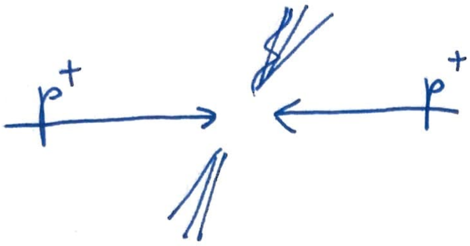
13. QCD @ Hadron colliders

We can address the pp & $p\bar{p}$ initial beams with the tools at hand.



$$\sigma_{pp} \sim \begin{cases} \text{SOFT} & \Delta p \sim \text{GeV} \\ \text{HARD} & \Delta p \gg \text{GeV} \end{cases}$$

SOFT processes dominate the σ. They give a large number of soft QCD particles, not well defined jets

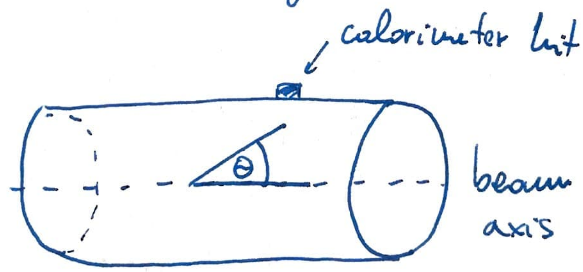
Pomeranchuk theorem $\quad \dfrac{\sigma(pp)}{\sigma(\bar{p}p)} \sim 1 \;, \quad \dfrac{\sigma(\pi^+ p)}{\sigma(\pi^- p)} \sim 1$

$$\sigma^{pp}_{SOFT}(\sqrt{s} \sim \text{TeV}) \sim 100\,\text{mb} = 0.1\,\text{b}$$

COLLIDER VARIABLES

Detectors are typically cyllindrical along the $\hat{z}$ axis, therefore one defines transverse energies / momenta

$$\underbrace{E_T}_{\text{transverse energy}} = E \sin\Theta$$



$\underbrace{p_x^2 + p_y^2} = p_T^2$ ... transverse momentum

$$p^\mu = (E, p_x, p_y, p_z) \xrightarrow[\text{TO } x]{\text{ROTATE}} p^\mu = (E, p_T, 0, p_\parallel)$$

$$p^2 = m^2 = E^2 - p_T^2 - p_\parallel^2$$

It is useful to define a Lorentz-additive variable, similar to the boost, called <u>rapidity</u>.

$$\left.\begin{aligned} E &= \sqrt{p_T^2+m^2}\ \mathrm{ch}\, y \\ p_\parallel &= \sqrt{p_T^2+m^2}\ \mathrm{sh}\, y \end{aligned}\right\} \quad \mathrm{th}\, y = \frac{p_\parallel}{E}$$

$$\text{or} \quad y = \mathrm{th}^{-1} \frac{p_\parallel}{E}$$

$$\downarrow$$

$$E^2 - p_\parallel^2 = p_T^2 + m^2 \ \checkmark$$

Remember the boost by β gives $p_\parallel' = \gamma(p_\parallel - \beta E)$

$$E' = \gamma(E - \beta p_\parallel)$$

with $\gamma = \mathrm{ch}\,\alpha$, $\gamma\beta = \mathrm{sh}\,\alpha$

we get $E' = \underbrace{\sqrt{p_T^2+m^2}}_{m_T}\ \mathrm{ch}(y+\alpha)$, $p_\parallel' = \sqrt{p_T^2+m^2}\ \mathrm{sh}(y+\alpha)$

For large momentum transfer, $m \sim 0$ & $m_T = p_T$

$$E = p$$

$$p^\mu = (p, p_T, 0, p_\parallel) \sim (p_T\,\mathrm{ch}\, y, p_T, 0, p_T\,\mathrm{sh}\, y)$$

& $p + p_\parallel = p_T e^y$ or $y \cong \ln \frac{p + p_\parallel}{p_T} = \ln \frac{p\cos\theta + 1}{p\sin\theta}$

