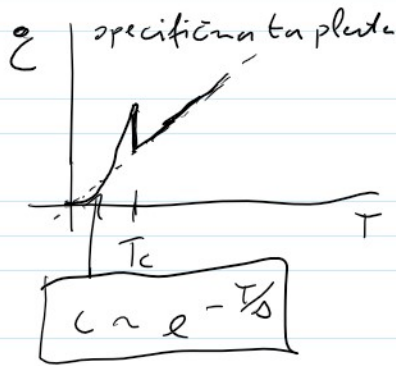
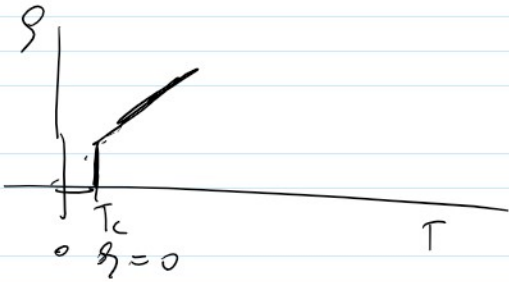


Coulombske blokade in superprevodnost



mikroskopski izvor

$$| \Psi_0 \rangle = \prod_{k \in \mathcal{B}} c_{k \downarrow}^+ | 0 \rangle$$



$$| \Psi_{BCS} \rangle = \prod_k ( u_k + v_k e^{i\phi} c_{k \uparrow}^+ c_{-k \downarrow}^+ ) | 0 \rangle$$

$$H = \sum_k \epsilon_k n_k + \sum_{k k' \dots} U_{k k' \dots} c_{k \downarrow}^+ c_{k' \downarrow}^+ c_{k' \uparrow} c_{k \uparrow}$$

$$n_k = c_{k \uparrow}^+ c_{k \uparrow} + c_{k \downarrow}^+ c_{k \downarrow}$$

$$\langle c_i^+ c_i \rangle_{n_i} \langle c_j^+ c_j \rangle$$

$$\langle c_i^+ c_i \rangle = \int 0 \quad \text{ni superprevodnosti}$$

$$H_{BdG} = \sum_k \epsilon_k n_k + \sum_k \Delta e^{i\phi} c_{k \uparrow}^+ c_{-k \downarrow}^+ + h.c.$$

$$H_{BdG} = \sum_k \tau_k m_k + \left( \sum_k \Delta e^{i\phi} (c_{k\uparrow}^\dagger) c_{-k\downarrow}^\dagger + h.c. \right)$$



$$c_{-k\downarrow}^\dagger = \tilde{d}_{k\uparrow} \quad U \langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \rangle$$

$$c_{-k\downarrow} = d_{k\uparrow} \quad \Delta$$

BdG

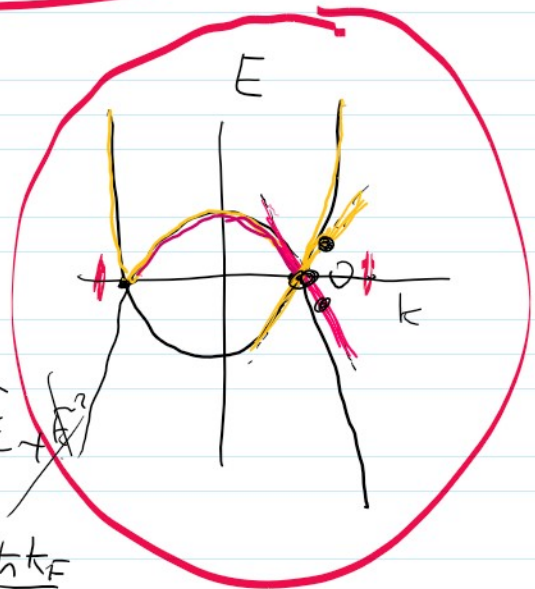
$$\begin{pmatrix} \hat{H} & \Delta e^{i\phi} \\ \Delta e^{-i\phi} & -\hat{H}^* \end{pmatrix} \begin{pmatrix} \psi_e(\vec{r}) \\ \psi_h(\vec{r}) \end{pmatrix} = E \begin{pmatrix} \psi_e(\vec{r}) \\ \psi_h(\vec{r}) \end{pmatrix}$$

