IJS

Lectures on

Dark matter, Phase transitions and Gravitational Waves

Miha Nemevšek

Outline

1) Dark matter/intro and review [~2hr]

- ☐ *basics* of cosmology [Kolb&Turner, Dodelson, Weinberg, Baumann, TASI lectures, PDG, Sec.21 BBCosmo]
- ☐ Dark matter properties [PDG, Sec. 26 DM]
- ☐ Candidates, lifetime
- ☐ Thermal freeze-out production of WIMPs
- ☐ Sterile neutrinos, freeze-in, entropy dilution
- ☐ A history of Dark matter [Bertone, Hooper '16]

Outline

2) Phase transitions and gravitational waves [~2hr]

- ☐ effective potential/basics of thermal field theory
- ☐ first order phase transitions
- ☐ false vacuum decay
- ☐ energy budget of gravitational waves
- ☐ GW spectra and sensitivities

Basics of cosmology

Expansion - metric

FRW metric: 3D isotropic & homogeneous

$$\begin{aligned} ds^2 &= dt^2 - a(t)^2 \left(dx^2 + dy^2 + dz^2\right) \\ &= dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right) \end{aligned}$$

$k \simeq 0$ for a flat universe

Expansion - metric

FRW metric: 3D isotropic & homogeneous

$$ds^2 = dt^2 - a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$
$$= dt^2 - a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$$

$k \simeq 0$ for a flat universe

$a(t) = \frac{1}{1+z} \ldots$ scale factor describing the expansion of space, *z* is redshift of (wave)lengths $a = \frac{\lambda'}{\lambda}$

$V \propto a^3$

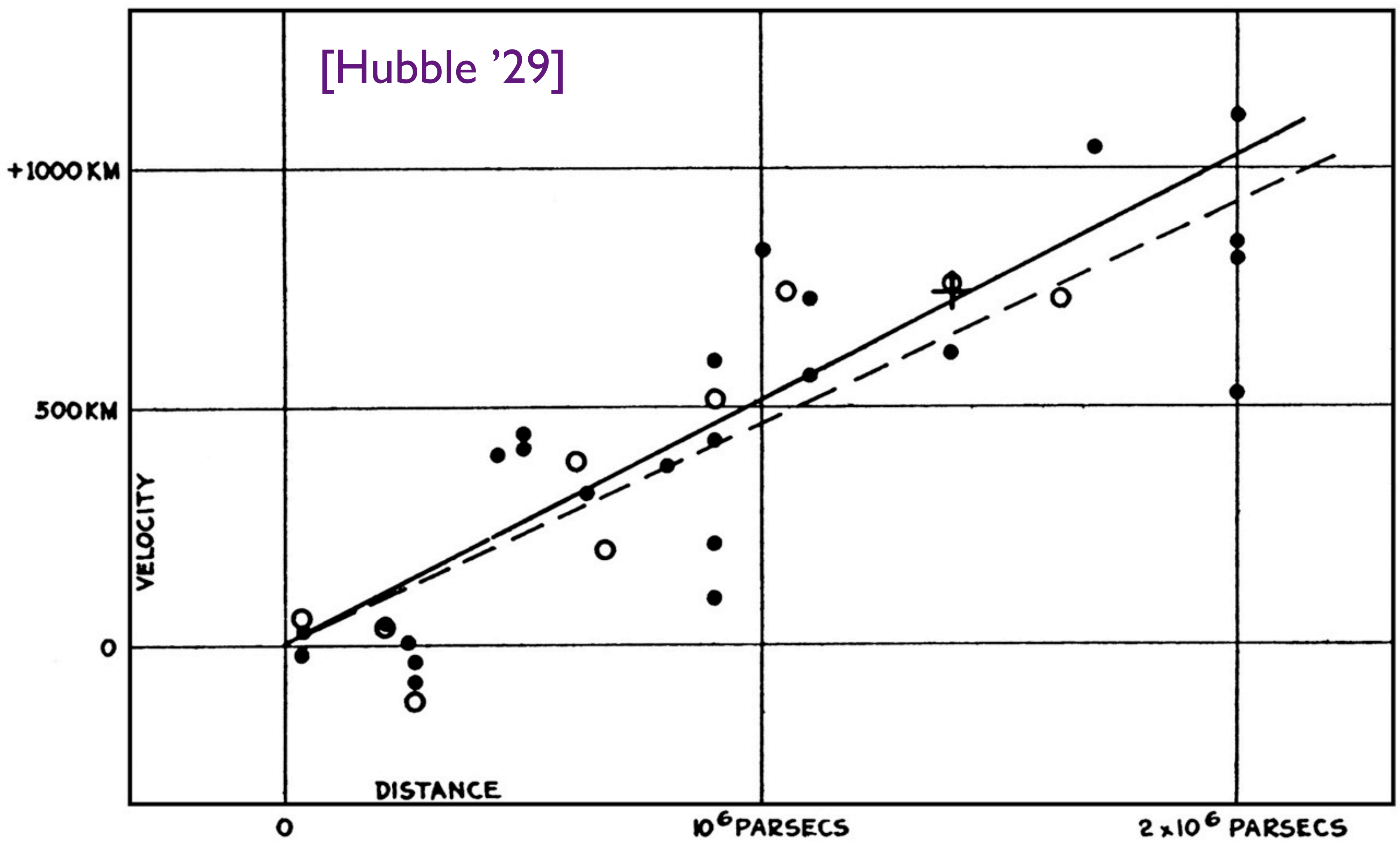
$a_0 = 1$ today, a is 1 by definition

Hubble parameter

$$H = \frac{\dot{a}}{a}$$

velocity(redshift)
proportional to the distance

$$v = H_0 D$$



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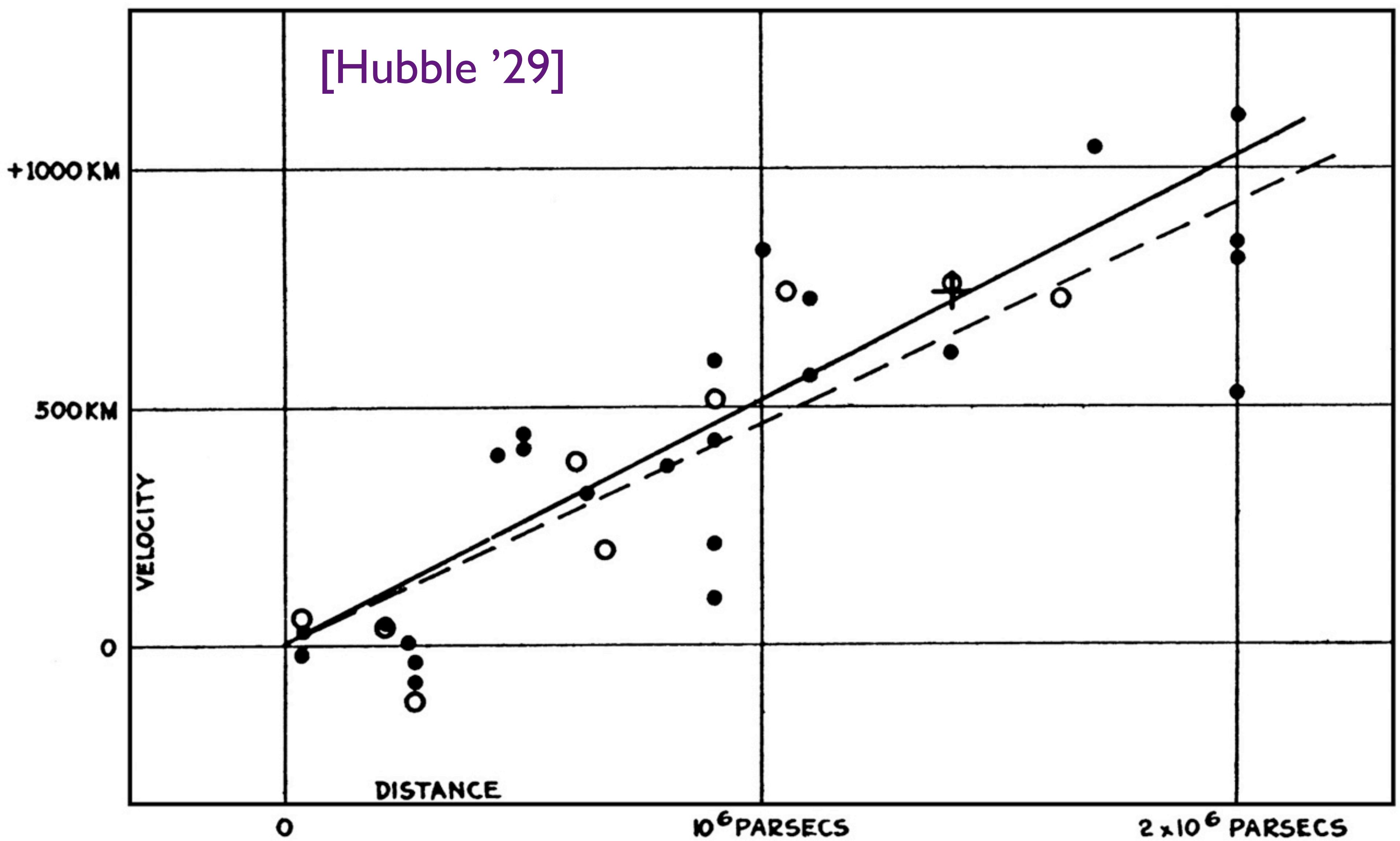
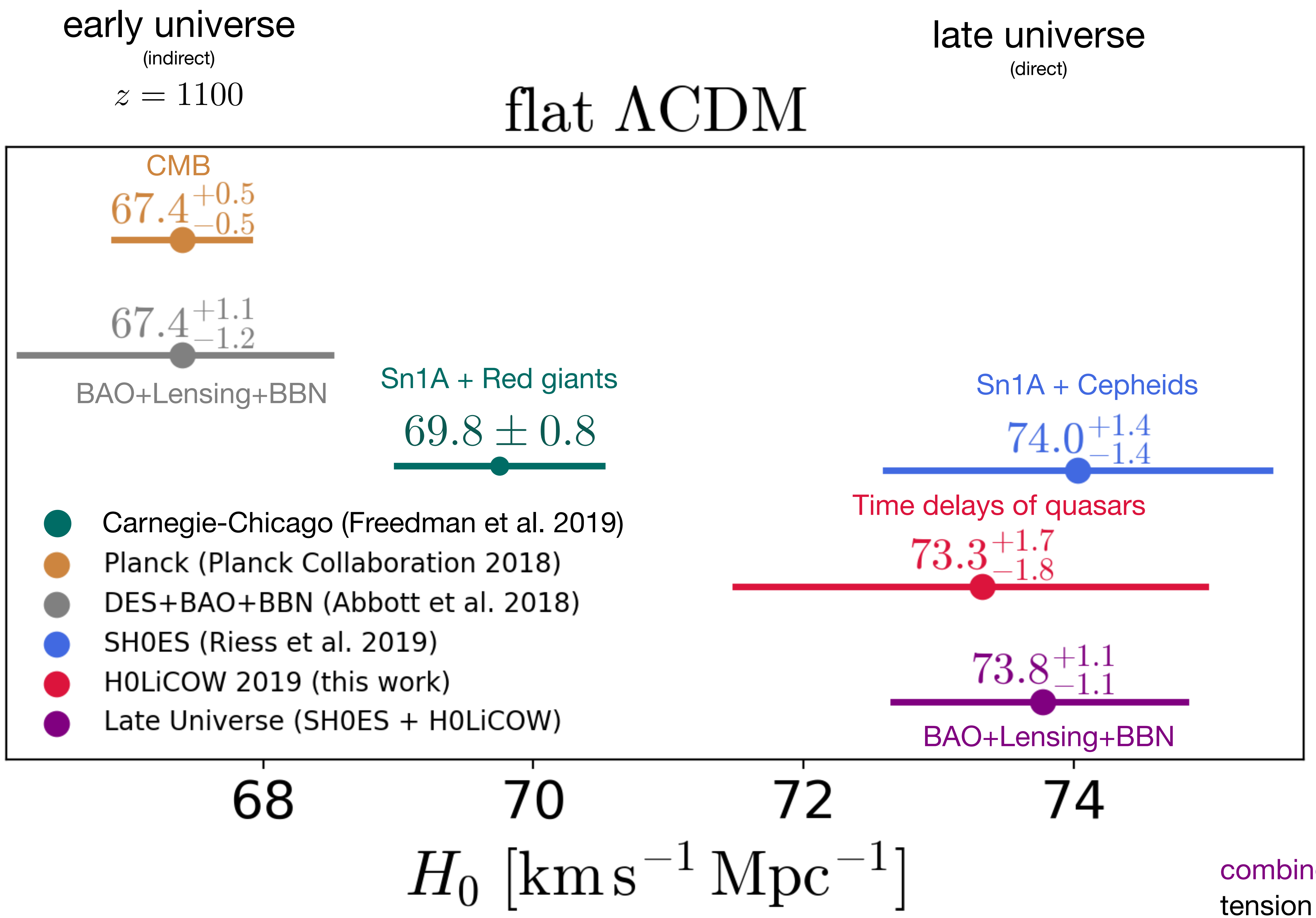