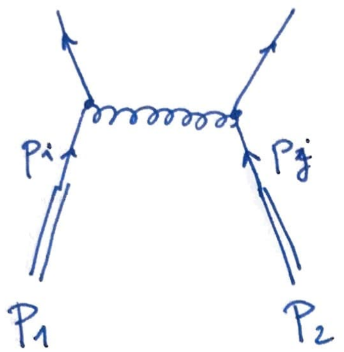
$$= \frac{1}{2} \ln \frac{1 + c_\theta}{1 - c_\theta}.$$

$\eta = \frac{1}{2} \ln \frac{1+c_\theta}{1-c_\theta}$ ... pseudo-rapidity.

$\parallel$

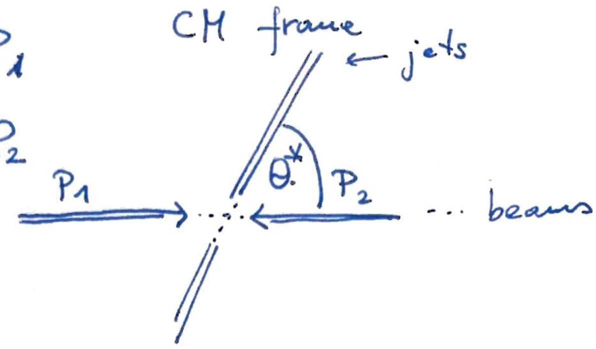
EASY to measure from trackers & calorimeters.

13.2. HARD scattering at large $\Delta p \gg$ GeV



$p_i = x_1 P_1$

$p_j = x_2 P_2$



Using the known PDFs and summing over all the partons, the pp cross-section to 2 jets is given by

$$\sigma_{pp\to jj} = \sum_{\substack{ijke \\ = \\ q,\bar{q},g}} \int dx_1\, dx_2 \underbrace{\sigma_{q_i q_j \to ke}}_{} f_i(x_1) f_j(x_2)$$

computable in QCD as we did perturbatively in QED

- all partons have $\hat{s}, \hat{t}, \hat{u}$ Mandelstam variables

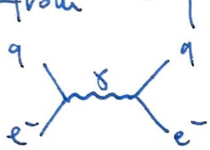
$*m_p \sim 0$

$$\hat{s} = (p_i+p_j)^2 = 2p_i p_j + \overset{0}{\cancel{m_{i,j}^2}} = 2x_1x_2P_1P_2 = x_1x_2 s$$

$$\hat{s} = x_1 x_2 s$$

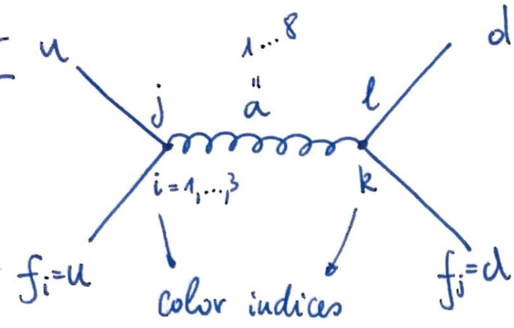
- from the jet momenta $\Rightarrow \hat{s}$
- from the jet θ^* $\Rightarrow \hat{t}$ & $\hat{u}$

from e^-q :



$$\left.\frac{d\sigma}{dc_\theta}\right|_{eq\to eq} = \frac{\pi Q_q^2 \alpha^2}{s} \frac{s^2+u^2}{t^2} \hookrightarrow$$