$$\hat{H} = \frac{\hbar^2}{2m} \nabla^2 + V(r) - E_F$$

$\Delta = 0$

$$\psi_e(\vec{r}) = e^{i\vec{k} \cdot \vec{r}}$$

$$\psi_h(\vec{r}) = e^{i\vec{k} \cdot \vec{r}}$$



$$\hat{H} : \tilde{E} = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 k_F^2}{2m} + \frac{\hbar^2 k_F \tilde{k}}{m}$$

$$k = k_F + \tilde{k}$$

$$v_F = \frac{\hbar k_F}{m}$$

$$= E_F + \hbar v_F \tilde{k}$$

$\Delta = 0$

$\Delta > 0$

$$\begin{pmatrix} \tilde{E} & \Delta e^{i\phi} \\ \Delta e^{-i\phi} & -\tilde{E} \end{pmatrix} \begin{pmatrix} \psi_e^0 \\ \psi_h^0 \end{pmatrix} = E \begin{pmatrix} \psi_e^0 \\ \psi_h^0 \end{pmatrix}$$

$$\psi_e = e^{i(k_F + \tilde{k}) \cdot \vec{r}} \psi_e^0$$

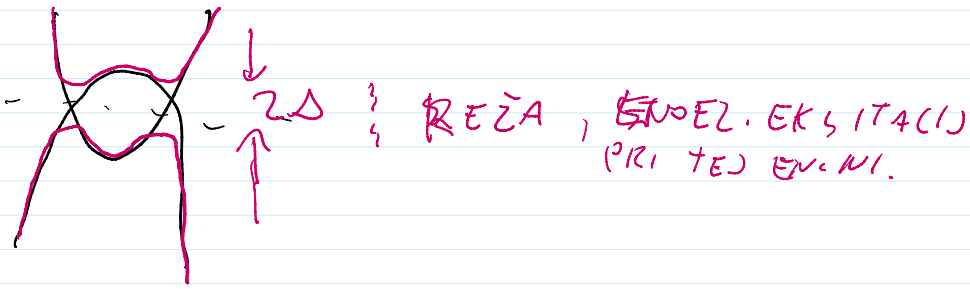
$$\psi_h = e^{i(k_F + \tilde{k}) \cdot \vec{r}} \psi_h^0$$

$$\tilde{E} = \hbar v_F \tilde{k}$$

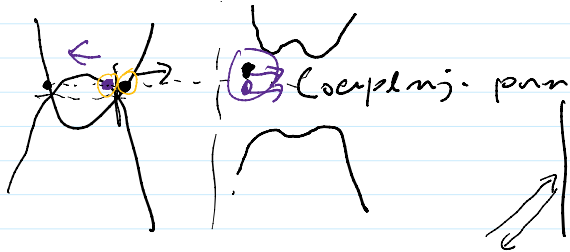
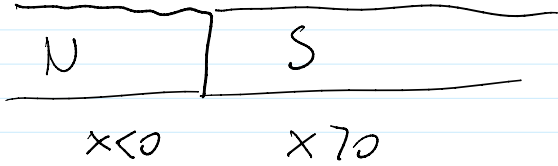
$$\begin{vmatrix} \tilde{E} - E & \Delta e^{i\phi} \\ \Delta e^{-i\phi} & -\tilde{E} - E \end{vmatrix} = 0$$

$$E^2 - \tilde{E}^2 - \Delta^2 = 0$$

$$E^2 = \tilde{E}^2 + \Delta^2 \quad E = \pm \sqrt{\tilde{E}^2 + \Delta^2}$$



## Andreeva tunneliranje



$$\left( \frac{-\hbar^2 \nabla^2}{2m} \right) \psi_e(x) = \frac{\hbar^2 k_F^2}{2m} \psi_e(x) + \frac{-\hbar^2 i k_F \nabla}{2m} \psi_e(x) + \frac{\hbar^2 \Delta^2}{2m} \tilde{\psi}_e(x)$$