$$H_0 = [68 - 74]\,\frac{\mathrm{km}}{\mathrm{s}}\frac{1}{\mathrm{Mpc}} = h \times 100\,\frac{\mathrm{km}}{\mathrm{s}}\frac{1}{\mathrm{Mpc}}$$

measured today

Hubble parameter



combined in 5.3 sigma tension with Planck

Dynamics

Friedman equations

Einstein equations $$G_{\mu\nu} = 8\pi\, G_N\, T_{\mu\nu}$$

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$$G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}$$

geometry of the FRW universe

Geometry

$$G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2} g_{\mu\nu}$$

$$R_{00} = -\delta_{ii} \frac{\partial}{\partial t} \left(\frac{\dot{a}}{a} \right) - \left(\frac{\dot{a}}{a} \right) \delta_{ij} \delta_{ij}$$

$$= -3 \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} \right) \left(\frac{\dot{a}}{a} \right)^2 = -3 \, \frac{\ddot{a}}{a}$$

[see e.g. Dodelson p. 28-33]

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$$R = g^{\mu\nu} R_{\mu\nu} = -R_{00} + \frac{1}{a^2} R_{ii} = 6 \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right)$$

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$$G_{00} = R_{00} - \frac{R}{2} g_{00} = 3 \left(\frac{\dot{a}}{a} \right)^2 = 3H^2$$

Dynamics

Friedman equations

Einstein equations $$G_{\mu\nu} = 8\pi\, G_N\, T_{\mu\nu}$$

$$T^{\mu\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

Stress-energy tensor for a perfect fluid

[see e.g. Lifschitz-Landau]

$\rho \ldots$ energy density, depends on the content of the universe

$p \ldots$ pressure

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$$G_{00} = 3H^2 = T_{00} = 8\pi G_N \rho = \frac{\rho}{M_{\rm Pl}^2}$$

expansion depends on the energy content

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$$G_{00} = 3H^2 = T_{00} = 8\pi G_N \rho = \frac{\rho}{\overline{M}_{\mathrm{Pl}}^2}$$

$$\rho_{\mathrm{crit}} = 3H_0^2\, \overline{M}_{\mathrm{Pl}}^2$$

expansion depends on the energy content

Units and constants

Gravity is weak, much weaker than the weak force

$$\overline{M}_{\rm Pl} = \frac{M_{\rm Pl}}{\sqrt{8\pi}} = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}\ \text{GeV} \gg v = 246\ \text{GeV}$$

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The critical energy density is rather small

$$\rho_{\rm crit} = \frac{3H_0^2}{8\pi G_N} = 3H_0^2\,\overline{M}_{\rm Pl}^2 = 10^{-5}\,h^2\,\frac{\text{GeV}}{\text{cm}^3} \simeq 4\,m_p\,\frac{1}{m^3}$$

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All the energy content normalized to the critical density

$$\Omega_x = \frac{\rho_x}{\rho_{\rm crit}} \qquad\qquad \Omega_{\rm tot} = \Omega_\Lambda + \Omega_m + \Omega_\gamma + \dots$$

Energy budget today

for a flat universe with k=0

$$\Omega_x = \frac{\rho_x}{\rho_{\rm crit}}$$

$$\Omega_{\rm tot} = \Omega_\Lambda + \Omega_m + \Omega_\gamma + \ldots$$

$$\Omega_{\rm tot}^{\rm exp} \simeq 1$$

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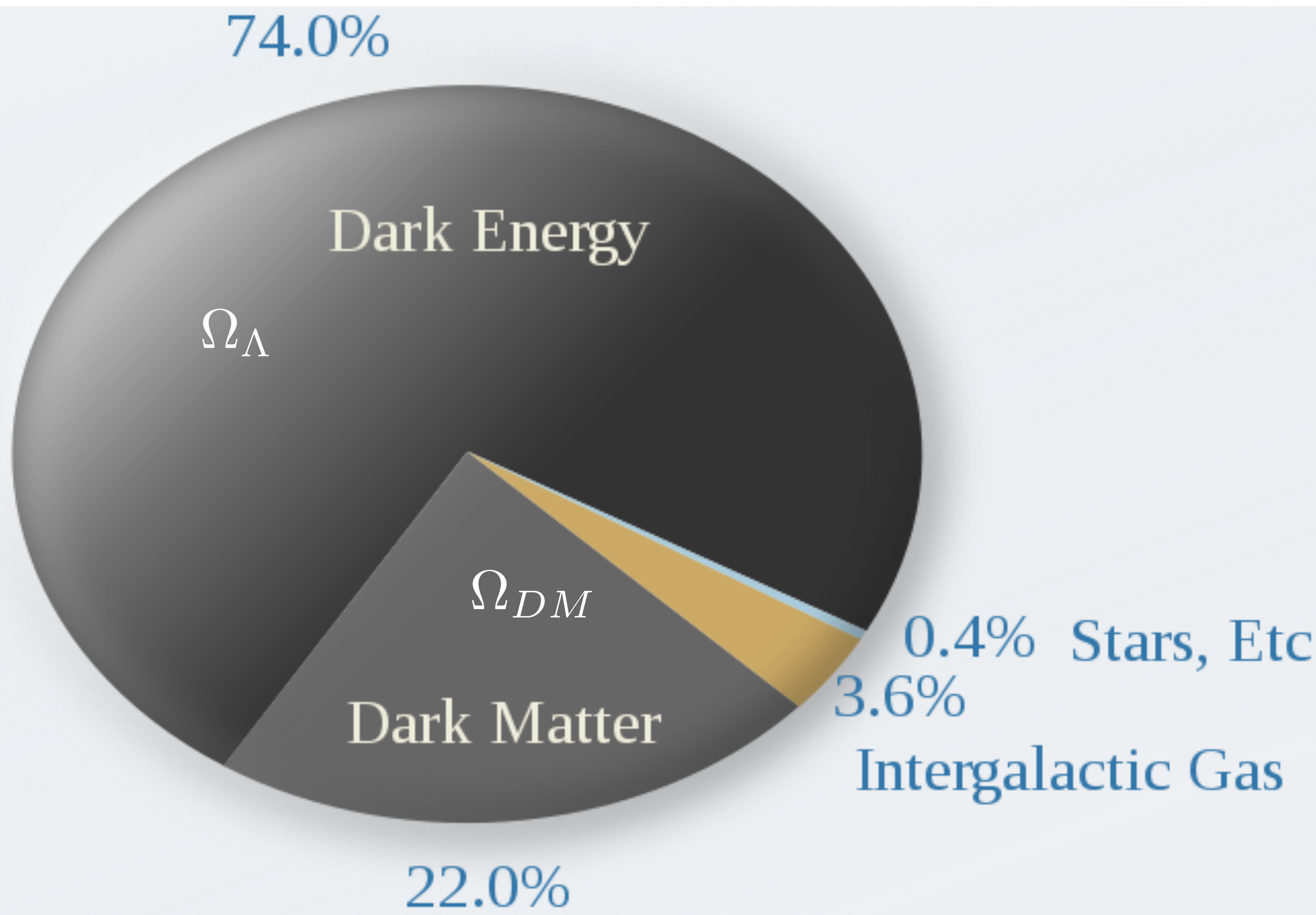

radiation

$$\Omega_\gamma = 5 \times 10^{-5}$$

neutrinos

$$\Omega_{C\nu B} \simeq 1.3 \times 10^{-5}$$

GWs

$$\Omega_{GW} \lesssim 10^{-11}$$

Expansion in matter/radiation universe

Number density

$N \ldots$ no. of particles in a co-moving volume $V \propto a^3$

$n \ldots$ number density

$$\frac{dN}{dt} = 0 \qquad \dot{n}a^3 + 3a^2\dot{a}n = 0$$

$$\frac{\dot{n}}{n} = -3\frac{\dot{a}}{a} \Rightarrow \boxed{n \propto a^{-3}}$$

Expansion in matter/radiation universe

Energy density

$\rho \ldots$ energy density per co-moving volume

$$\rho_m = m\,n \propto \boxed{a^{-3}}$$

non-relativistic matter scales with volume

$$\rho_\gamma = E_\gamma\,n \propto \frac{1}{\lambda} a^{-3} \propto \boxed{a^{-4}}$$

light gets stretched and redshifts, energy depletes faster

Expansion in matter/radiation universe

Energy density

$\rho \ldots$ energy density per co-moving volume

More formally, using the 2nd Friedman equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \qquad \Rightarrow \qquad a^{-3}\frac{d}{dt}\left(\rho a^3\right) = -3\frac{\dot{a}}{a}p$$

matter $_{v \ll c}$ $\qquad p = 0 \qquad \frac{d}{dt}\boxed{\left(\rho_m a^3\right)} = 0$

Expansion in matter/radiation universe

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matter $\quad p = 0 \quad \frac{d}{dt}\left(\rho_m a^3\right) = 0$

$v \ll c$

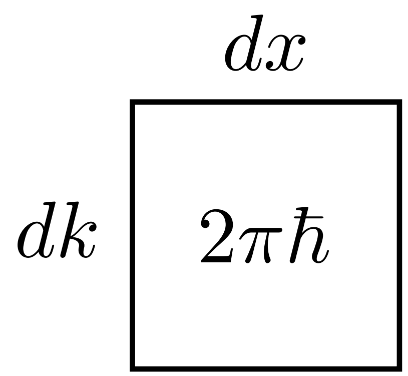
radiation $\quad p = \frac{\rho}{3} \quad \frac{d\rho_\gamma}{dt} + \frac{\dot{a}}{a}4\rho_\gamma = a^{-4}\frac{d(\rho_\gamma a^4)}{dt} = 0$

Thermodynamics

Particles meet & thermalize to equilibrium

interaction rate $\Gamma \gg H$ expansion rate

Box of particles in phase space



$$\rightarrow \int \frac{d^3x d^3k}{2\pi\hbar}$$

k ... proper momentum

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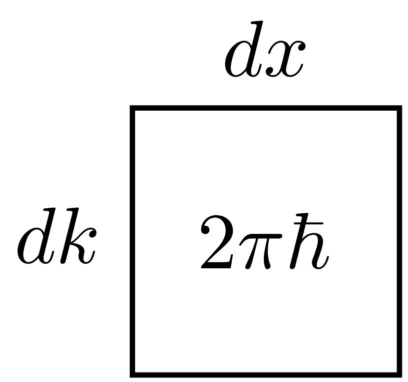

$$\rightarrow \int \frac{d^3x d^3k}{2\pi\hbar}$$

k ... proper momentum

$$n_i = g_i \int \frac{d^3k}{(2\pi)^3} f_i(x,k)$$

$$\rho_i = g_i \int \frac{d^3k}{(2\pi)^3} f_i(x,k) E_i(k)$$

$$E_i(k)^2 = k^2 + m_i^2$$

on-shell

degrees of freedom

distribution function

$$\rho_i = g_i \int \frac{d^3k}{(2\pi)^3} f_i(x,k) E_i(k)$$

distribution function

$$f_{F/B} = \frac{1}{e^{\frac{E_i}{T}} \pm 1}$$

$+1$ Fermi blocking

-1 BE condensation

Fermions/Bosons

here we neglect the chemical potential/number changing interactions, more about that later on

