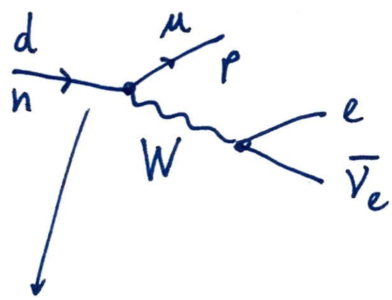
15. Weak Interaction

- Most QCD-mediated decays are fast $\Gamma \sim \Lambda_{QCD}$ or $\sim$ GeV $\Rightarrow \tau \sim 10^{-24}$ s.

- Lightest mesons / baryons ($\pi/K/B$ / Λ/n) are "stable" wrt QCD they decay through weak interactions

$$\tau_{\pi,K} \sim 10^{-8}\,s, \quad \tau_B \sim 10^{-12}\,s, \quad \tau_{\Lambda^0} \sim 10^{-10}\,s$$

$$\tau_n \sim 880\,s \quad * \text{ phase space } \Delta m \ll m$$



... weak interaction $M_W \gg \Lambda_{QCD}$

the quark flavor changes w. weak interactions, not QCD; so any FC process involves the weak interaction and is "slow" wrt QCD.

- First try: weak current, just like QED & QCD

$$j_W^\mu = \bar{\psi}_i \gamma^\mu \psi_j \qquad \text{Fermi '34}$$

• <u>History</u> : – Chadwick measures β decay e^-

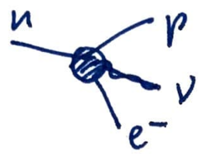
lightning review

$\Downarrow$

not a sharp line 

$\Rightarrow$ 3 body decay

– Pauli postulates the neutrino in '32



– Fermi proposes a current-current interaction $j^\mu_{np}\, j_{\mu\, e\nu}$ '34

$$\overset{\parallel}{\bar p \gamma^\mu n} \quad \bar e \gamma^\mu \nu \; \ldots$$ parity conserving

– '56 Lee & Yang propose parity violation in weak processes; Wu et al.; Lederman et al. follow up w. exp. ✓
Co decay; μ decay

– '58 Out of all possible (scalar, pseudo-scalar, ...) combinations, Sudarshan & Marshak; Feynman & Gell-Mann '58 propose / single-out the V−A ; $P_L = (1-\gamma_5)/2$ interaction.

$$\psi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix} = \underbrace{\overset{P_L}{\overset{\parallel}{\left(\frac{1-\gamma_5}{2}\right)}}\psi}_{\begin{pmatrix} \chi_L \\ 0 \end{pmatrix}} + \underbrace{\overset{P_R}{\overset{\parallel}{\left(\frac{1+\gamma_5}{2}\right)}}\psi}_{\begin{pmatrix} 0 \\ \chi_R \end{pmatrix}}$$

• only one chirality participates in weak interactions

- so instead of $\bar{\Psi}\gamma^\mu\Psi = \bar{\chi}_L\bar{\sigma}^\mu\chi_L + \cancel{\bar{\chi}_R\sigma^\mu\chi_R}$
- this holds for quarks and leptons

$$j_L^{\mu+} = u_L^+\bar{\sigma}^\mu d_L + \nu_L^+\bar{\sigma}^\mu e_L$$
$$j_L^{\mu-} = d_L^+\bar{\sigma}^\mu u_L + e_L^+\bar{\sigma}^\mu \nu_L$$

- the amplitude is a jj product $\times$ weak Fermi constant

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}}\, j_L^{\mu+}\, j_{L\mu}^{-}\, .$$

- $G_F \cong 10^{-5}\,\mathrm{GeV}^{-2}$ ... $[G_F] = -2 \Rightarrow \exists$ cut-off of about 100 GeV
  $\sim \dfrac{1}{(100\,\mathrm{GeV})^2}$
- This theory is a priori non-renormalizable with an expected upper bound on the region of validity.

15.2. PREDICTIONS FOR LEPTONIC PROCESSES

- $j^\mu_L = e_L^+ \bar{\sigma}^\mu \nu_L$
- Let us understand the polarization of e^- in β decay. Since only e_L^- participates, we expect the high E e^- to be completely polarized. ~~and~~

The low energy ones can flip the chirality only through ~~the~~ m_e mass insertion. This effect is $\propto \frac{m_e}{Q}$.