DIFFERENT FLAVORS



• average over the initial colors

$$\frac{1}{3}\cdot\frac{1}{3}\cdot\sum_{i,k}$$

• sum over the intermediate gluons $\sum_{a=1}^{8}$

• sum over the final colors $\sum_{j,l}$

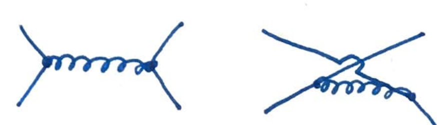
$$\frac{1}{9}\cdot\sum_{ijkla}\left|t^a_{ji}\,t^a_{kl}\right|^2 = \frac{1}{9}\underbrace{\mathrm{tr}\,t^b t^a}_{\frac{1}{2}\delta^{ab}}\;\underbrace{\mathrm{tr}\,t^b t^a}_{\frac{1}{2}\delta^{ab}}$$

$$= \sum_a \frac{1}{9}\,\frac{1}{2}\,\frac{1}{2}\,\delta^{aa} = \frac{2}{9}$$

$$\frac{d\sigma_{ud\to ud}}{dc_{\theta^*}} = \frac{2}{9}\,\frac{\pi\alpha_s^2}{\hat{s}}\,\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}.$$

SAME FLAVOR $f_i=u$, $f_j=u$

• additional diagrams:



$$\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}-\frac{2}{3}\,\frac{\hat{s}^2}{\hat{u}\hat{t}}$$

• remember: $\hat{t} = -\frac{1}{2}\hat{s}(1-c_{\theta^*})$, $\hat{u} = -\frac{1}{2}\hat{s}(1+c_{\theta^*})$

forward region: $\theta^*\simeq 0 \Rightarrow \hat{t}\to 0$, $\hat{u}\to -\hat{s}$

$$\frac{d\sigma}{dc_{\theta^*}} \sim \frac{4\pi\alpha^2}{9\,\hat{s}}\,\frac{1}{s^4_{\theta^*/2}}$$ ... Coulomb scattering on Vaco.

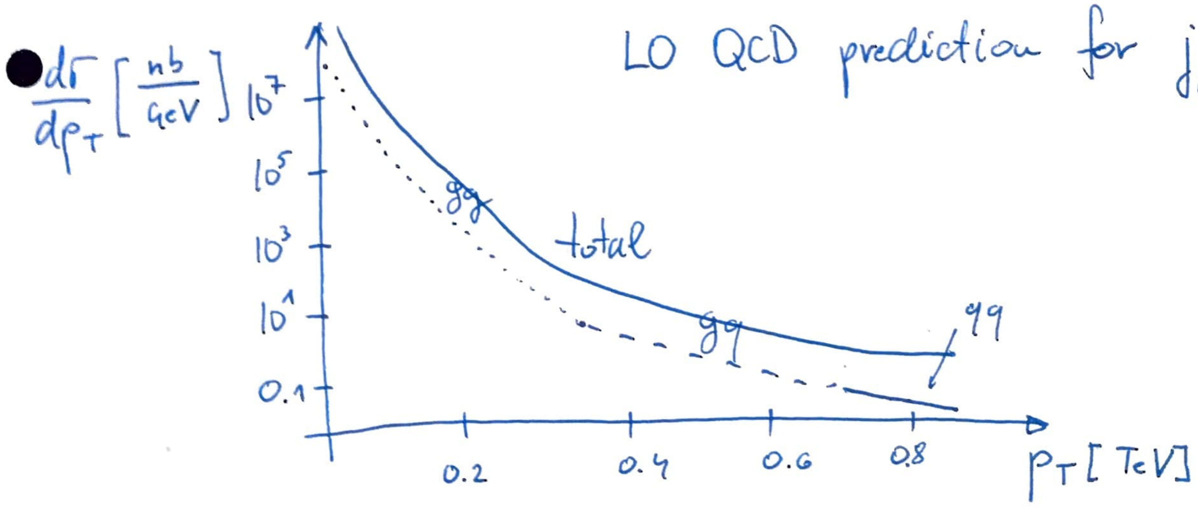
• similarly we get the qg & gg σ's

$$\frac{d\sigma}{dc_{\theta^*}} = \frac{\pi \alpha_s^2}{2\hat{s}} \begin{cases} \frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} - \frac{4}{9}\frac{\hat{u}^2+\hat{s}^2}{\hat{u}\hat{s}} & \text{for } qg \to qg \\ \sim \frac{9}{2}\frac{\hat{s}^2}{\hat{t}^2} & \text{for } gg \to gg \end{cases}$$