Boltzmann suppression $k \ll m$ $E \simeq m$ $f \propto e^{-\frac{m}{T}}$

Presence of particles heavier than T is exponentially suppressed

$$\rho_i = \boxed{g_i} \int \frac{d^3k}{(2\pi)^3} \boxed{f_i(x,k)} E_i(k)$$

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degrees of freedom

$g_i(\gamma) = 2$ two polarizations

more on this later

Adiabatics

Pressure and conservation of entropy

Pressure

$$p_i = g_i \int \frac{d^3k}{(2\pi)^3} f_i(x,p) \frac{k^2}{3E_i}$$

$$\frac{dp}{dT} = \frac{\rho + p}{T} \equiv s$$

Entropy density conserved with expansion

$$a^{-3} \frac{d(\rho+p)a^3}{dt} - \frac{dp}{dt} = 0$$

chain rule and the above gives

$$a^{-3} T \frac{d}{dt}\left(\frac{(\rho+p)a^3}{T}\right) = 0$$

$$s \propto a^3$$

Cosmic inventory

$$\rho_i = g_i \int \frac{d^3k}{(2\pi)^3} f_i(x,k) E_i(k)$$

Photons $g_\gamma = 2$ $m = 0 \Rightarrow E = k$

Cosmic inventory

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$$\rho_\gamma = 2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\frac{k}{T}} - 1} k = \frac{8\pi T^4}{(2\pi)^3} \int_0^\infty \frac{\mathrm{d}x\, x^3}{e^x - 1}$$

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Stefan-Boltzman's law

$$\rho_\gamma = \frac{\pi^2}{15} T^4$$

Cosmic inventory

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Stefan-Boltzman's law

$$\rho_\gamma = \frac{\pi^2}{15} T^4$$

Present-day CMB: $T_0 = 2.7 \ \mathrm{K}$ $\Omega_\gamma = \frac{\rho_\gamma}{\rho_{\mathrm{crit}}} = \frac{2.5 \times 10^{-5}}{h^2}$

Cosmic inventory

$$\rho_i = g_i \int \frac{d^3k}{(2\pi)^3} f_i(x,k)\, E_i(k)$$

Matter

$$\rho^m_{eq} = \begin{cases} m\, n^m_{eq}, & m \gg T \\ \frac{\pi^2}{30}\, g\, T^4 \left(1, \frac{7}{8}\right), & m \ll T \end{cases}$$

Cosmic inventory

$$\rho_i = g_i \int \frac{d^3k}{(2\pi)^3} f_i(x,k) E_i(k)$$

Matter

$$\rho_{eq}^m = \begin{cases} m\, n_{eq}^m, & m \ll T \\ \frac{\pi^2}{30}\, g\, T^4 \left(1, \frac{7}{8}\right), & m \gg T \end{cases}$$

and the number density

$$n_{eq} = g \int \frac{d^3k}{(2\pi)^3} f(x,k)$$

$$= \begin{cases} g \left(\frac{mT}{2\pi}\right) e^{-m/T}, & m \gg T \\ \frac{g}{\pi^2} T^3 \zeta(3) \left(1, \frac{3}{2}\right), & m \gg T \end{cases}$$

Cosmic inventory

Degrees of freedom

$$\rho = g_*(T)\frac{\pi^2}{30}T^4$$

SM

$$T \gg m_T : \quad g_* = 106.75$$

Cosmic inventory

Calculation of g_*

$$\rho = g_*(T)\frac{\pi^2}{30}T^4$$

In[96]:=
```
me = 0.51 × 10 ^ -3; mμ = 0.105; mτ = 1.776;
mu = 2.5 × 10 ^ -3; md = 5 × 10 ^ -3; ms = 0.1;
mc = 1.29; mb = 4.19; (*MSbar*) mt = 172.9;
MW = 80.399; MZ = 91.1876; mh = 125.5;
mπ = 0.1396; mπ0 = 0.135; mK = 0.49368; mK0 = 0.4976;
(* ρ is vector with spin = 1 *)
mρ = 0.77549;
```

g_* for massless particles (m << T and μ << T)

In[118]:=
```
(* g* from a massless real boson and a massless Dirac fermion *)
{(30 / π^2) 1 / (2 π^2) Integrate[u^3 / (Exp[u] - 1), {u, 0, Infinity}],
 (30 / π^2) 1 / (2 π^2) Integrate[u^3 / (Exp[u] + 1), {u, 0, Infinity}]}
(* gluons + photons, massless = 2 d.o.f, for SU(n), we have n^2 - 1 generators *)
gSgf = (8 + 1) 2;
(* neutrinos *)
gSnu = 3 × 2 × 7 / 8;
```

Out[118]= $\left\{1, \frac{7}{8}\right\}$

Temperature dependence, again (μ << T)

In[106]:=
```
gSf[m_, T_] := 15 / π^4 NIntegrate[Sqrt[u^2 - (m / T) ^2] u^2 / (Exp[u] + 1), {u, m / T, Infinity}]
gSb[m_, T_] := 15 / π^4 NIntegrate[Sqrt[u^2 - (m / T) ^2] u^2 / (Exp[u] - 1), {u, m / T, Infinity}]
(* g*(T) in the SM *)
gSSMabvQCD[T_] := gSgf + gSnu + 2 × 2 (gSf[me, T] + gSf[mμ, T] + gSf[mτ, T]) +
  3 × 2 × 2 (gSf[mu, T] + gSf[md, T] + gSf[ms, T] + gSf[mc, T] + gSf[mb, T] + gSf[mt, T]) + 6 gSb[MW, T] + 3 gSb[MZ, T] + gSb[mh, T]
```

Cosmic inventory

Degrees of freedom

$$\rho = g_*(T)\frac{\pi^2}{30}T^4$$

SM

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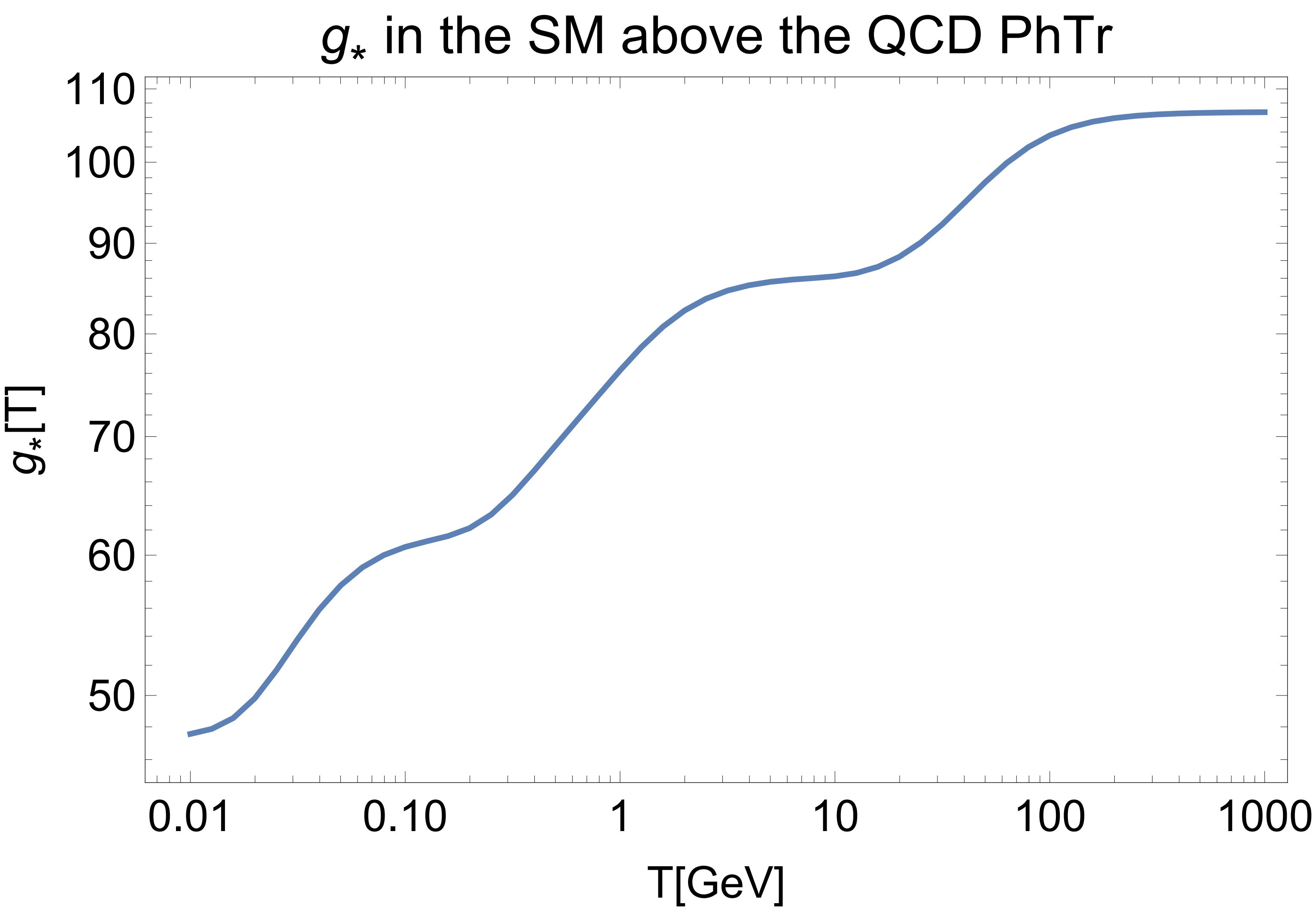


Radiation dominated

$$H = 0.3\sqrt{g_*}\frac{T^2}{\overline{M}_{Pl}}$$

$$\rho = g_*\frac{\pi^2}{30}T^4 = 3H^2\overline{M}_{\rm Pl}^2$$

Entropy and decoupling

$$\rho = g_*(T)\frac{\pi^2}{30}T^4 \qquad\qquad s = \frac{\rho + p}{T}$$

matter $p_m \simeq 0$ radiation $p_\gamma = \frac{\rho}{3}$

entropy
$$\boxed{s = g_{*s}\frac{2\pi^2}{45}T^3}$$

$$g_* \simeq g_{*s}$$

mass effects

$$0 = \frac{dS}{dt} = \frac{g_{*s}T^3a^3}{dt} \quad \Rightarrow \quad T \propto g_{*s}^{-1/3}a^{-1}$$

Entropy and decoupling

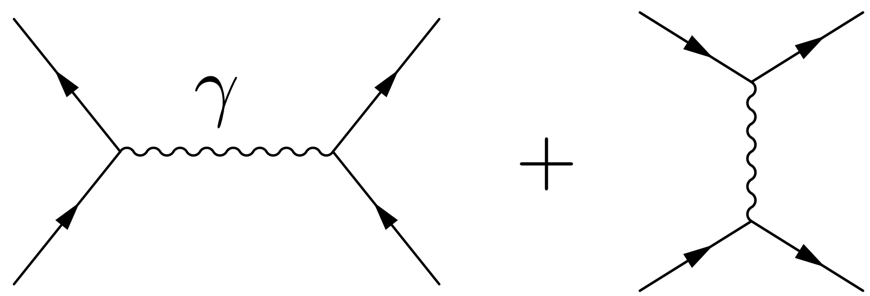
Thermal equilibrium $\Gamma \sim H$

Entropy and decoupling

QED thermal equilibrium $\qquad \Gamma \sim H$

$$n_{eq} \propto T^3$$

more on this later $\qquad \langle \sigma v \rangle \propto \frac{\alpha^2}{T^2}$



$\mathcal{A} \propto \frac{\alpha}{k^2} \qquad m_\gamma = 0$

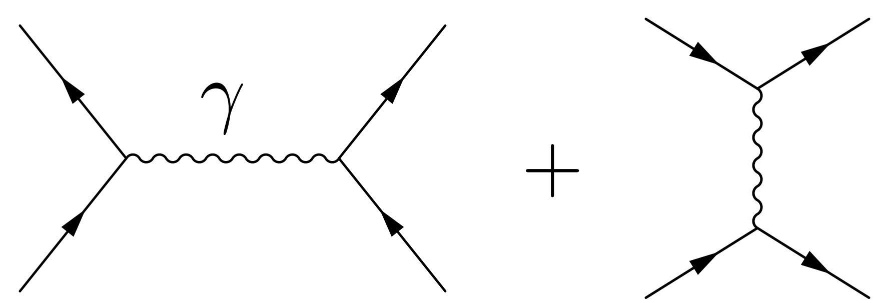
$$\Gamma \simeq n_{eq} \langle \sigma v \rangle \sim 10\, \alpha^2\, T$$