- For low E e^- we use the chiral ~~not. rel.~~ γ^μ basis.

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \& \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\overset{e^-}{\circ} \xrightarrow{\hat{3}} \qquad p^\mu = (E, 0, 0, p) \quad , \quad E^2 - p^2 = m_e^2$$

DIRAC eq. :
$$\begin{pmatrix} 0 & E - p\sigma^3 \\ E + p\sigma^3 & 0 \end{pmatrix} \underbrace{u(p)}_{\text{4-comp. spinor}} = 0$$

$p_1, p_2 = 0$

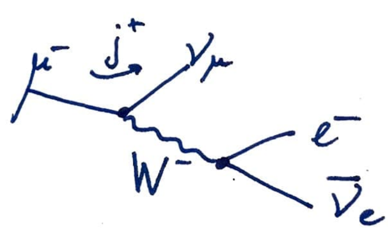
$$\Rightarrow \text{RH}: \; u_{e_R} = \begin{pmatrix} \sqrt{E-p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \sqrt{E+p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix}, \quad u_{e_L}(p) = \begin{pmatrix} \sqrt{E+p} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \sqrt{E-p} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix}$$

- The measured polarization is defined by the L-R asymmetry factor $\mathcal{A}_{LR} = Pol = \dfrac{P_{e_L} - P_{e_R}}{P_{e_L} + P_{e_R}}$

- In weak interactions the e_L^+ part of the field operator acts only on the upper component

$$\Rightarrow \quad Pol = \frac{(\sqrt{E+p})^2 - (E-p)}{E+p + E-p} = \frac{p}{E} = \frac{\gamma m v}{\gamma m c} = \beta$$

3) <u>MUON DECAY</u>



$m_\nu \sim 0$

$\mu^- \to \nu_\mu e^- \bar{\nu}_e$

$$\mathcal{M} = \langle \nu_\mu e^- \bar{\nu}_e | \frac{4G_F}{\sqrt{2}} \nu_\mu^+ \bar{\sigma}^\mu \mu_L \, e_L^+ \bar{\sigma}_\mu \nu_e | \mu \rangle$$

$$= \frac{4G_F}{\sqrt{2}} u_L^+(p_\nu) \bar{\sigma}^\mu u_L(p_\mu) \, u_L^+(p_e) \bar{\sigma}_\mu u_L(p_{\bar{\nu}})$$

$$\overline{\mathcal{M}}^2 = \frac{1}{2} \frac{16 G_F^2}{2} \cdot 4 (2 p_e p_\nu)(2 p_\mu p_{\bar{\nu}})$$

average over the spin of the muon

- To derive the $\overline{|\mathcal{M}|^2}$ we had to sum over all the spins of the final state fermions.

Note : $(\bar{\sigma}^\mu)_{\alpha\beta}\,(\bar{\sigma}_\mu)_{\gamma\delta} = 2\,\varepsilon_{\alpha\gamma}\,\varepsilon_{\beta\delta} \quad \forall \alpha,\beta,\gamma,\delta$

This gives $2\,\underbrace{\left(u^\dagger_\alpha(p_\nu)\,\varepsilon_{\alpha\gamma}\,u^\dagger_\gamma(p_e)\right)}_{\text{L.I. }(\star)}\underbrace{\left(u_\beta(p_\mu)\,\varepsilon_{\beta\delta}\,u_\delta(p_{\bar\nu})\right)}_{\text{L.I. }(\star\star)}$

- For $(\star)$, go to the $e^-_L \xleftarrow{\;\text{spin up}\;\Rightarrow\;} \bullet \xrightarrow[\;\Leftarrow\;]{\nu_\mu}$ CM frame $\cdots\rightarrow \hat{3}$

$m_e \simeq 0 \Rightarrow \quad u_L(p_\nu) = \sqrt{2E_\nu}\begin{pmatrix}0\\1\end{pmatrix}$ spin down, $\quad u_L(p_e) = \sqrt{2E_e}\begin{pmatrix}1\\0\end{pmatrix}$

$$|(\star)|^2 = 2E_\nu\,2E_e = 2\,p_e\,p_\nu$$

$$p_e = (E_e,0,0,E_e) \qquad p_\nu = (E_\nu,0,0,E_\nu)$$

$$2\,p_e\cdot p_\nu = (p_e+p_\nu)^2 = (E_e+E_\nu)^2 - (E_e-E_\nu)^2 = 4E_eE_\nu$$

