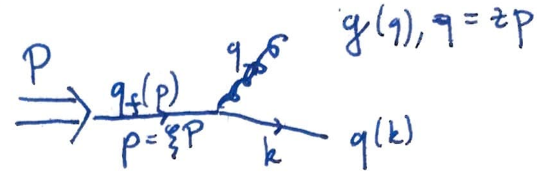
12. PARTONS and JETS

- pQCD @ $E \gg$ GeV allows for more detailed understanding by studying QCD jets

12.1. Splitting functions



$$\Gamma(q_f^* \to q g) = \int dz \int^{Q} \frac{dq_\perp}{q_\perp} \frac{4}{3} \frac{\alpha_s(q_\perp)}{\pi} \frac{1+(1-z)^2}{z}$$

$p \to k\, q$

$\Downarrow$ integrate over PDFs to get the emission from protons

$$\int dx\, f_g(x) = \sum_f \int d\xi\, \Gamma(q_f^* \to q g) \times f_f(\xi),$$

$\Downarrow$ gluon PDF ; $x = z\xi$, $d\xi = \frac{dx}{z}$

$q = xP$
$= zp = z\xi P$

$$\Rightarrow f_g(x) = \int \frac{dz}{z} \int_0^Q \frac{dq_\perp}{q_\perp} \frac{4}{3} \frac{\alpha_s(q_\perp)}{\pi} \frac{1+(1-z)^2}{z} f_f\left(\frac{x}{z}\right)$$

$$\frac{df_g}{d\ln Q} = \frac{\alpha_s}{\pi} \int_x^1 \frac{dz}{z} \underbrace{\frac{4}{3} \frac{1+(1-z)^2}{z}}_{P_{g\leftarrow q}(z)} f_f\left(\frac{x}{z}\right)$$

$P_{g\leftarrow q}(z)$ splitting functions

• From the splitting  we got the evolution equation for the gluon PDF. Adding the other possible strahlung processes  & 

GLUONS : $$\frac{df_g}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_x^1 \frac{dz}{z}\left(P_{g\leftarrow g}(z)\, f_g\left(\frac{x}{z}\right) + \sum_f P_{g\leftarrow q}\,(f_f + f_{\bar f})\left(\frac{x}{z}\right)\right)$$

QUARKS : $$\frac{df_q}{d\ln Q} = \frac{\alpha_s(Q)}{\pi}\int_x^1 \frac{dz}{z}\left(P_{q\leftarrow g}\, f_g + P_{q\leftarrow q}\, f_q\right)$$

ANTI-QUARKS : —"— $\Big|_{q\to\bar q}$

Quark splitting :

$$P_{g\leftarrow q} = \frac{4}{3}\,\frac{1+(1-z)^2}{z} \xrightarrow{z\to 0} \frac{4}{3}\,\frac{2}{z}$$

$$P_{q\leftarrow q} = \frac{4}{3}\left(\frac{1+z^2}{1-z} + \underbrace{A\,\delta(z-1)}_{*}\right),$$

Gluon splitting :

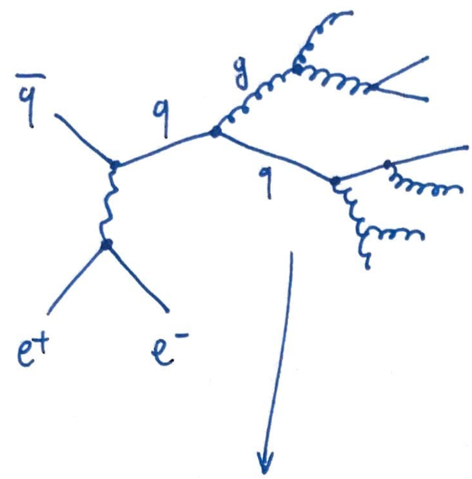
$$P_{q\leftarrow g} = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

$$P_{g\leftarrow g} = 3\left(\frac{1+z^4+(1-z)^4}{z(1-z)} + B\,\delta(z-1)\right) \overset{z\to 0}{\simeq} 3\,\frac{2}{z}$$

*The δ term control the behaviour at ~~small~~ $z\sim 1$ to have $\int_0^{1-\varepsilon} dz\, P_{q\leftarrow q}(z) = 0$

• $z\sim 0$ P_i grow $\Rightarrow$ $q, \bar q$ & g acquire a lot of soft gluons (= jets)

12.2. JETS! STRUCTURE, BEHAVIOR & CLUSTERING



* SOFTER & COLLINEAR

momentum softens with each splitting ⇒ α_s grows and eventually confines. We are left with $\underbrace{p, n, \ldots}_{\text{baryons}}$ $\underbrace{\pi, K, \ldots}_{\text{mesons}}$

HADRONIC jets

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* Here, the "naive" infinities appear that are properly regulated in QCD but signal $\alpha_s \sim 1$ & nonperturbativity.

- To define a well-behaved jet algorithm / observable, it has to be insensitive to these soft = IR issues.

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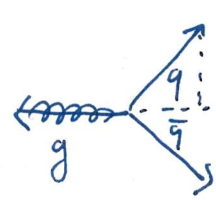
e.g. thrust $\quad T = \max\limits_{\hat{n}} \dfrac{\sum |\hat{n} \cdot p_i|}{\sum |p_i|}$

$\xrightarrow{\hat{n}} T = 1$



$\xrightarrow{\hat{n}} \dfrac{1 + \overbrace{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}^{2}}{3} = \dfrac{2}{3}.$

- Jet algorithms collected in FASTJET package
  - anti k_T
  - k_T
  - SISCONE
  - ... (JADE)