Entropy and decoupling

QED thermal equilibrium $\Gamma \sim H$ $H \sim \sqrt{3g_*}\frac{T^2}{\overline{M}_{Pl}^2}$

$n_{eq} \propto T^3$

$\langle \sigma v \rangle \propto \frac{\alpha^2}{T^2}$



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Entropy and decoupling

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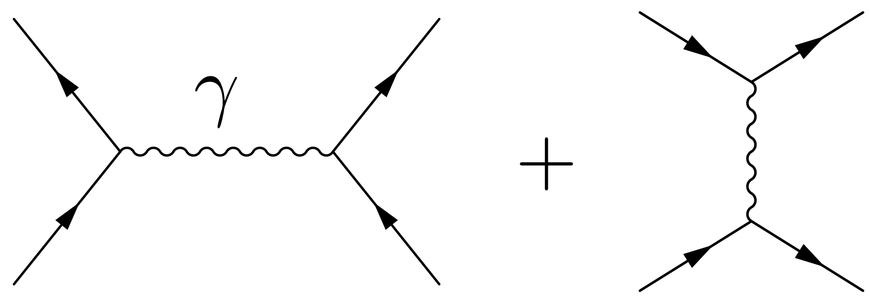


$\mathcal{A} \propto \frac{\alpha}{k^2}$ $m_\gamma = 0$

$$\Gamma \simeq n_{eq}\langle \sigma v \rangle \sim 10\,\alpha^2\,T$$

$$T \lesssim \alpha^2 M_{\rm Pl} \sim 10^{16}\ {\rm GeV}$$

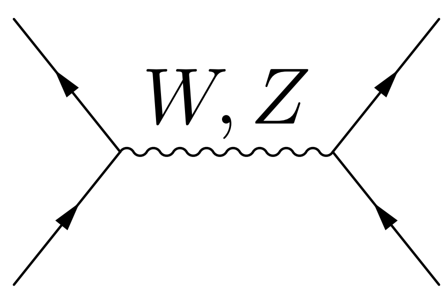
SM QED in thermal equilibrium

Entropy and decoupling

EW thermal equilibrium $\quad \Gamma \sim H \quad H \sim \sqrt{3g_*}\frac{T^2}{\overline{M}_{Pl}^2}$

$$n_{eq} \propto T^3$$

$\langle \sigma v \rangle \sim G_F^2 T^2$



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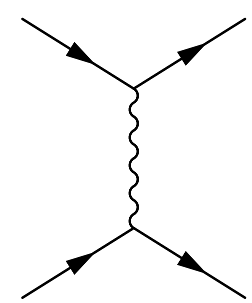

$\mathcal{A} \propto \frac{\alpha_2}{M_W^2} \qquad m_{W,Z} > T > 0$

$$\Gamma \simeq n_{eq} \langle \sigma v \rangle \sim G_F^2 T^5$$

Neutrino freeze-out, decoupling

$$T_\nu^{\rm dec} \simeq {\rm MeV}$$

Entropy and decoupling

Cosmic neutrino background ($C\nu B$) neutrino decoupling = freeze-out, later DM

$$\gamma \qquad e^- \quad e^+ \qquad \overline{\nu}_L \times 3$$

$$s_< = \frac{2\pi^2}{45} T_<^3 \left(2 + \frac{7}{8} \left(2 + 2 + 3 + 3 \right) \right)$$

$$= \frac{43\pi^2}{90} T_<^3 \qquad \nu_L \times 3$$

Entropy and decoupling

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neutrinos do nothing

entropy conserved when electrons annihilate $T \simeq m_e < \mathrm{MeV}$

$$s_< a_<^3 = s_> a_>^3$$

$$a_< T_< = a_> T_\nu$$

$$s_> = \frac{2\pi^2}{45} \left(2T_\gamma^3 + 6\frac{7}{8} T_\nu^3 \right)$$

$$\frac{T_\nu}{T_\gamma} = \sqrt[3]{\frac{4}{11}}$$

Entropy and decoupling

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$$s_> = \frac{2\pi^2}{45}\left(2T_\gamma^3 + 6\frac{7}{8}T_\nu^3\right) \qquad\qquad \frac{T_\nu}{T_\gamma} = \sqrt[3]{\frac{4}{11}}$$

$$\frac{43}{2}(a_< T_\nu)^3 = 4\left(\frac{T_\gamma^3}{T_\nu^3} + \frac{21}{8}\right)(a_> T_\nu)^3$$

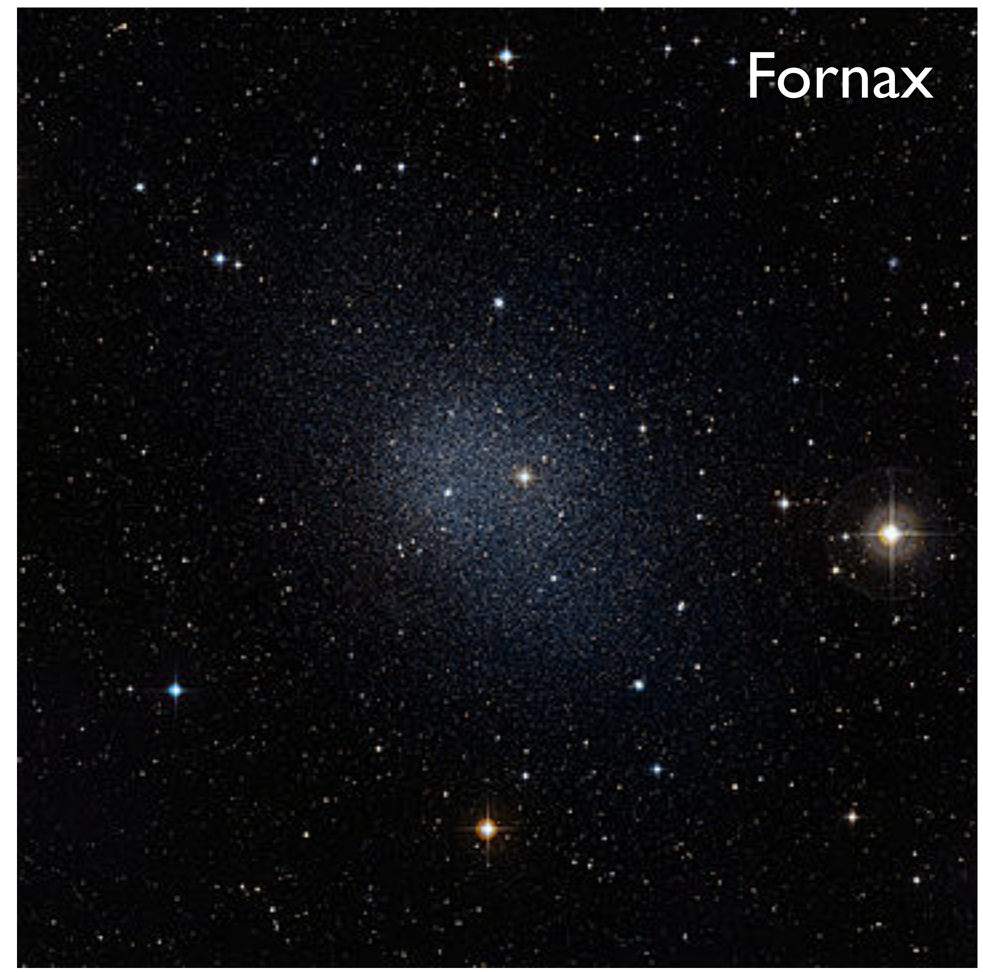
$$T_\gamma^0 = 2.7\ \mathrm{K}$$

$$T_\nu^0 = 1.94\ \mathrm{K} = 0.17\ \mathrm{meV}$$

some DM Properties

Mass range from observations of dwarf spheroidal galaxies

dSph are small, low luminosity galaxies with no dust and few old stars, gravitationally bound and thus likely dominated by DM



$R_{\rm dSph} \sim$ kpc

$M_{\rm dSPh} \gtrsim 10^7 M_\odot$

$$v_{DM} \simeq \sqrt{\frac{G_N M_{\rm dSph}}{R_{\rm dSph}}} \sim 10^{-5}$$

Mass range

Fermions lower limit due to the Pauli exclusion principle [Tremaine, Gunn '79]

$$f_{DM}(x,p) \leq h^{-3}$$ limited phase space

volume estimate

$$d^3x \sim R_{\rm dSph}^3$$

$$d^3p \sim p^3 = (m_{DM} v_{DM})^3$$

$$M_{\rm dSph} = m_{DM} \int f_{DM}(x,p)\, d^3x\, d^3p$$

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$f_{DM}(x,p) \leq h^{-3}$ limited phase space

$$\begin{aligned} M_{\mathrm{dSph}} &= m_{DM} \int f_{DM}(x,p)\, d^3x\, d^3p \\ &\leq m_{DM} R^3_{\mathrm{dSph}} \left(m_{DM} v_{DM}\right)^3 h^{-3} && \text{volume estimate} \\ &= m^4_{DM} R^3_{\mathrm{dSph}} \left(\frac{G_N M_{\mathrm{dSph}}}{R_{\mathrm{dSph}}}\right)^{3/2} h^{-3} && \text{velocity from gravity} \end{aligned}$$

lower bound $m^F_{DM} > \left(G^3_N M_{\mathrm{dSph}} R^3_{\mathrm{dSph}} h^{-6}\right)^{-1/8}$

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volume estimate

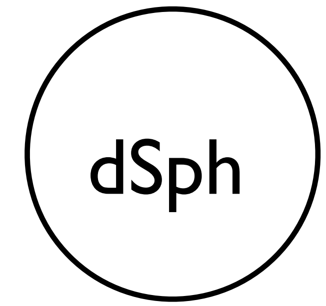
velocity from gravity

lower bound $m_{DM}^F > \left(G_N^3 M_{\rm dSph} R_{\rm dSph}^3 h^{-6}\right)^{-1/8}$

Fornax only $m_{DM}^F > 70 \text{ eV}$ [Randall, Gunn '79]

Combined + uncertainties $m_{DM}^F \gtrsim 0.4 \text{ keV}$ [Domcke & Urbano '14, di Paolo, Nesti, Villante '17]

Mass range

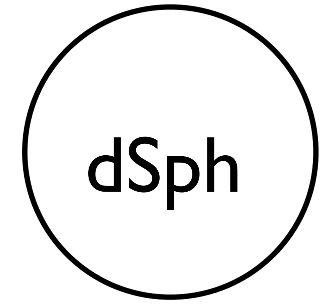


$$\lambda_{dB} < R_{dSph}$$

Bosons lower limit due to free-streaming

de Broglie wavelength should not exceed the size of the dSph

Mass range



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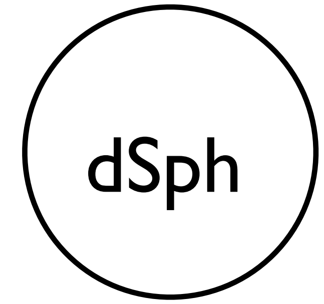
de Broglie wavelength should not exceed the size of the dSph

$$\lambda_{dB} = \frac{h}{m_{DM} v_{DM}} = 2 \text{ kpc} \left(\frac{10^{-22} \text{ eV}}{m_{DM}} \right) \left(\frac{3 \times 10^{-5}}{v_{DM}} \right) < R_{dSPh} \simeq 2 \text{ kpc}$$

$$m_{DM}^{B} \gtrsim 10^{-22} \text{ eV}$$

'fuzzy' dark matter

Mass range



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'fuzzy' dark matter

see the recent study and references [Hui, Ostriker, Tremaine, Witten '16]

Fermions and bosons $m_{DM} < 5\, M_{\odot}$ upper limit from tidal disruption

[Audren et al. 1407.2418]