$\psi_e = e^{ik_F x} \tilde{\psi}_e(x)$

$$\begin{pmatrix} -i\hbar v_F \nabla & \Delta e^{i\ell} \\ \Delta e^{-i\ell} & i\hbar v_F \nabla \end{pmatrix} \begin{pmatrix} \tilde{\psi}_e(x) \\ \tilde{\psi}_h(x) \end{pmatrix} = E \begin{pmatrix} \tilde{\psi}_e(x) \\ \tilde{\psi}_h(x) \end{pmatrix}$$



$$\tilde{\psi} = \begin{pmatrix} e^{i\tilde{k}x} \\ r e^{-i\tilde{k}x} \end{pmatrix}$$

$$E = E = \hbar v_F \tilde{k}$$

$$\begin{pmatrix} +i\hbar v_F k - E & \Delta e^{i\ell} \\ \Delta e^{-i\ell} & -i\hbar v_F k - E \end{pmatrix} \begin{pmatrix} \tilde{\psi}_e \\ \tilde{\psi}_h \end{pmatrix} = 0$$

na S:

$$\tilde{\psi} = \begin{pmatrix} \tilde{\psi}_0 \\ \tilde{\psi}_2 \\ \tilde{\psi}_h \end{pmatrix} e^{-Kx}$$

$$K = \frac{\sqrt{\Delta^2 - E^2}}{\hbar v_F}$$

$$(K^2 \hbar^2 v_F^2 - \Delta^2) = -E^2$$

$$E^2 = \Delta^2 - \hbar^2 v_F^2 K^2$$

$$(i\hbar v_F k - E) \tilde{\psi}_e + \Delta \tilde{\psi}_h e^{i\ell} = 0$$

$$\tilde{\psi}_h = \frac{(E - i\sqrt{1 - \frac{E^2}{\Delta^2}}) \tilde{\psi}_e}{e^{i\ell}} = \frac{(\cos 2\theta - i \sin 2\theta) \tilde{\psi}_e}{e^{i\ell}}$$

$$\frac{\psi_0}{\psi_e} = e^{-i\ell} e^{-i\alpha} = e^{i\chi_e} \quad E/\Delta = \cos 2$$

$$\psi(0^-) = \psi(0^+)$$

$$\begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix} = \begin{pmatrix} e^{ikx} \\ r e^{-ikx} \end{pmatrix} \Big|_{0^-} = \begin{pmatrix} 1 \\ r \end{pmatrix} =$$

$$\frac{\psi_h}{\psi_e} = r = e^{i\chi_e} \quad \chi_e = -\ell - \alpha$$

$$r_e = e^{i\chi_e} \quad R_A = |r|^2 = 1$$

D.N. 1:



$$r_h = e^{i\ell} e^{-i\alpha} = e^{i\chi_h}$$

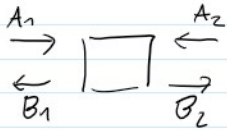
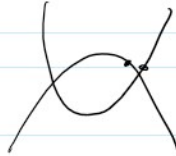
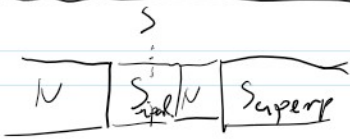
D.N. 2:



$E > \Delta$

$$r_A = e^{-i\ell} \left( \frac{E}{\Delta} - \frac{\sqrt{E^2 - \Delta^2}}{\Delta} \right)$$

### Prevalent Andreev



$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = S \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \quad S_e = \begin{pmatrix} r_e & t_e \\ t_e & r_e' \end{pmatrix}$$

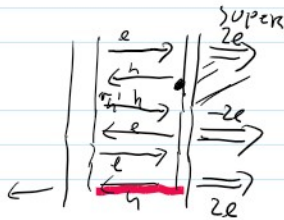
$$S_e = S_e(E) = S(E_F + E) \doteq S(E_F) = \begin{bmatrix} r & t \\ t & r' \end{bmatrix}$$

$$S_h = S^*(E_F - E) \doteq S^*(E_F) \quad S_h = \begin{pmatrix} r_h & t_h' \\ t_h & r_h' \end{pmatrix}$$



$$= \begin{bmatrix} r^* & t^* \\ t^* & r'^* \end{bmatrix}$$

$$r_A = t_e e^{i\chi_e} t_h' + t_e \left( e^{i\chi_e} r_h' e^{i\chi_h} \right) e^{i\chi_e} t_h' + t_e \left( \quad \right)^2 e^{i\chi_e} t_h' + \dots$$



$$r_A = \frac{t_e e^{i\chi_e} t_h'}{1 - e^{i(\chi_e + \chi_h)} r_h' r_e'} = \frac{|t|^2 e^{i\chi_e}}{1 - (-1)|r|^2}$$