- For $(\star\star)$, go to the rest frame of the muon

$p_\mu = (m_\mu,0,0,0)$, $\quad U(p_\mu) = \sqrt{m_\mu}\begin{pmatrix}\boxed{\xi}\\ \xi\end{pmatrix}$ ← V-A sees only the χ_L part

$p_{\bar\nu} = (E_{\bar\nu},0,0,E_{\bar\nu})$, $\quad v_L(p_{\bar\nu}) = \sqrt{2E_{\bar\nu}}\begin{pmatrix}0\\1\end{pmatrix}$ ← $\bar\nu$ is RH

$\mu^-\ \bullet \xrightarrow[\;\Rightarrow\;]{\bar\nu} \cdots \hat{3}$

$$|\star\star|^2 = 2m_\mu E_{\bar\nu}\left|\xi^\dagger\begin{pmatrix}0\\1\end{pmatrix}\right|^2 \xrightarrow{\text{avg.}} \frac{2}{2}m_\mu E_{\bar\nu} = p_\mu\cdot p_{\bar\nu}$$

- In order to get the total Γ we need to integrate over the 3-body phase space $d\Pi_3$.

We have $x_i = \frac{2E_i}{Q = m_\mu}$, $x_e = \frac{2E_e}{m_\mu}$, x_ν, $x_{\bar\nu}$

$$\sum x_i = 2.$$

$$\overline{|\mathcal{M}|^2} = 2(2p_e p_\nu)(2p_\mu p_{\bar\nu}) = 2\, m_\mu^2 x_{\bar\nu}\, m_\mu^2 (1-x_{\bar\nu})$$

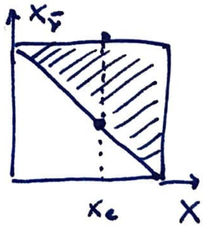
$$p_\mu = p_e + p_\nu + p_{\bar\nu} \Rightarrow (p_e + p_\nu)^2 = 2 p_e p_\nu$$

$$= (p_\mu - p_{\bar\nu})^2 = m_\mu^2 - 2 \underbrace{p_\mu p_\nu}_{m_\mu E_\nu \text{ rest frame}}$$

$$= m_\mu^2 (1 - x_{\bar\nu})$$

- To get the final rate, we integrate over the phase space $\int d\Pi_3 = \frac{E_{CM}^2}{128\pi^3} \int dx_1 \int dx_2$, $E_{CM} = m_\mu$

$dx_1 = dx_e$, $dx_2 = dx_{\bar\nu}$



$x_{\bar\nu} = 1 - y$

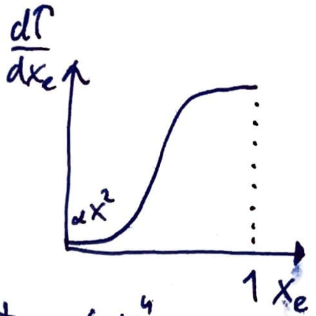
$$\Gamma = \frac{1}{2M_A} \int d\Pi_n\, |\mathcal{M}(A \to f)|^2$$

$$= \frac{1}{\cancel{2m_\mu}} \cdot \frac{m_\mu^{\cancel{2}}}{\underset{16}{\cancel{128}}\,\pi^3} \int_0^1 dx_e \int_{1-x_e \,\to\, x_e}^{1 \,\to\, 0} dx_{\bar\nu}\; \overset{8}{\cancel{16}}\, G_F^2\, m_\mu^{\cancel{4}}\; x_{\bar\nu}(1-x_{\bar\nu})$$

$$= \frac{m_\mu^5 G_F^2}{16\pi^3} \int_0^1 dx_e \underbrace{\int_0^{x_e} dy\, (1-y)\, y}_{x_e^2/2 \,-\, x_e^3/3} = \frac{m_\mu^5 G_F^2}{16\pi^3} \underbrace{\left(\frac{1}{6} - \frac{1}{12}\right)}_{\frac{1}{12}} = \frac{m_\mu^5 G_F^2}{192\pi^3}$$