Electromagnetic charge

dark structures don't shine like stars, no O(1) e.m. interactions

may be milli-charged, coupling to photons suppressed by mixing/small U(1) charge

Electromagnetic charge

dark structures don't shine like stars, no O(1) e.m. interactions

may be milli-charged, coupling to photons suppressed by mixing/small U(1) charge

main constraint from CMB, DM (single component) should be there at recombination

$m_{DM} < \text{ GeV}$

$$Q_{DM} < 4 \times 10^{-7} \left(\frac{m_{DM}}{\text{GeV}}\right)^{0.35}$$

$m_{DM} > \text{ GeV}$

$$Q_{DM} < 3.5 \times 10^{-7} \left(\frac{m_{DM}}{\text{GeV}}\right)^{0.58}$$

[McDermott, Yu, Zurek 1011.2907]

Electromagnetic charge

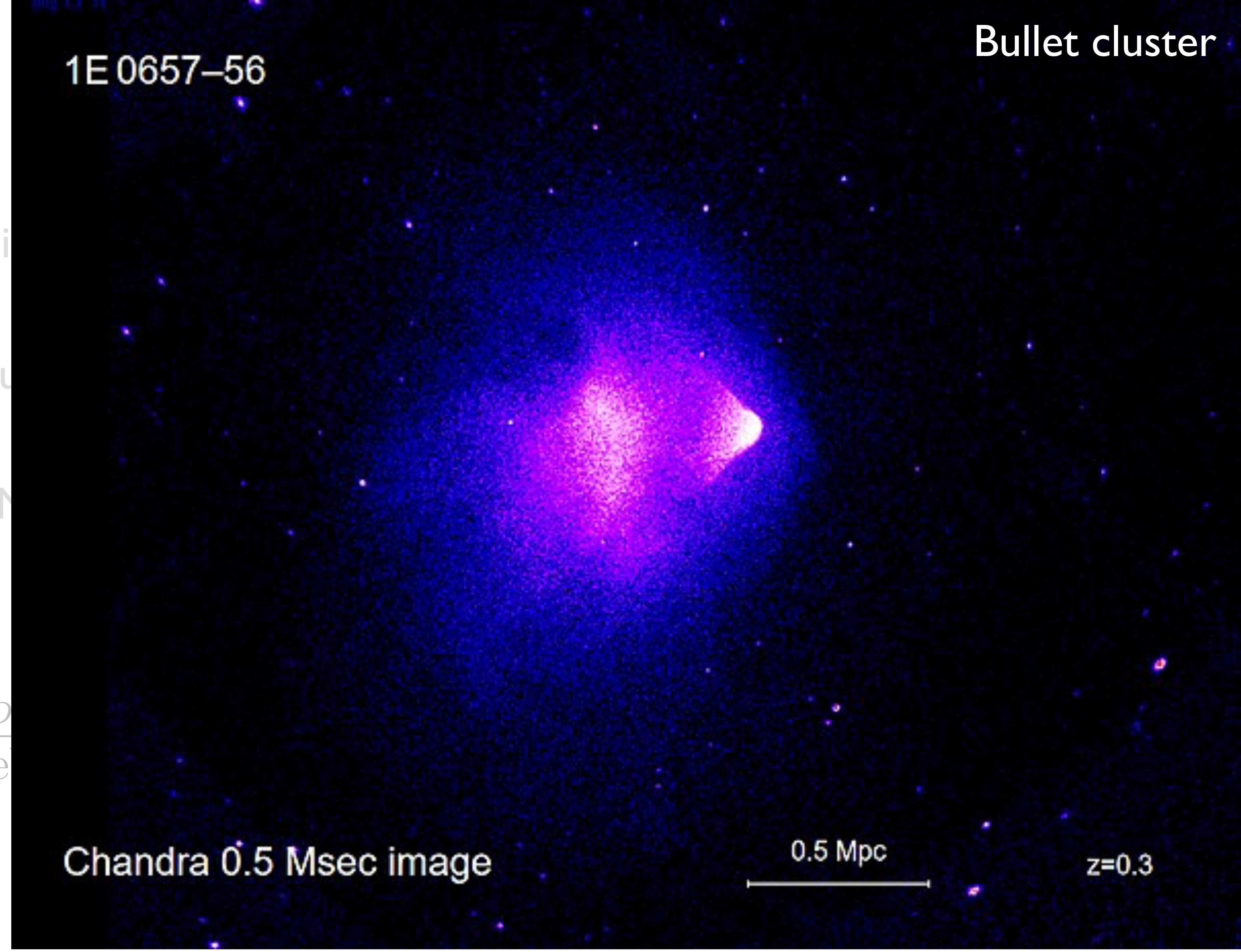
dark structures don't shi

may be milli-charged, cou

main constraint from CN

$m_{DM} <$ GeV

$Q_{DM} < 4 \times 10^{-7} \left(\frac{m_D}{\text{Ge}} \right.$



Self-interactions

instead DM can interact quite strongly with itself [Randall et al. 0704.0261]

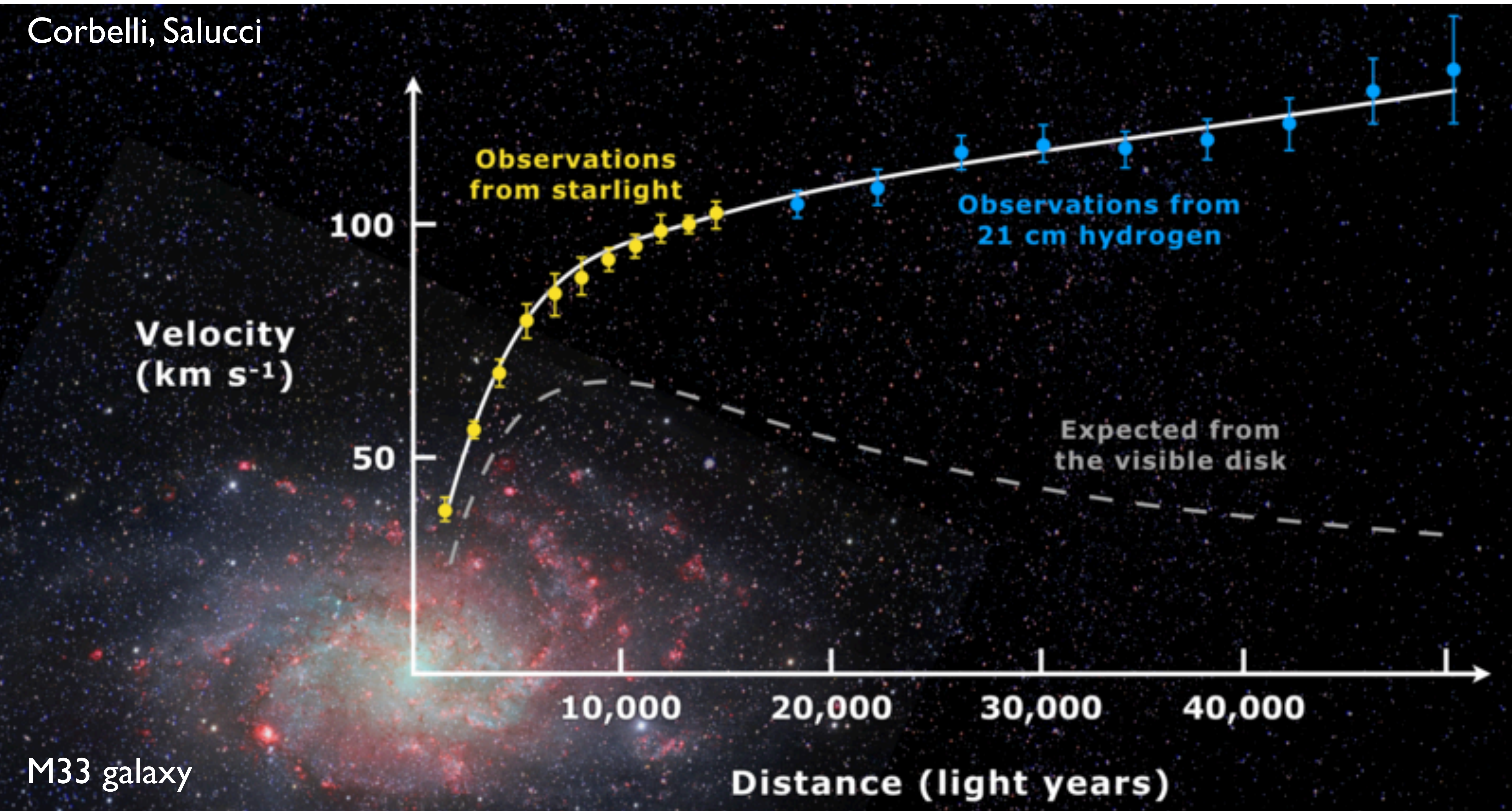
$$\frac{\sigma_{DM-DM}}{m_{DM}} < 0.8 \frac{\text{barn}}{\text{GeV}}$$

nucleons $\frac{\sigma_{n-n}}{m_n} \sim \frac{\text{few barns}}{\text{GeV}}$ [Tulling, Yu 1705.02358]

Dark matter density profile

direct observations of radial velocity vs. distance from the center of the galaxy

DM (other interpretations such as MOND less likely, involve ~DM as well)



Dark matter density profile

velocity flattens out at large distances, what does that say about the DM density?

$$\frac{G_N M_{\rm halo}(R)}{R^2} = \frac{v_c^2}{R} \qquad\qquad v_c = \sqrt{\frac{G_N M_{\rm halo}(R)}{R}} \sim \text{const.}$$

$$M_{\rm halo} \propto R \qquad\qquad M_{\rm halo} = \int d^3x\, \rho_{DM} = 4\pi \int_0^R dr\, r^2 \rho_{DM}(r)$$

$$\rho_{DM}(r) \propto \frac{1}{r^2}, \quad r > R$$

[Navarro, Frenk, White '96]

$$\rho_{DM}^{NFW} = \frac{\rho_0}{\frac{r}{R_s}\left(1 + \frac{r}{R_s}\right)^2}$$

common DM profile,
fit to simulations

Dark matter density profile

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$$M_{\rm halo} \propto R \qquad\qquad M_{\rm halo} = \int d^3x\, \rho_{DM} = 4\pi \int_0^R dr\, r^2 \rho_{DM}(r)$$

Milky Way, at our local vicinity, relevant for direct detection

local density $\quad \rho_{DM}\left(R_\odot\right) \simeq 0.4\, \frac{\rm GeV}{\rm cm^3}$

local velocity $\quad v_{DM}\left(R_\odot\right) \simeq \sqrt{\frac{G_N M_{MW}(R_\odot)}{c^2 R_\odot}} \simeq 10^{-3}$

local flux $\quad \phi_{DM} = \frac{\rho_{DM} v_{DM}}{m_D M} = \left(\frac{\rm TeV}{m_{DM}}\right) \frac{10^4}{\rm cm^2\ s}$

Candidates

DM candidates

WIPMs

SUSY - MSSS & beyond motivated by hierarchy, stabilized by R-parity

Singlets, additional multiplets (Inert Higgs doublet/multiplet)

Extended/Hidden interactions (Left-Right, Twin sectors, dark QCD, ...)

Extra dimensions, Kaluza-Klein stabilized with KK-parity

Right-handed/sterile neutrinos motivated by massive neutrinos

Axions motivated by strong CP

Many others...

Primordial black holes

Stability

$$\tau_U \gtrsim 14.7 \text{ byrs} \qquad \Rightarrow \qquad \Gamma_{DM} \lesssim 10^{-42} \text{ GeV}$$

Generic issue for heavy particles

$$\Gamma = |\mathcal{A}|^2 \times \text{phase space}$$

$$\Gamma_{2-\text{body}} = \frac{m}{18\pi} y^2 = \left(\frac{m}{100 \text{ GeV}}\right)\left(\frac{y}{10^{-21}}\right)^2 \Gamma_{DM}$$

Stabilize by: symmetries $Z_2: \quad \text{SM} \to \text{SM}, \quad \text{DM} \to -\text{DM}$

decouple by hand (issues with corrections)

light enough, lightest in the spectrum

DM Production

=

Dark genesis

some Production mechanisms

Freeze-out

a DM particle is thermally coupled to the primordial plasma, just like neutrinos. After some time, the expansion of the universe is faster than the equilibration rate and DM decouples, freezes-out. From then on, it follows the expansion of the universe (red-shifts).

