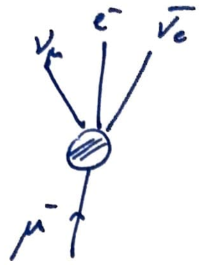
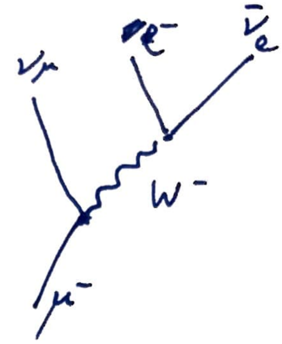
16. Gauge theories with Spontaneous Symmetry Breaking

- V-A theory suggests $\bar{\psi}_L \gamma^\mu \psi_L$ via W exchange

Muon decay



from W exchange



- $[G_F = 10^{-5}\,\mathrm{GeV}^{-2} \Rightarrow M_W \sim 100\,\mathrm{GeV}]$

also q^2 dependence up to $\sim \mathcal{O}(10)\,\mathrm{GeV}$ $\sim \frac{1}{q^2}$

16.1. Maxwell Equations ⟹ Massive Proca Fields

- gauge invariance forces the photon to be massless. How do we keep it and at the same time accommodate $M_W \sim 100\,\mathrm{GeV}$?

- The solution is spontaneous symmetry breaking.

- Start with a massive Abelian $U(1)$

$$\mathcal{L}_{U(1)} = -\frac{1}{4} FF + (D_\mu \phi)^\dagger (D_\mu \phi) - V(\phi)$$

- We work with scalars that can get a vev, fermions and vector bosons cannot (Lorentz inv.)

$$D_\mu \phi - \partial_\mu \phi - ieQA_\mu \phi$$

- We can have the potential for ϕ such that

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4 \quad w \quad U(1) \text{ symm.}$$

but the $\phi = 0$ is unstable, thus the minimum is at $\langle \phi \rangle = v \neq 0$

$$\phi = \frac{1}{\sqrt{2}} (v + \underset{\text{reals scalar}}{\chi} + i \overset{\text{Goldstone}}{\eta})$$

- The kinetic term gives us

$$|D_\mu \phi|^2 \supset \frac{1}{2} (eQv)^2 A_\mu A^\mu = \frac{1}{2} m_A^2 A_\mu A^\mu$$

$$m_a = eQv$$

- The η field is a would-be-Goldstone that gets eaten up by A_μ to become the third degree of freedom. This is the longitudinal polarization supplied by the η on top of the two transverse polarization of A_μ.

16.2. From Abelian => Non-Abelian

$SU(2) \sim SO(3)$

$\downarrow$

adjoint rep. dim 3 ; $\frac{n(n-1)}{2}$

A_μ^a, $a = 1,2,3$; reals scalar triplet ϕ^a

$$D_\mu \phi^a = \partial_\mu \phi^a + g \underbrace{f^{abc}}_{\varepsilon^{abc}} A_\mu^b \phi^c$$

$$\langle \phi^a \rangle = (0, 0, v)$$

linear σ model

$$\phi^a = (\underbrace{\pi^1, \pi^2}_{\text{two Goldstones}}, v + \underbrace{h}_{\text{one "Higgs"}})$$

G.B. masses: $$|D_\mu \phi|^2 \supset \frac{g^2}{2} \varepsilon^{ab3} \varepsilon^{ac3} A_\mu^b A^{\mu c} v^2 + \dots$$
$$= \frac{g^2 v^2}{2} (A_\mu^a A^{\mu a} - A_\mu^3 A^{\mu 3})$$

$\Rightarrow m_{A^1} = m_{A^2} = \frac{1}{\sqrt{2}} g v$, $m_{A^3} = 0$ ... remains massless

- The reason A^3 remains massless because the $\hat{3}$ direction remains symmetric, a $U(1)$ symmetry around the $\hat{3}$ axis remains. $W^{\pm} = \frac{1}{\sqrt{2}}(A_\mu^1 \pm i A_\mu^2)$

- This model would account for a massive $W_\mu^{\pm}$ and a massless A_μ. However, the charge Q has to be identified with the only diagonal operator I_3 of $SO(3)$.