$$e^{i\chi_e} = e^{-i\ell} e^{-i\alpha}$$

$$\alpha = \arccos \frac{E}{\Delta}$$

$$|r|^2 = T$$

$$e^{ix_e} = e^{-i\ell} e^{-i\alpha}$$

$$\alpha = \arccos \frac{E}{\Delta}$$

$$|t|^2 = T$$

$$e^{ix_h} = e^{i\ell} e^{-i\alpha}$$

$$E \ll \Delta$$

$$|r|^2 = |t|^2 = 1 - T$$

$$e^{i(x_e + x_h)} = e^{-2i\alpha}$$

$$\alpha = \pi/2$$

$$= e^{-i\pi} = -1$$

$$r_A = e^{ix_e} T$$

$$R_A = |r_A|^2 = \frac{T^2}{(2 - T)^2}$$

$$G = \frac{2e^2}{h} \cdot 2 R_A = 2G_0 R_A$$

$$T \sim 1 \quad G = 2G_0$$

maximalna

$$G = G_0$$

$$T \text{ mali} \quad G = 2G_0 \frac{T^2}{4}$$

$$G = G_0 T$$

Andreana vezana stanija



$$\underline{B}^e = \begin{pmatrix} B_1^e \\ B_2^e \end{pmatrix} = S \begin{pmatrix} A_1^e \\ A_2^e \end{pmatrix} \Rightarrow \underline{A}^e \quad \underline{B}^h = \begin{pmatrix} B_1^h \\ B_2^h \end{pmatrix} = S^* \begin{pmatrix} A_1^h \\ A_2^h \end{pmatrix} = S^* \underline{A}^h$$

$$\begin{pmatrix} \underline{B}^e \\ \underline{B}^h \end{pmatrix} = \begin{bmatrix} S & 0 \\ 0 & S^* \end{bmatrix}_{4 \times 4} \begin{pmatrix} \underline{A}^e \\ \underline{A}^h \end{pmatrix}$$

$$\begin{pmatrix} A_1^e \\ A_1^h \end{pmatrix} = \begin{pmatrix} 0 & e^{ix_1^e} \\ e^{ix_1^e} & 0 \end{pmatrix} \begin{pmatrix} B_1^e \\ B_1^h \end{pmatrix} \quad \begin{pmatrix} A_2^e \\ A_2^h \end{pmatrix} = \begin{pmatrix} 0 & e^{ix_2^e} \\ e^{ix_2^e} & 0 \end{pmatrix} \begin{pmatrix} B_2^e \\ B_2^h \end{pmatrix}$$

$$\begin{pmatrix} A_1^e \\ A_2^e \end{pmatrix} = \underbrace{\begin{pmatrix} e^{ix_1^e} & \\ & e^{ix_2^e} \end{pmatrix}}_{S_3} \begin{pmatrix} B_1^h \\ B_2^h \end{pmatrix}$$

$$\begin{pmatrix} A_1^h \\ A_2^h \end{pmatrix} = \underbrace{\begin{pmatrix} e^{ix_2^e} & \\ & e^{ix_1^e} \end{pmatrix}}_{\tilde{S}_3} \begin{pmatrix} B_1^e \\ B_2^e \end{pmatrix}$$

$$\begin{pmatrix} \underline{A}^e \\ \underline{A}^h \end{pmatrix} = \underbrace{\begin{pmatrix} S_3 & \\ & \tilde{S}_3 \end{pmatrix}}_{S_{2 \times 4 \times 4}} \begin{pmatrix} \underline{B}^e \\ \underline{B}^h \end{pmatrix}_{4 \times 4}$$

$$\underline{A} = S_{3 \times 4 \times 4} \cdot S_{4 \times 4} \underline{B}$$

$$\det(S_{3 \times 4 \times 4} \cdot S_{4 \times 4} - 1) = 0$$

$$e^{i\mathcal{K}} = e^{-i\alpha} e^{-i\varphi} \quad \cos \alpha = E/\Delta$$

energija vez. stanja

$$E = \Delta \sqrt{1 - T \sin^2 \left( \frac{\varphi_L - \varphi_0}{2} \right)}$$