Freeze-in, oscillations/matter effects

sterile neutrinos may be produced through the mixing with active ones, either in vacuum or in presence of lepton number. Axions may also be produced from unfrozen oscillations of a classical axion field.

Non-thermal

decays from an inflation, gravitational production.

some

Production mechanisms

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Non-thermal

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Asymmetric DM production

Phase transitions (defects)

Multiple annihilations

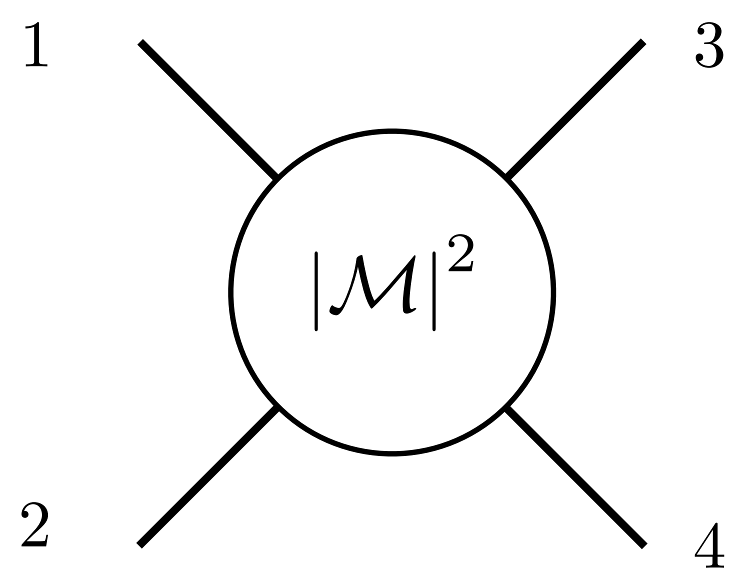
Multiple annihilations

...

Freeze-out

Boltzmann equation

[see e.g. Dodelson's book]

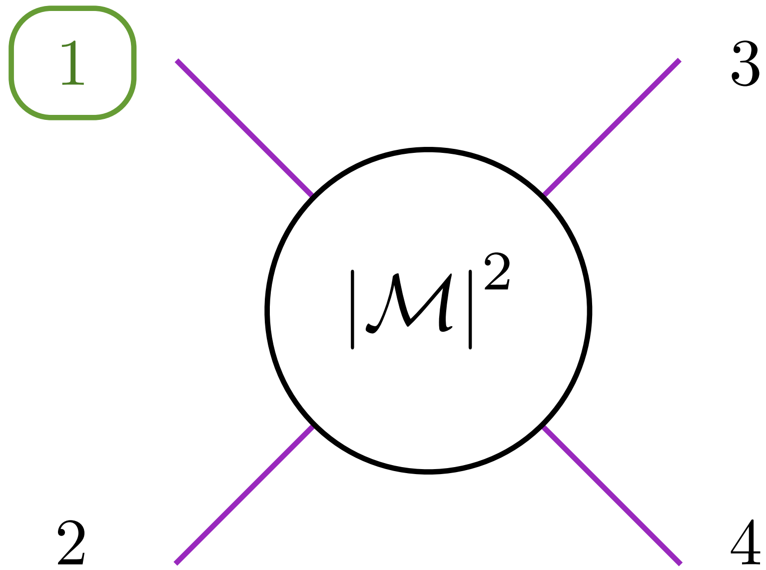


$$\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = \prod_{i=1}^{4}\int \frac{d^3 p_i}{(2\pi)^3 2E_i}\,(2\pi)^4\,\delta\left(\sum p_i\right)\delta\left(\sum E_i\right)$$
$$\times\,|\mathcal{M}|^2 \times \{f_3 f_4 (1\pm f_1)(1\pm f_2) - f_1 f_2 (1\pm f_3)(1\pm f_4)\}$$

Relevant for BBN, recombination, DM production, ...

Freeze-out

Boltzmann equation



[see e.g. Dodelson's book]

entropy conservation in absence of interactions

kinematics, relativistic phase space integration

$$\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = \prod_{i=1}^{4}\int \frac{d^3 p_i}{(2\pi)^3 2E_i}\,(2\pi)^4\,\delta\left(\sum p_i\right)\delta\left(\sum E_i\right)$$

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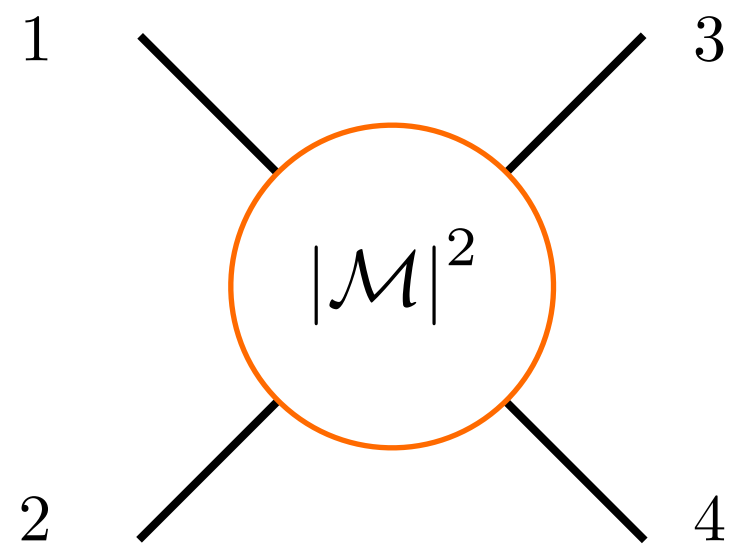

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kinematics, relativistic phase space integration $\equiv \int_{p_i}$

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$$\times\, |\mathcal{M}|^2 \times \{f_3 f_4 (1 \pm f_1)(1 \pm f_2) - f_1 f_2 (1 \pm f_3)(1 \pm f_4)\}$$

particle properties, interactions

statistics, Fermi vs. Bose

$$f(T < E_i) \rightarrow e^{\frac{\mu_i - E_i}{T}}$$

solve for the chemical equilibrium

Boltzmann equation

number density

$$n_i = g_i e^{\mu_i/T} \int \frac{d^3k}{(2\pi)^3}\, e^{-E_i/T}$$

equilibrium

$$n_i^{(0)} = g_i \int \frac{d^3k}{(2\pi)^3}\, e^{-E_i/T} = \begin{cases} \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T}, & m_i \gg T \\ g_i \frac{T^3}{m_i} & m_i \ll T \end{cases}$$

Boltzmann suppression

non-relativistic number in eq.

Boltzmann equation

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Boltzmann suppression

non-relativistic number in eq.

$$\frac{n_i}{n_i^{(0)}} = e^{-\mu_i/T}$$

thermally averaged cross-section

$$\langle \sigma v \rangle = \frac{1}{n_1^{(0)} n_2^{(0)}} \int_{p_i} |\mathcal{M}|^2\, e^{-(E_1+E_2)/T}$$

Boltzmann equation

number density

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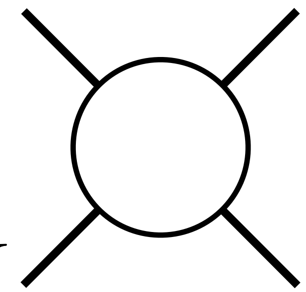
$$\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = n_1^{(0)} n_2^{(0)} \langle \sigma v \rangle \left\{ \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right\} \xrightarrow{\langle \sigma v \rangle \gg n^2} 0$$

strong interactions = equilibrium

WIMP

Thermal DM relic

$m = m_X$



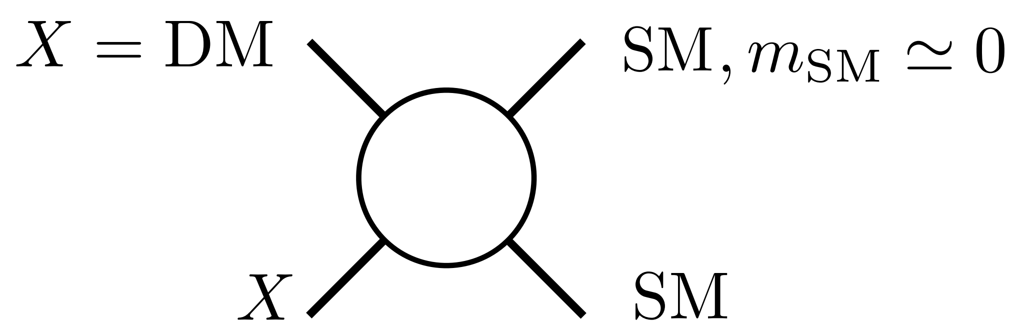
$n_{\mathrm{SM}} = n_{\mathrm{SM}}^{(0)}$

Boltzmann eq. becomes

$$\frac{1}{a^3}\frac{d(n_X a^3)}{dt} = \langle \sigma v \rangle \left(n_X^{(0)2} - n_X^2 \right)$$

WIMP

Thermal DM relic



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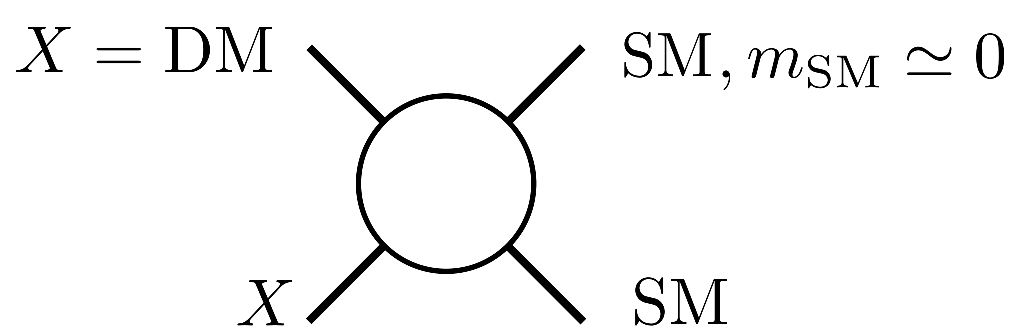
Yield

$$Y \equiv \frac{n_X}{T^3} \qquad \Rightarrow \qquad \frac{dY}{dt} = T^3 \langle \sigma v \rangle \left(Y_{\mathrm{eq}}^2 - Y^2 \right)$$