- Now, fermions have to be added. The smallest rep. is 3 of $SO(3)$ e.g. $I_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \Rightarrow \begin{pmatrix} \blacksquare \\ \nu \\ e^- \end{pmatrix}$ $\to E^+$

- This is not phenomenologically viable, no E^+ exists.

16.3. Mixed Abelian - non-Abelian = SM

$$\underset{\substack{\| \\ A_\mu^a}}{SU(2)} \times \underset{B_\mu}{U(1)} \leftarrow \text{hypercharge}$$

- Here we can put the ν and e into a single doublet $L_L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$ with $\underset{\substack{\| \\ \frac{\sigma_3}{2}}}{I} = \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$

$$Q = T_{3L} + \frac{\overset{-1}{\overset{\|}{Y}}}{2}$$

- It transforms as $L_L \to U_2 U_1 L_L$ — $-\frac{1}{2}$ needed for charges

$$= e^{i\alpha^a\sigma^a/2} e^{-i\beta/2} \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix} \begin{matrix} 0 \\ -1 \end{matrix}$$

- Similarly, the "Higgs" field that breaks the symmetry spontaneously will be a doublet with

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \longrightarrow e^{i\alpha^a\sigma^a/2} e^{i\beta/2} \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \quad \begin{matrix} Q = \frac{1}{2} + \frac{1}{2} = 1 \\ = -\frac{1}{2} + \frac{1}{2} = 0 \end{matrix}$$

$Y = +1$

- Breaking is triggered from the potential

$$V(\varphi) = -\mu^2 |\varphi|^2 + \lambda |\varphi|^4$$

$$\frac{dV}{d\varphi^*} = -2\mu^2\varphi + 4\lambda\varphi|\varphi|^2 = 0$$

- This gives the vev $|\varphi|^2 = \frac{\mu^2}{2\lambda}$

and thus $\langle\varphi\rangle = \frac{\mu}{\sqrt{2}\sqrt{\lambda}} = \frac{v}{\sqrt{2}}$, $v = \frac{\mu}{\sqrt{\lambda}}$

$$\varphi = \begin{pmatrix} (\Pi_1 + i\Pi_2)/\sqrt{2} = \Pi^+ \\ \frac{1}{\sqrt{2}}(v + h + i\Pi_3) \end{pmatrix}$$

would-be-Goldstones
↓
can be removed by an SU(2) rotation

- Gauge boson masses

→ as before, the covariant derivatives contains

$$|D_\mu\varphi|^2 \ni \left|\left(g A_\mu^a \frac{\sigma^a}{2} + g' B_\mu \frac{Y}{2}\right)\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}\right|^2$$

for $a = 1,2$: $\frac{1}{4}\frac{g^2 v^2}{2}\left(A_\mu^{1\,2} + A_\mu^{2\,2}\right) = \left(\frac{gv}{2}\right)^2 W_\mu^+ W^{\mu-}$

$$M_W = \frac{gv}{2}$$

for $a = 3$ and B_μ we have a 2×2 matrix

$$(A_\mu^3 B_\mu)\left(\frac{v}{2}\right)^2\begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}\begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix}$$

$\Rightarrow \det M_{AB}^2 = 0$ $\quad \mathrm{tr}\, M^2 = (g^2 + g'^2)\left(\frac{v}{2}\right)^2$

- Rotating to $\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix}\begin{pmatrix} A_\mu^3 \\ B_\mu \end{pmatrix}$

- The Z and A are physical, propagating states in the mass basis.

$$M_Z^2 = \left(\frac{v}{2}\right)^2 (g^2 + g'^2) \quad , \quad M_A = 0$$

the photon remains massless because the U(1) group remains unbroken

- The mixing angles are given by

$$c_W = \frac{g}{\sqrt{g^2+g'^2}} \;, \quad s_W = \frac{g'}{\sqrt{g^2+g'^2}} \;, \quad t_W = \frac{g'}{g}$$