$\Downarrow$

• all of these are then convoluted with the q & g PDFs to obtain the $pp \to jj$ hard scattering events

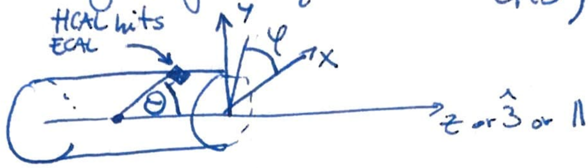
- @ low p_T : gg dominates due to $\frac{9}{2}$ factor
- @ higher p_T: g PDF diminishes, qg takes over and finally qq is dominant at very high p_T



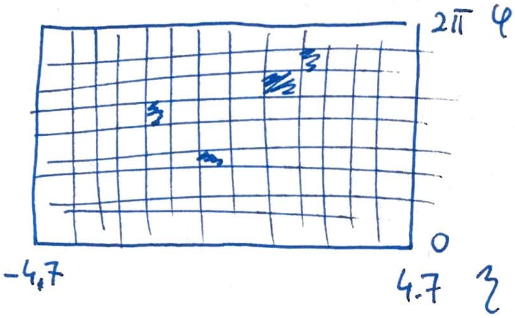
LO QCD prediction for jj events

13.3. JET OBSERVABLES AND ALGORITHMS

- need to cluster the tracks and calorimeter hits into well defined bundles = JETS
- first we eliminate all the SOFT activity that goes down the beam pipe anyways $|\eta| \gtrsim 4.7$ (ATLAS & CMS)



- look at $\dfrac{dE_T}{d\eta\, d\varphi}$
- spread out the hits in the $\left(\eta = \frac{1}{2}\ln\left(\frac{1+c_\theta}{1-c_\theta}\right), \varphi\right)$ plane and define the distance



$$\Delta R^2 = \Delta\eta^2 + \Delta\varphi^2$$

∥

distance between two events

- now combine all events w. $p_T > p_{T\,\mathrm{jcut}}$ & $\Delta R > R_{\mathrm{cut}}$ $\quad 0.5 \sim 0.4$

∥

JET CONE SIZE

- this is further weighted (to be IR safe) by E_T, e.g. in the anti-k_T algorithm

$$\lambda_{ij} = \Delta R_{ij}^2 \min\left(\frac{1}{E_{Ti}^2}, \frac{1}{E_{Tj}^2}\right)$$

- The JET width variable $\Psi(\rho) = \dfrac{\sum_{\Delta R<\rho R} E_T}{\sum_{\Delta R<R} E_T} \in [0,1]$

LO: q NLO q

$1-\Psi \sim 0$

HADRONIZATION (PYTHIA, HERWIG)

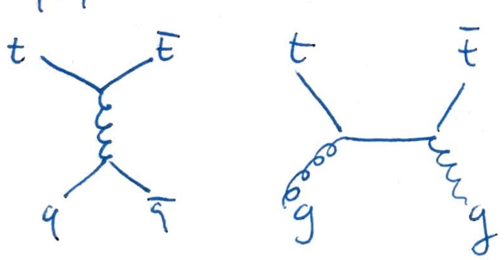
- q-jets are typically more narrow and dominate at high p_T, while gluon jets are broader and are prevalent at low jet p_T.

- top quark pair production, $m_t \sim 173$ GeV, $2m_t \sim 350$ GeV

$$\sigma^{LHC}_{pp\to t\bar{t}}(13\text{ TeV}) \sim 100 \times \sigma^{Tevatron}_{p\bar{p}\to t\bar{t}}(\sqrt{s} = 2\text{ TeV})$$

while $\dfrac{\sqrt{s}=13}{\sqrt{s}=2} \sim 6.5$ how such a large increase is explained

1) High $Q^2 \sim (350\text{ GeV})^2$ is only possible with $q,\bar{q}$ at the TeVatron, much easier @ LHC



+ u + s - channels

2) Gluon σ higher by a factor of 5.