WIMP

Thermal DM relic



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Yield

$$Y \equiv \frac{n_X}{T^3} \qquad \Rightarrow \qquad \frac{dY}{dt} = T^3 \langle \sigma v \rangle \left(Y_{\mathrm{eq}}^2 - Y^2 \right)$$

x measures temperature

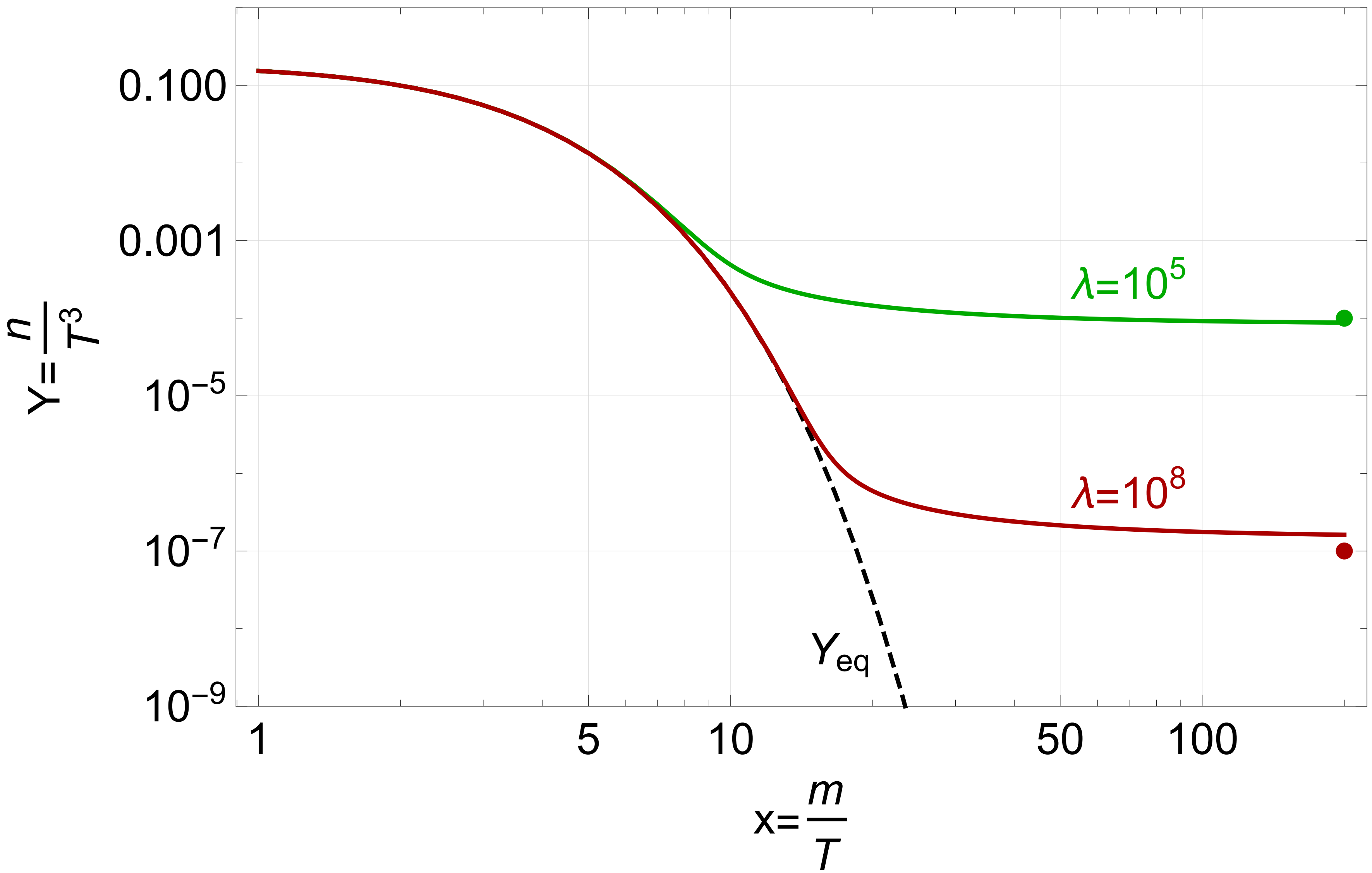
very simple differential equation

$$x \equiv \frac{m}{T} \qquad \Rightarrow \qquad \frac{dY}{dx} = -\frac{\lambda}{x^2} \left(Y^2 - Y_{\mathrm{eq}}^2 \right) \qquad \lambda \equiv \frac{m^3 \langle \sigma v \rangle}{H\, x^2}$$

WIMP

$$\frac{dY}{dx} = -\frac{\lambda}{x^2}\left(Y^2 - Y_{\text{eq}}^2\right)$$

```
Yeq[x_] := 1 / π^2 NIntegrate[y^2 / (Exp[Sqrt[y^2 + x^2]] + 1), {y, 0, Infinity}]
NDSolve[{Y'[x] == -λ / x^2 (Y[x]^2 - Yeq[x]^2), Y[x0] == Yeq[x0]}, Y[x], {x, x0, 100}]
```

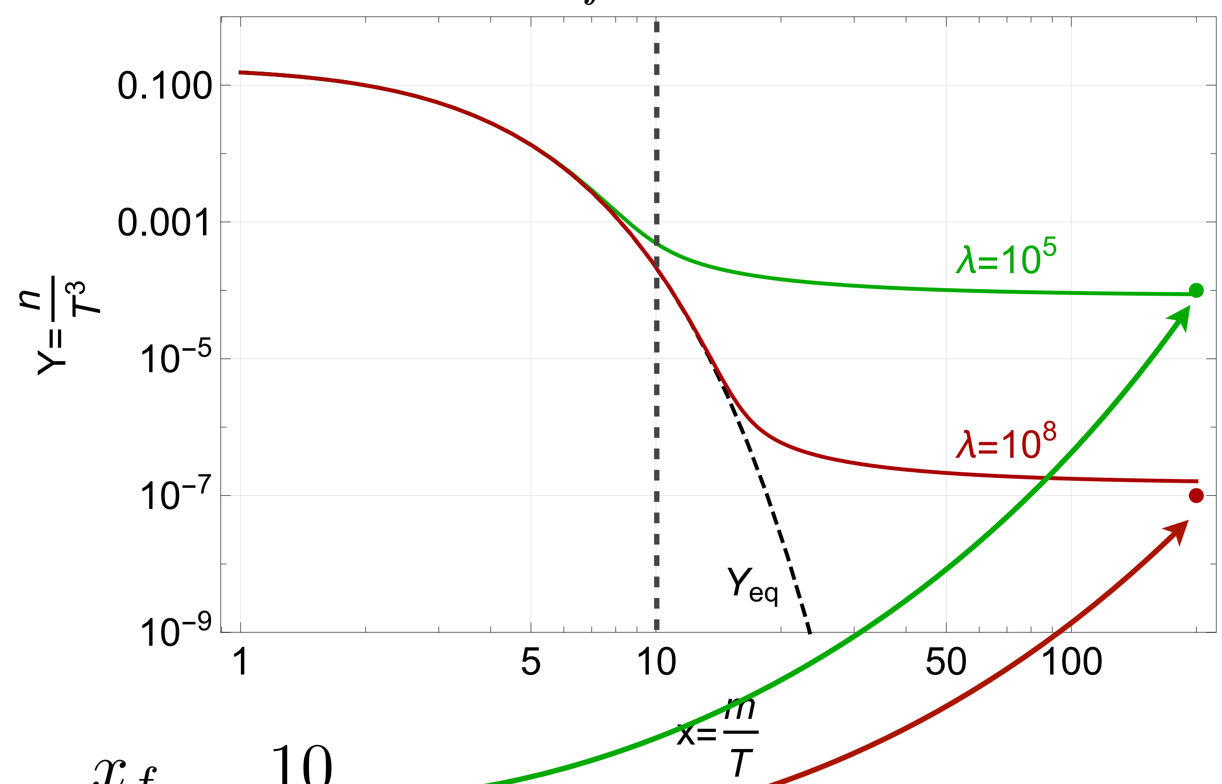


weaker annihilation,
freeze-out early,
higher abundance

stronger coupling,
annihilates more,
freezes-out later,
abundance drops

WIMP

$x_f \simeq 10$



Freeze-out analytic estimates can be useful

late times, low T freeze-out value $x \gg 1$

$$\frac{dY}{dx} \simeq -\lambda \frac{Y^2}{x^2}$$

$$\frac{1}{Y_\infty} - \frac{1}{Y_f} = \frac{\lambda}{x_f} \qquad Y_\infty = \frac{x_f}{\lambda} \simeq \frac{10}{\lambda}$$

improved estimate $\Gamma(x_f) = n_X^{(0)} \langle \sigma v \rangle = H(x_f)$ $\quad \frac{e^{x_f}}{\sqrt{x_f}} = \underbrace{\frac{3 g_1}{\sqrt{(2\pi)^3 g_*}}}_{0.1} \langle \sigma v \rangle m \overline{M}_{\rm Pl}$

improved semi-analytic approach [Beacom, Dasgupta, Steigman 1204.3622]

tools [DarkSUSY, micrOMEGAs]

exceptions: co-annihilations, up-scattering, resonances [Griest, Seckel PRD 43, '91]

WIMP

Dark matter abundance   exp. value $\Omega_{DM} \simeq 0.12 h^{-2} = 0.265$

DM energy density $\rho_{DM}(T_f) = m\, n_\infty = m\, Y_\infty\, T_f^3$ at freeze-out

today: $$\rho_{DM} = m\, Y_\infty\, T_0^3 \left(\frac{a_1 T_1}{a_0 T_0}\right)^3 \sim \frac{m\, Y_\infty\, T_0^3}{30}$$

Normalized density $$\Omega_{DM} = \frac{\rho_{DM}}{\rho_{cr}} = \frac{Y_\infty\, m\, T_0^3}{30\, \rho_{cr}} = \frac{x_f\, m\, T_0^3}{\lambda\, 30\, \rho_{cr}}$$

$$\Omega_{DM} \simeq 0.3\, h^{-2} \left(\frac{x_f}{10}\right) \left(\frac{g_*}{100}\right)^{1/2} \boxed{\frac{10^{-39}\ \mathrm{cm}^2}{\langle \sigma v \rangle}}$$

weak cross-section = WIMP

Sterile neutrinos

See-saw

$$\mathcal{L}_{\nu_s} = \frac{i}{2}\,\overline{\nu}_s \partial\!\!\!/ \nu_s - \frac{1}{2} m_s \nu_s^T\, C\, \nu_s - y_D \overline{\nu}_s\, H\, L$$

$$m_D = y_D\, v$$

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_s \end{pmatrix}$$

$$m_\nu \simeq \frac{m_D^2}{m_s}$$

$$m_{\nu_4} \simeq m_s$$

masses

M_ν has to be diagonalized, therefore sterile and active neutrinos mix

$$\nu \simeq \nu_L - \theta\, \nu_s, \qquad \nu_4 \simeq \nu_s + \theta\, \nu_L$$

mixings

$$\theta \simeq \frac{m_D}{m_s} \simeq \sqrt{\frac{m_\nu}{m_{\nu_4}}} = 3 \times 10^{-3} \left(\frac{m_\nu}{0.01 \text{ eV}}\right)^{1/2} \left(\frac{m_{\nu_4}}{\text{keV}}\right)^{-1/2}$$

Sterile neutrinos

$\langle \sigma v \rangle \simeq \theta^2 G_F^2 T^2$

$\Gamma_{\nu_4} \simeq n_{eq} \langle \sigma v \rangle \simeq \theta^2 G_F^2 T^5$ [Dodelson, Widrow '93]

Sterile neutrinos

$$\langle \sigma v \rangle \simeq \theta^2 G_F^2 T^2 \qquad \Gamma_{\nu_4} \simeq n_{eq} \langle \sigma v \rangle \simeq \theta^2 G_F^2 T^5$$

[Dodelson, Widrow '93]

Freeze-in

$$\frac{dY_{\nu_4}}{dT} = -\frac{\langle \sigma v \rangle n_{SM}^2}{HTs} \qquad \theta \to \theta_{\rm eff} \sim \frac{\theta}{1+(T/T_0)^6} \qquad T_0 \sim 100 \text{ MeV}$$

$$\Omega_\nu h^2 \sim 0.1 \left(\frac{\theta^2}{3 \times 10^{-9}} \right) \left(\frac{m_{\nu_4}}{3 \text{ keV}} \right)^{1.8}$$

Sterile neutrinos

$$\langle \sigma v \rangle \simeq \theta^2 G_F^2 T^2 \qquad \Gamma_{\nu_4} \simeq n_{eq} \langle \sigma v \rangle \simeq \theta^2 G_F^2 T^5$$ [Dodelson, Widrow '93]