- Couplings to leptons are now fixed due to their known quantum numbers $\frac{\sigma^3}{2} = I_3$ & $\frac{Y}{2}$ with $Q = I_3 + \frac{Y}{2}$

$$\mathcal{L}_\psi \ni \bar{\psi} D_\mu \psi = \bar{\psi}\Big(\partial_\mu - i\frac{g}{\sqrt{2}}\left(W_\mu^+ \gamma^\mu P_L + h.c.\right)$$

$$- ig\left(c_W Z_\mu + s_W A_\mu\right) I_3 - ig'\left(-s_W Z_\mu + c_W A_\mu\right)\frac{Y}{2}\Big)\psi$$

$$-ie A_\mu Q - i\frac{g}{c_W} Z_\mu Q_Z \quad , \quad e = g s_W = g' c_W = \frac{g g'}{\sqrt{g^2+g'^2}} .$$

- The charges are given by

$$Q = I_3 + \frac{1}{2}Y \quad , \quad Q_Z = I_3 - s_W^2 Q$$

- While the I_3 and Y follow from the reps.

	I_3	Q	$\frac{Y}{2}$
$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$\frac{1}{2}$	0	$-\frac{1}{2}$
	$-\frac{1}{2}$	-1	$-\frac{1}{2}$
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\frac{1}{2}$	$\frac{2}{3}$	$+\frac{1}{6}$
	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$

these have the same $\frac{Y}{2}$; they are in the same doublet of $SU(2)_L$

	I_3	Q	$\frac{Y}{2}$
e_R	0	-1	-1
u_R	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	0	$-\frac{1}{3}$	$-\frac{1}{3}$

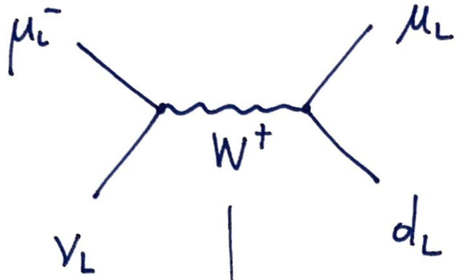
- This applies to the entire generation. The pattern gets repeated three times $e\,\mu\,\tau$, $u\,c\,t$, $d\,s\,b$.

- Note that the W couplings are purely chiral, the A_μ couplings are vector-like (same for L & R), while Z_μ (and the unphysical B_μ) have mixed chiralities, different charges for L & R.

16.4. Neutral current (NC) weak interaction

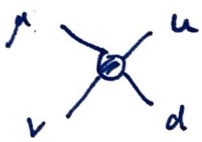
CC currents :

e.g. ν-DIS



$$\frac{g}{\sqrt{2}} j^{\mu -} \frac{1}{q^2 - M_W^2} \frac{g}{\sqrt{2}} j_\mu^{+}$$

$$\text{propagator} \quad \frac{1}{q^2 - M_W^2} \begin{cases} \frac{1}{M_W^2} & \text{for } q^2 \ll M_W^2 \\ \frac{1}{q^2} & \text{for } q^2 \gg M_W^2 \end{cases}$$

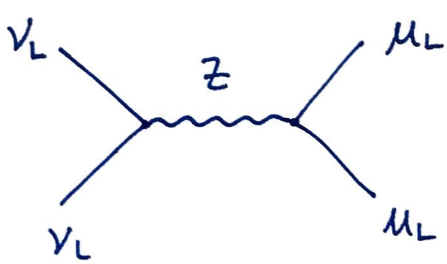
- at low q^2 :  ... contact operator ~ Fermi

  high q^2 :  ... ~ massless mediator ~ photon

- comparing to the Fermi operator

$$\frac{4 G_F}{\sqrt{2}} = \frac{g^2}{2 M_W^2} \Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{1}{v^2}$$

- on top of the CC, we also have the NC events



- Mixed-chiral current-current interaction

$$\left(\frac{g}{\sqrt{2} c_W}\right)^2 \left(j_L^{3\mu} - s_W^2 j_Q^{\mu} \right) \frac{1}{q^2 - M_Z^2} \left(j_L^{3\mu} - s_W^2 j_Q^{\mu} \right)$$

acts only on f_L

- while $$j_Q^\mu = \sum_f \underbrace{Q_f}_{\text{remains vector-like, non-chiral, with same } Q_f \text{ for L \& R.}} \left(\bar{f}_L \bar{\sigma}^\mu f_L + \bar{f}_R \sigma^\mu f_R \right)$$

