^h \end{pmatrix} = S^* \begin{pmatrix} A_1^h \\ A_2^h \end{pmatrix} = S^* \underline{A}^h$$

$$\begin{pmatrix} \underline{B}^e \\ \underline{B}^h \end{pmatrix} = \underbrace{\begin{bmatrix} S & 0 \\ 0 & S^* \end{bmatrix}_{4\times4}}_{S_{4\times4}} \begin{pmatrix} \underline{A}^e \\ \underline{A}^h \end{pmatrix}$$

$$\begin{pmatrix} A_1^e \\ A_1^h \end{pmatrix} = \begin{pmatrix} 0 & e^{i\chi_h^L} \\ e^{i\chi_e^L} & 0 \end{pmatrix} \begin{pmatrix} B_1^e \\ B_1^h \end{pmatrix} \qquad \begin{pmatrix} A_2^e \\ A_2^h \end{pmatrix} = \begin{pmatrix} 0 & e^{i\chi_h^0} \\ e^{i\chi_e^0} & 0 \end{pmatrix} \begin{pmatrix} B_2^e \\ B_2^h \end{pmatrix}$$

$$\begin{pmatrix} A_1^e \\ A_2^e \end{pmatrix} = \underbrace{\begin{pmatrix} e^{i\chi_h^L} & \\ & e^{i\chi_h^0} \end{pmatrix}}_{S_s} \begin{pmatrix} B_1^h \\ B_2^h \end{pmatrix}$$

$$\begin{pmatrix} A_1^h \\ A_2^h \end{pmatrix} = \underbrace{\begin{pmatrix} e^{i\chi_e^L} & \\ & e^{i\chi_e^0} \end{pmatrix}}_{\tilde{S}_s} \begin{pmatrix} B_1^e \\ B_2^e \end{pmatrix}$$

$$\begin{pmatrix} \underline{A}^e \\ \underline{A}^h \end{pmatrix} = \underbrace{\begin{pmatrix} & S_s \\ \tilde{S}_s & \end{pmatrix}_{4\times4}}_{S_{s\,4\times4}} \begin{pmatrix} \underline{B}^e \\ \underline{B}^h \end{pmatrix}$$

$$\boxed{\underline{A} = S_{s\,4\times4} \cdot S_{4\times4}\, \underline{A}}$$

$$\det\left(S_{s\,4\times4} \cdot S_{4\times4} - 1\right) = 0$$

$$e^{i\chi} = e^{-i\alpha} e^{-i\varphi} \qquad \cos\alpha = E/\Delta$$

energija vez. stanja

$$\boxed{E = \Delta \sqrt{1 - T \sin^2\left(\frac{\varphi_L - \varphi_0}{2}\right)}}$$