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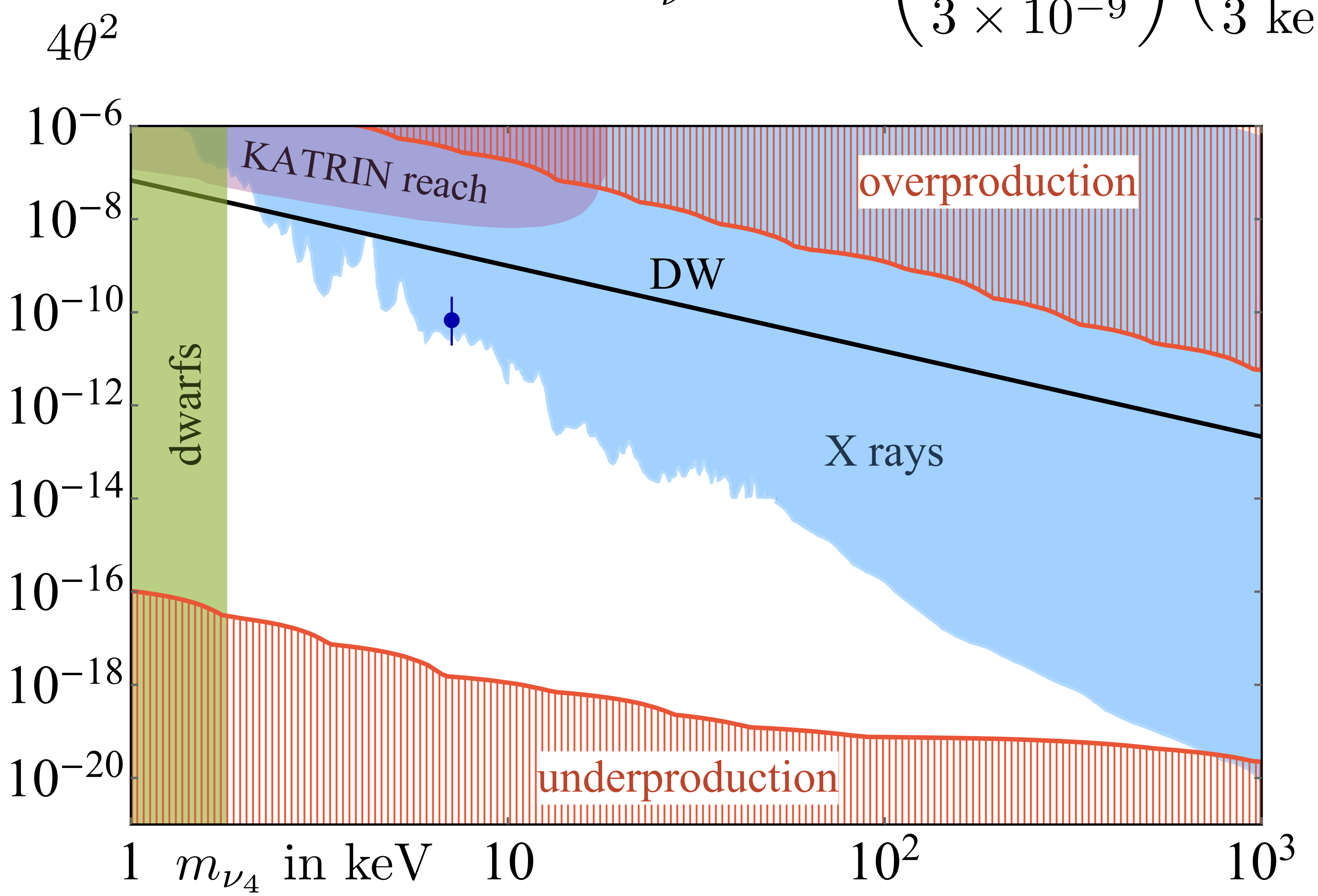
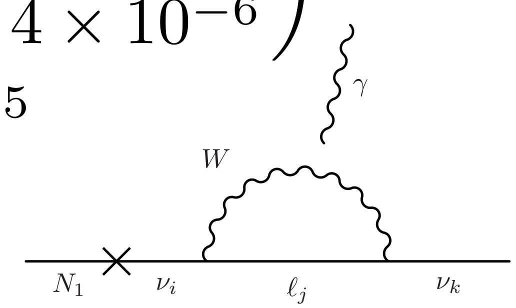


[de Gouvêa, Sen, Tangarife, Zhang '19]

ruled out by X-ray searches

$$\Gamma \sim 10^{-28} \text{ s}^{-1} \left(\frac{\theta}{4 \times 10^{-6}} \right)^2 \times \left(\frac{m_{\nu_4}}{7 \text{ keV}} \right)^5$$



'saved' with additional interactions

anomaly at 7 keV

Right-handed neutrinos

Additional gauge interactions [Bezrukov et al. ’09, Nemevšek et al. ’11]

Left-right symmetry and parity restoration [Pati, Salam ’74, Mohapatra, Pati ’75, Mohapatra, Senjanović, ’75…’79]

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

matter comes in symmetric representations, three right-handed neutrinos required by gauge anomaly cancellation

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad L_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

extra gauge bosons $W_R^{\pm}, Z_{LR}$ $\qquad M_{Z_{LR}} \sim \sqrt{3} M_{W_R}$

Additional gauge interactions

RH neutrinos in equilibrium

$$\langle \sigma v \rangle \simeq \left(\frac{M_{W_L}}{M_{W_R}} \right)^4 G_F^2 T^2$$

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thermal freeze-out

$$\left(\frac{M_{W_L}}{M_{W_R}} \right)^4 G_F^2 T^5 \simeq \sqrt{2 g_*} \frac{T^2}{\overline{M}_{\rm Pl}^2}$$

weaker coupling = earlier freeze-out, around the QCD phase transition

$$T_f \simeq 400 \text{ MeV} \left(\frac{g_*}{70} \right)^{1/6} \left(\frac{M_{W_R}}{5 \text{ TeV}} \right)^{4/3}$$

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$$T_f \simeq 400 \text{ MeV} \left(\frac{g_*}{70} \right)^{1/6} \left(\frac{M_{W_R}}{5 \text{ TeV}} \right)^{4/3} > m_{\nu_R} \simeq \text{ keV}$$

relativistic at freeze-out, no Boltzmann suppression

$$Y_{\nu_R} = \frac{n_{\nu_R}}{s} \simeq \frac{135 \zeta(3)}{4 \pi^4 g_*}$$

Over-abundance and entropy dilution

un-suppressed additional species leads to over-production [Lee, Weinberg '77]

$$\Omega_{\nu_R} = Y_{\nu_R} m_{\nu_R} \frac{s}{\rho_c} = 3.3 \left(\frac{m_{\nu_R}}{\mathrm{keV}}\right) \left(\frac{70}{g_*}\right) \simeq 13\,\Omega_{DM}$$

either many d.o.f suppress the abundance or there's explicit dilution

similar to electron annihilation, heavy particle (diluter) dominates the universe, decays to radiation and heats up the photons w.r.t. to DM [Scherrer, Turner '85]

$$\Gamma_D^2 = H^2 = \frac{Y_D m_D}{\overline{M}_{\mathrm{Pl}}} \frac{2\pi^2 g_* T_<^3}{45}$$

$$T_< = 0.65 \left(\frac{\Gamma_D M_{\mathrm{Pl}}}{Y_D m_D g_*}\right)^{1/3}$$

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$$T_< = 0.65 \left(\frac{\Gamma_D M_{\rm Pl}}{Y_D m_D g_*}\right)^{1/3} \qquad T_> \simeq 0.78 \sqrt{\frac{\Gamma_D M_{\rm Pl}}{\sqrt{g_*}}} \simeq 1.22\ \mathrm{MeV} \sqrt{\frac{1\ \mathrm{s}}{\tau_D}}$$

Over-abundance and entropy dilution

[Nemevšek, Senjanović, Zhang '12]

final dilution factor and DM abundance

sudden decay approx

$$m_{\nu_R} Y_{\nu_R} s_< = \frac{3}{4} s_> T_r$$

reheating temperature above MeV for BBN

Over-abundance and entropy dilution

[Nemevšek, Senjanović, Zhang '12]

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reheating temperature above MeV for BBN

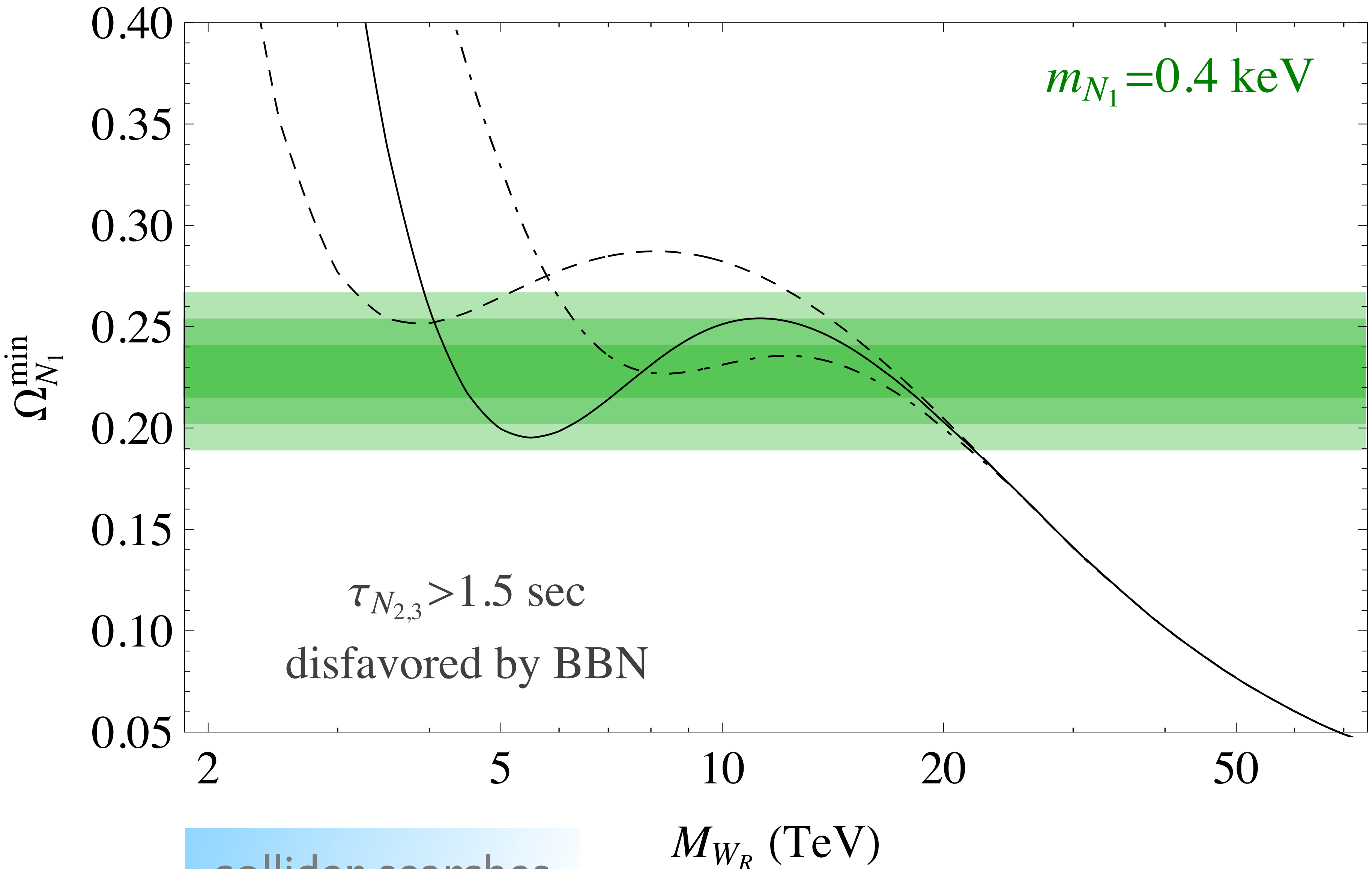
dilution factor $$\mathcal{S} \equiv \frac{S_{\rm after}}{S_{\rm before}} \simeq \frac{s_{\rm after}}{s_{\rm before}} \simeq 1.8 \left(g_* \left(T_r\right)\right)^{1/4} \frac{Y_{\nu_R} m_{\nu_R}}{\sqrt{\Gamma_D M_{\rm Pl}}}$$

final DM abundance $$\hat{\Omega}_{\nu_R} \simeq 0.23 \left(\frac{m_{\nu_R}}{1{\rm keV}}\right) \left(\frac{1.85{\rm GeV}}{m_D}\right) \left(\frac{1{\rm sec}}{\tau_D}\right)^{1/2} \left(\frac{g_* \left(T_{fD}\right)}{g_* \left(T_{f\nu_R}\right)}\right)$$

Warm dark matter in Left-Right

[Nemevšek, Senjanović, Zhang '12]

rich phenomenology, connections to other searches/experiments



collider searches

neutrino-less double beta

EDMs

Outline

2) Phase transitions and gravitational waves [~2hr]

- ❑ effective potential/basics of thermal field theory [Quiros ICTP lectures, Laine&Vuorinen, Kapusta & Gale]
- ❑ first order phase transitions
- ❑ false vacuum decay [Coleman lectures]
- ❑ energy budget of gravitational waves
- ❑ GW spectra and sensitivities