- Together, the CC and NC (at low q^2) give

$$\mathcal{M} = \left\langle \frac{4 G_F}{\sqrt{2}} \left(\underbrace{j_L^{\mu +} j_L^{\mu -} + \left(j_L^{\mu 3}\right.}_{\text{symmetric under SU(2), chiral}} - \underbrace{s_w^2 j_Q^\mu}_{\text{U(1) piece}} \Big)^2 \right\rangle$$

- Transcribing the DIS formulae onto the above matrix element and integrating over the PDFs we get the ν-DIS cross-sections.

$$\frac{d^2\sigma}{dx\,dy} \sim \frac{G_F^2 s}{\pi} \cdot x f \quad , \quad \sum_{u,d,\ldots}$$

helicity flip for $\bar{u}_L$

$$\frac{d^2\sigma}{dx\,dy}(\nu p \to \nu X) = \frac{G_F^2 s}{\pi} \Big[\left(\tfrac{1}{2} - \tfrac{2}{3} s_w^2\right)^2 \left(x f_u + (1-y)^2 \, x f_{\bar{u}} \right)$$
$$+ \left(-\tfrac{2}{3} s_w^2 \right)^2 \left(x f_u (1-y)^2 + x f_{\bar{u}} \right)$$

up quarks, anti-quarks

down (anti) quarks

$$+ \left(-\tfrac{1}{2} + \tfrac{1}{3} s_w^2\right)^2 \left(x f_d + x f_{\bar{d}} (1-y)^2 \right)$$
$$+ \left(\tfrac{1}{3} s_w^2 \right)^2 \left(x f_{\bar{d}} (1-y)^2 + x f_d \right) \Big]$$

- When the target consists of A nucleons there is an additional (non-coherent) enhancement

$$f_q = A\,(f_u + f_d)$$

NC : $\dfrac{d^2\sigma_{NC}}{dx\,dy}(\nu A \to \nu X) = \dfrac{G_F^2 s}{\pi}\Big(x f_q \big((1/2 - s_w^2) + \tfrac{5}{9} s_w^4 (1-y)^2 \big) + \tfrac{\ }{9}$

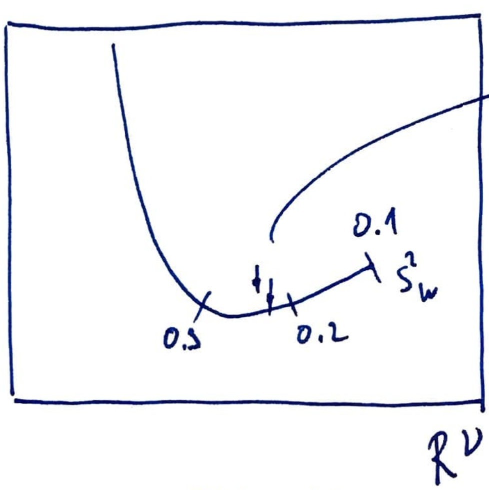
CC : $\dfrac{d^2\sigma_{CC}}{dx\,dy}(\bar\nu A \to \mu^- X) = \dfrac{G_F^2 s}{\pi}\Big(x f_q + x f_{\bar q} (1-y)^2 \Big)$

$$\Rightarrow \quad r = \frac{\sigma_{CC}^{\bar\nu}}{\sigma_{CC}^{\nu}} = \int_{xy} \frac{x f_q (1-y)^2 + x f_{\bar q}}{x f_q + x f_{\bar q} (1-y)^2}$$

$$R^\nu = \frac{\sigma_{NC}^{\nu}}{\sigma_{CC}^{\nu}} = \frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4 (1+r)$$

$$R^{\bar\nu} = \frac{\sigma_{NC}^{\bar\nu}}{\sigma_{CC}^{\nu}} = \frac{1}{2} - s_w^2 + \frac{5}{9} s_w^4 \left(1 + \frac{1}{r}\right)$$

EXPERIMENT : $R^{\bar\nu}$



$s_w^2 \sim 0.23$ $\left(\sim \frac{1}{4}\right)$ for theorists :)