Josephsonov stik / efekt

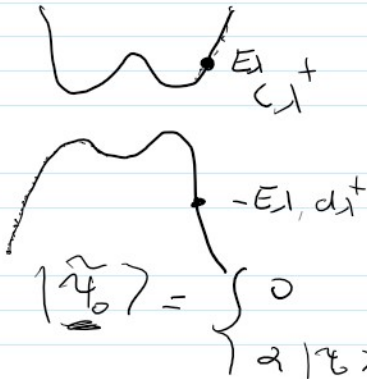
$$I, V, P, E(\varphi)$$



$$P = \dot{\varphi}_0 - \dot{\varphi}_1$$

$$|\psi_0\rangle =$$

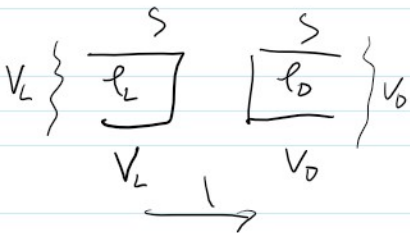
$$|\psi_0\rangle = \prod_{E_{\lambda} > 0} (c_{\lambda})$$



$$E_0 = \sum_{\lambda; E_{\lambda} < 0} E_{\lambda} = -\sum_{\lambda; E_{\lambda} > 0} E_{\lambda}$$

$$E_0(\varphi) = -\sum_{\lambda} E_{\lambda} = -\Delta \sum_n \sqrt{1 - T_n \sin^2 \left( \frac{\varphi}{2} \right)} = -\Delta \sum_n \frac{1 - T_n \sin^2 \frac{\varphi}{2}}{2}$$

↑  
Andreeva vezuma st.



$$V = V_L - V_0$$

$$P = VI$$

$$\frac{dE}{dt} = VI$$

$$\frac{dE}{d\varphi} = +\Delta \sum_n \frac{T_n \sin \varphi \cos \frac{\varphi}{2}}{2} \left( \frac{dE}{d\varphi} \right) \left( \frac{d\varphi}{dt} \right) = VI \quad i\varphi = H\varphi = eV$$

$$= \text{const} + \frac{\Delta T_n}{4} (-\cos \varphi)$$

$$E_0(\varphi) = E_0 (-\cos \varphi)$$

$$\frac{dE}{dt} = + \Delta \sum_n \frac{T_n \sin \frac{\phi}{2} \cos \frac{\phi}{2}}{\sqrt{1 - T_n \sin^2 \frac{\phi}{2}}} \left| \frac{dE}{d\phi} \left( \frac{d\phi}{dt} \right) \right| \Rightarrow VI$$

$$- \Delta \sum_n \frac{T_n \sin \phi}{4 m \sqrt{1 - T_n \sin^2 \frac{\phi}{2}}}$$

$$\frac{i\hbar \partial \psi}{\partial t} = H\psi = eV_L$$

$$i\hbar \frac{d e^{i\phi}}{dt} = 2e V_L e^{i\phi}$$

$$-\hbar \frac{d\phi}{dt} = 2e V_L$$

$$\dot{\phi} = -\frac{2e}{\hbar} V_L, \quad \dot{\phi}_0 = -\frac{2e}{\hbar} V_0$$

$$\phi = \phi_0 - \phi_L$$

$$V = V_L - V_0$$

$$\left[ \dot{\phi} = \frac{2e}{\hbar} \cdot V \right]$$

konstr. map.

$$\phi = \frac{2e}{\hbar} V t$$

$$\frac{\Delta}{4} \sum_n \frac{T_n \sin \phi}{m} \cdot \frac{2e}{\hbar} V = VI$$