· We thus got

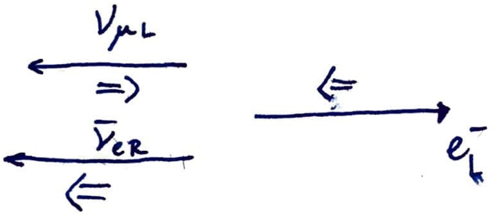
$$\text{i)} \quad \frac{d\Gamma}{dx_e} = \overbrace{\frac{m_\mu^5 G_F^2}{192\pi^3}}^{\Gamma_0} \cdot 2x_e^2 (3 - 2x_e)$$



$$\text{ii)} \quad \Gamma_{tot} = \Gamma_0 \Rightarrow \tau_\mu \cong \frac{\hbar}{\Gamma_0} = \frac{\overbrace{192\pi^3}^{10^4} \hbar c \quad GeV^4}{(0{,}1\ GeV)^5 \cdot 10^{-10}\ 3\cdot 10^8 \frac{m}{s}}$$

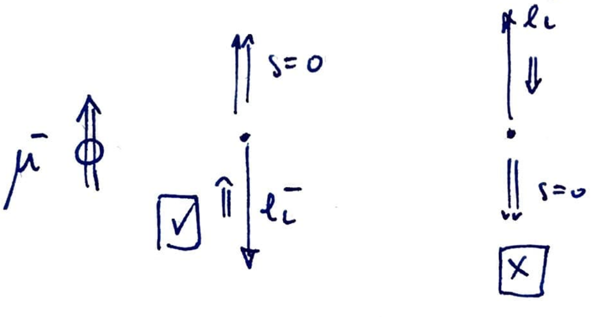
$$\simeq 2\,\mu s \quad ✓$$

· At the kinematic endpoint when $x_e \sim 1$, we have



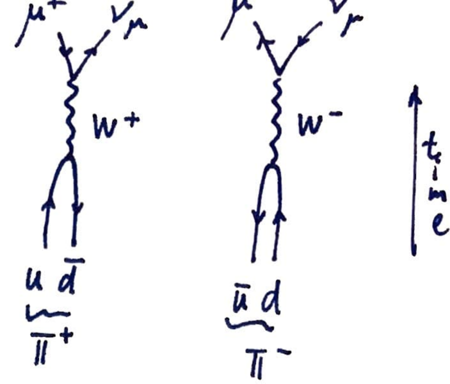
$$\frac{d\Gamma}{dc_\theta} = 1 - c_\theta$$

· If we have a polarized μ^- at rest (say with spin up along the z-axis), then at $x_e \simeq 1$



ALLOWED $\theta \simeq \pi$, FORBIDDEN $\theta \cong 0$

15.3. π DECAY



$$\pi^- \longrightarrow \mu^- \bar\nu_\mu \,,\; e^- \bar\nu_e$$
$$p \quad = p_\mu + p_\nu$$

- Weak interactions are universal

but exp. : $\dfrac{Br(\pi^- \to e^- \bar\nu_e)}{Br(\pi^- \to \mu^- \bar\nu_\mu)} = 10^{-4} \neq 1$. How do we understand this?

- Let's look at the μ-channel : $\mathcal{M} = \dfrac{4G_F}{\sqrt{2}} \langle \mu_L^+ \bar\sigma^\mu \nu_\mu \; u_L^+ \bar\sigma^\mu d_L \rangle$

- We know the matrix element for the leptonic part. For the π we have

$$|\pi^-\rangle = \frac{1}{\sqrt{2}}\left(|\pi^1\rangle - i\,|\pi^2\rangle\right)$$

axial part $\neq 0$

$$\langle 0 | u_L^+ \bar\sigma^\mu d_L | \pi^-\rangle = \frac{1}{\sqrt{2}} \langle 0 | \bar\Psi_u \gamma^\mu \frac{1-\gamma_5}{2} \Psi_d \,|\, \pi^1 - i\pi^2 \rangle$$

$$\underset{}{\frac{\sigma_1 + i\sigma_2}{2}}$$

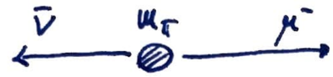
$$= -\frac{1}{2\sqrt{2}} \langle 0 | \underbrace{\bar\Psi \gamma^\mu \gamma^5 \tau \Psi}_{j_5^{\mu 1} + i j_5^{\mu 2}} | \pi^1 - i\pi^2 \rangle = -\frac{i}{\sqrt{2}} f_\pi p^\mu$$