$T_n$  mali

$$I = \frac{\Delta |e| \sum_n T_n \sin \phi}{2\hbar} =$$

$$I = -I_c \sin \phi$$

$$= - \frac{\Delta \pi (2kl)^2}{2R |2\pi\hbar} \sum_n T_n \sin \phi$$

$$I = \left( - \frac{\Delta \pi}{2 |e|} \right) \sin \phi$$

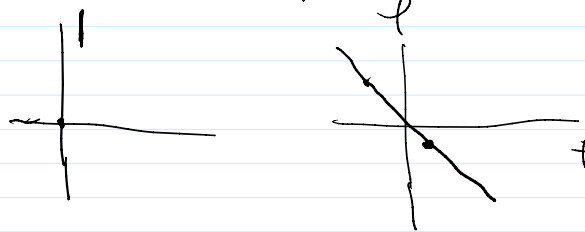
$$I = -I_c \sin \phi$$

izmenicini Josephsonov pojav

$$E(\phi) = -E_J \cos \phi$$

$$I = -E_J \frac{2|e|}{\hbar} \sin \phi \Rightarrow -I_c \sin \phi$$

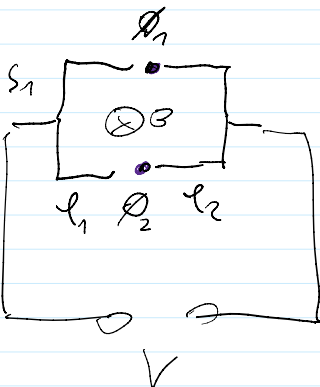
$$I_c = E_J \frac{2|e|}{\hbar}$$



energetični J. pojav

$$V = 0; \quad \phi = \text{konst}; \quad I = -I_c \sin \phi$$

SQUID



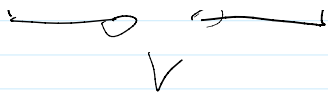
$$I = -I_c (\sin \phi_1 + \sin \phi_2)$$

$$\Phi_m = \int \vec{A} \cdot \frac{2e}{\hbar} d\vec{s} = \int \vec{B} \cdot d\vec{s} \frac{2e}{\hbar} = \frac{2 \cdot 2\pi \Phi}{\phi_0}$$

$$\phi_1 = \phi_2 - \phi_1 + \Phi_m / 2$$

$$\phi_2 = \phi_2 - \phi_1 - \Phi_m / 2$$

$$\phi_0 = h/2e$$



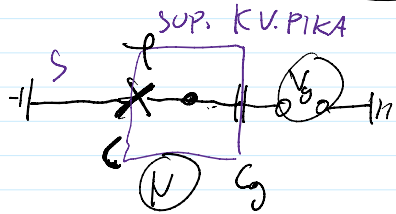
$$\phi_2 = \phi_1 - \frac{\pi}{2}$$

$$I = -I_0 \left( \sin(\ell + \phi_m/2) + \sin(\ell - \phi_m/2) \right)$$

$$I = -2I_0 \cos \phi_m/2 \sin \ell$$

$$\phi_0 = 10^{-15} \text{ Tm}^2$$

### Cooperjevna skutla



$$E = \frac{1}{2(C+C_g)} (eN - q)^2 + (-E_J) \cos \varphi$$

$$q = C_g V_g$$

$$V_g = 0$$

$$\downarrow$$

$$1 - e^2/2$$

$$E = \frac{1}{2\tilde{C}} (eN)^2 + E_J \left( \frac{e^2}{2} + c \right)$$

$$E = E_C N^2/2 + E_J e^2/2$$

$$I = -I_c \sin \varphi$$

$$e\dot{N} = -I_c \sin \varphi$$

### KVANTIZACIJA OPIKA

$$\psi_{\text{BCS}}(\varphi) = \prod_k (u_k + v_k e^{i\varphi}) \langle \dots \rangle$$

$$\langle \dots \rangle = \int_0^{2\pi} \psi_{\text{BCS}}(\varphi) e^{-iN\varphi/2} d\varphi$$

$$\int_0^{2\pi} e^{iM\varphi - \frac{1}{2}\varphi} d\varphi = \begin{cases} 2\pi; & M = \frac{N}{2} \\ 0; & \text{riče} \end{cases}$$

$N, \varphi$  ... kanj. kalicini  $\varphi/x$

$$H = E_C \frac{N^2}{2} + E_J \frac{\hat{\varphi}^2}{2}$$

$\hat{\varphi}$  ... konv. kalicini  
 $N$  ... gib. kalicini

$$[N, \hat{\varphi}] = -2i$$

$$I = -I_c \sin \varphi$$

$$i\hbar \dot{N} = [N, H]$$



$[N, p] = -2i$   
 $i\hbar \hat{N} = [N, H]$

$l = -l_c \sin \varphi$

$l = \langle \hat{N} \rangle = -l_c \sin \varphi = -l_c \langle \varphi \rangle$

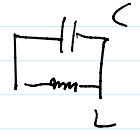
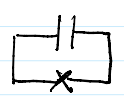
$\dot{l} = -l_c \langle \dot{\varphi} \rangle = -l_c \frac{2eV}{\hbar} = -l_c \frac{2e^2 N}{c \hbar}$

$V = \frac{eN}{C} \implies = -l_c \frac{E_c 4N}{\hbar}$

$\dot{N} = -\frac{l_c}{e} \dot{\varphi} \quad \dot{\varphi} = \frac{E_c 4N}{\hbar}$