π decay constant, to be measured

π momentum, the only L.I variable

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \langle \mu^- \nu_\mu | \mu_L^+ \bar\sigma^{\mu} \nu_\mu | 0 \rangle \underbrace{\langle 0 | u_L^+ \bar\sigma^\mu d_L | \pi^- \rangle}_{-i \frac{f_\pi}{\sqrt{2}} p^\mu}$$

• For the leptonic part, in the π^- rest frame we have $p = (m_\pi, \vec{0})$, $\mu = 0$, $\underline{i=1,2,3}$ $\Downarrow$ 0



$$\bar{\sigma}^\mu = (1, \cancel{-\sigma^i})$$

$$\Rightarrow \; i p_\mu \langle \mu^- \nu | \mu_L^\dagger \bar{\sigma}^\mu \nu | 0 \rangle = m_\pi \, u^\dagger(p) v_L(p_\nu)$$

$$\mathcal{M} = \frac{4 G_F}{\sqrt{2}} \frac{f_\pi}{\sqrt{2}} m_\pi \, u^\dagger(p_\mu) v_L(p_{\bar{\nu}})$$

$$p_\mu = (E, 0, 0, k) \quad , \quad p_{\bar{\nu}} = (k, 0, 0, -k) \quad , \quad E^2 - k^2 = m_\mu^2$$

$$E + k = m_\pi$$

• let us evaluate the spinors for μ & $\bar{\nu}$

$$\left.\begin{array}{lll} \text{muon} & : & u_R(p_\mu) = \sqrt{E-k} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \bar{\nu} & : & v_L(p_{\bar{\nu}}) = \sqrt{2\underset{\substack{\parallel \\ E_{\bar{\nu}}}}{k}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{array}\right\} \quad |u_R^\dagger v_L|^2 = 2k(E-k)$$

$$\underbrace{\left| (1\;0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right|^2}_{1}$$

• Finally we have $\overline{|\mathcal{M}|}^2 = \frac{1}{1} 4 G_F^2 f_\pi^2 m_\pi^2 \, 2k(E-k)$

spin of $\pi^- = 0$

• From the general 2-body $d\Pi_2$ kinematics, setting $m_\nu = 0$ we have $E = \dfrac{m_\pi^2 + m_\mu^2}{2m_\pi}$, $k = \dfrac{m_\pi^2 - m_\mu^2}{2m_\pi}$

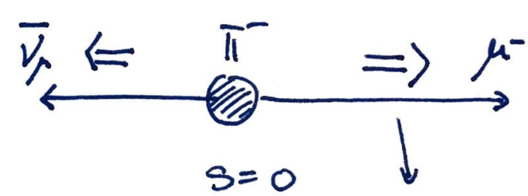
↪

$$\Gamma_{\pi \to \mu\nu} = \frac{1}{2m_\pi} \frac{1}{8\pi} \frac{2k}{m_\pi} 8 G_F^2 f_\pi^2 m_\mu^2 m_\pi \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

$$= \frac{G_F^2 f_\pi^2 m_\pi^3}{4\pi} \left(\frac{m_\mu}{m_\pi}\right)^2 \underbrace{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}_{\text{phase space suppression}}$$

$$\frac{\Gamma_{\pi \to \mu\nu}}{\Gamma_{\pi \to e\nu}} = \left(\frac{m_\mu}{m_e}\right)^2 \times \text{phase space} \sim 10^{-4}$$

- The mass suppression (proportionality) is apparent from V-A :