$E$   
 $|l| = \frac{\hbar^2 k^2}{2} + \frac{\hbar^2 p^2}{2m}$

$\dot{l} = LV$   $\varphi$  mala  $X \dots$  rom



$l_c = \frac{2eE_0}{\hbar}$

$\dot{N} = \frac{2E_0}{\hbar} \dot{\varphi} \quad \dot{\varphi} = \frac{E_c 4N}{\hbar}$

$[N, \varphi] = -2i$

$\mathcal{L} \langle N \rangle = e^{i\varphi N/2}$   $\mathcal{L} x |k\rangle = e^{ikx}$

$\hat{H}$  u fuzni repr.  $\hat{N} = -2i \partial_\varphi$

$H = \frac{E_c}{2} (-2i \partial_\varphi)^2 + \frac{E_0}{2} (e^{i\varphi}) = \frac{E_c}{2} (-2i \partial_\varphi)^2 + E_0 (-\cos \varphi)$

$H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + x^2 \dots$

$\hat{H}$  u mal. reprezentaciji

$H = \frac{E_c}{2} N^2$   $\cos \varphi = \frac{e^{i\varphi} + e^{-i\varphi}}{2}$

$e^{i\varphi} e^{i\varphi N/2} = e^{i\varphi(N+2)}$

$2 \cos \varphi |N\rangle = |N+2\rangle + |N-2\rangle$

$$H = \frac{E_C}{2} N^2 + \left( E_J \sum_N \right) |N+2\rangle \langle N| + |N-2\rangle \langle N|$$

Kvantna računalništvo

$$\phi \overset{N}{000000}$$

$\phi_1$

stanje  $2^N$  različnih stanj

$$|\psi\rangle = \sum_n |n\rangle$$

$$\begin{aligned} |0\rangle &= |0000100\rangle \\ |1\rangle &= |0001000\rangle \\ |2\rangle &= |0010000\rangle \end{aligned}$$

ca. 10 biti

$10 \cdot 2^N$  bitar za zapis kvantnega stanja

Enota kv. informacije kubit  
dvodimenzijski kv. sistem

$\uparrow, \downarrow$

$$|\psi\rangle = \cos \frac{\theta}{2} |\uparrow\rangle + \sin \frac{\theta}{2} e^{i\phi} |\downarrow\rangle$$

$\dots = 1 \dots \dots = 0 \dots \dots$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) = \left( \cos \frac{\theta}{2} |\uparrow\rangle + \sin \frac{\theta}{2} e^{i\phi} |\downarrow\rangle \right)$$

$$\langle \psi | \hat{S}_y | \psi \rangle = \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} = \begin{pmatrix} \langle \uparrow | \\ \langle \downarrow | \end{pmatrix} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

①  $|\psi\rangle = \hat{R}_z(\theta) \hat{R}_y(\phi) |\uparrow\rangle$

$$\hat{R}_z(\alpha) = e^{-i \hat{S}_z \frac{\alpha}{\hbar}} = \cos \frac{\alpha}{2} \hat{1} - i \hat{S}_z \sin \frac{\alpha}{2}$$

D.k. potkazi zvezto ①.

Izrek o nekloniranju (No-cloning TH.)

$$|\psi\rangle \dots \dots |\psi\rangle |\psi\rangle |\psi\rangle \dots$$

$|\psi\rangle$  -- znano zrač. stanje

Ali  $\nexists U$  ?

$$U(|\psi\rangle)|\psi\rangle = |\psi\rangle|\psi\rangle$$

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$$

$$U(a|\uparrow\rangle + b|\downarrow\rangle)|\psi\rangle = (a|\uparrow\rangle + b|\downarrow\rangle) | (a|\uparrow\rangle + b|\downarrow\rangle) |$$

$$Ua|\uparrow\rangle|\psi\rangle + Ub|\downarrow\rangle|\psi\rangle = a|\uparrow\rangle|\uparrow\rangle + b|\downarrow\rangle|\downarrow\rangle$$

$$b=0 \quad a=1$$

Operacije na kubitih

• elementarne operacije

$U|\psi\rangle \dots$  elek. potencial  $V$

$$H = \hat{1} eV + \mu_B \vec{B} \cdot \vec{S}$$

$$U = e^{-iHt}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \dots \text{SWAP} = e^{i\pi/4} \hat{R}_x(\pi)$$

$\sigma_x$  matritka  $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$