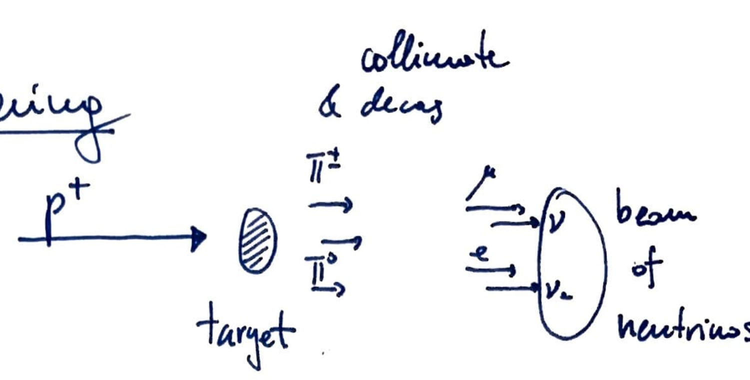


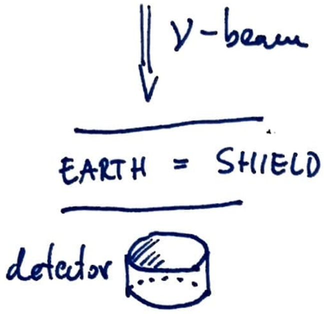
needs to be RH (massless)

spin projection needs to cancel $\Rightarrow$ μ is RH thus $\mathcal{M} \propto m_\mu$ (helicity flip $\propto m_\mu$)

15.4. Neutrino scattering

- DIS ν scattering





$$\nu_L d_L \to \mu_L^- u_L \quad , \quad \nu_L \bar{u}_R \to \mu_L^- \bar{d}_R$$

$$\bar{\nu}_R u_L \to \mu_R^+ d_L \quad , \quad \bar{\nu}_R \bar{d}_R \to \mu_R^+ \bar{u}_R$$

- these CC (charged current) events produce a charged leptons and hadronic depositions (inverse β decay) NuTeV exp.

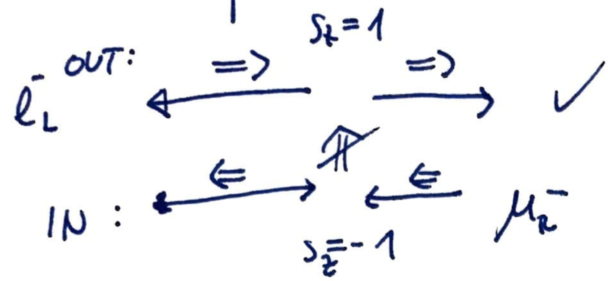
- This is very similar to e^- DIS except that they are mediated by W and ($M_W \sim 80$ GeV) and not γ.

@ $E_\nu \gg M_W$: $\dfrac{d\sigma}{d\cos\theta} = \dfrac{\pi Q_f^2 \alpha^2}{s} \dfrac{s^2+u^2}{t^2}$

$$|\mathcal{M}(e_L^- q_L \to e_L^- q_L)|^2 = 4 Q_f^2 e^4 \frac{s^2}{t^2} \to \text{same helicity}$$

$$|\mathcal{M}(e_L^- q_R \to e_L^- q_R)|^2 = \text{---}\,''\text{---} \frac{u^2}{t^2} \to \text{different (opposite) helicities}$$

- the u-amplitude is suppressed by helicity



- change of DIS variables $s^2 \to 1$, $u^2 \to (1-y)^2$

similarly for LH νs and RH νs

$$\frac{d\sigma}{dc_\theta}(\nu_L d_L \to \mu_L^- u_L) = \frac{G_F^2}{2\pi s}\left(\frac{s}{t}\right)^2$$

$$\frac{d\sigma}{dc_\theta}(\bar{\nu}_R d_L \to \mu_R^+ u_L) = \frac{G_F^2}{2\pi s}\left(\frac{u}{t}\right)^2$$

- to get the νp formula, we have to integrate over the PDFs

$$\Rightarrow \frac{d^2\sigma}{dx\,dy}(\nu p \to \mu^- X) = \frac{G_F^2}{\pi} s \left[x f_d + x f_{\bar{u}} (1-y)^2 \right]$$

$$\frac{d^2\sigma}{dx\,dy}(\bar{\nu} p \to \mu^+ X) = \frac{G_F^2}{\pi} s \left(x f_u (1-y)^2 + x f_{\bar{d}} \right)$$

- for valence quarks only

$$\frac{d\sigma}{dy}(\nu p) \sim 1 \quad ; \quad \frac{d\sigma}{dy}(\bar{\nu} p) \sim (1-y)^2$$