17. THE W AND Z BOSONS

- The CC and NC should be completed in the UV by W and Z boson exchange. How do we predict and test the properties of these gauge bosons.

- Produce at colliders:
  - e^+e^- (LEP)
  - $p\bar{p}$ (SPS, discovery in '83, Tevatron), pp (LHC)

- Look for decay products

$$W \to \ell\nu \,,\; q\bar{q}'$$

$$Z \to \ell^+\ell^- \,,\; q\bar{q}$$

- To understand these, we need to work out the <u>DECAYS</u> and <u>CROSS-SECTIONS</u>.

• Let us begin with the simplest rate $W \to e\bar{\nu}_e$

$$W^- \to \ell_L^- \bar{\nu}_{eR} \quad \text{or} \quad W^+ \to \ell_R^+ \nu_{eL}$$

weak coupling

only left-handed CC

create a RH anti-particle

create W^+ w. this pol.

$$\mathcal{M}_{W^+ \to \ell_R^+ \nu_L} = \frac{g}{\sqrt{2}} u_L^+(p_\nu) \bar{\sigma}^\mu v_L(p_e) \varepsilon_{W\mu}$$

creation of ν_L, a particle w P_L

comes from the $W_\mu^+ = (A_1 + i A_2)_\mu / \sqrt{2}$ assignment

$$= \frac{g}{\sqrt{2}} 2\sqrt{2} E \, \varepsilon_-^* \varepsilon_W \quad \text{(see } e^+e^- \text{ annihilation)}$$

CM energy $= M_W$

$$\overline{|\mathcal{M}|^2} = \frac{1}{3} \frac{g^2}{\cancel{2}} M_W^2 \cdot \cancel{2}$$

three d.o.f. from $s=1$

• Finally :

$$\Gamma_{W^+ \to e_R^+ \nu_L} = \frac{1}{2\cancel{M_W}} \frac{1}{8\pi} \frac{\overset{4\pi\alpha_W}{\overset{\parallel}{g^2}} M_W^{\cancel{2}}}{3} \quad \left(\begin{array}{l}\text{2 body ph. sp.} \\ \text{for } m_e = m_\nu = 0\end{array}\right)$$

$$= \frac{\alpha_W}{12} M_W .$$

• For quarks, we need the factors of N_c and the QCD correction and the CKM mixing $V_{ud}, \dots$

$$\Gamma_{W^+ \to u\bar{d}} = \frac{3\alpha_W}{12} \cdot |V_{ud}|^2 M_W \left(1 + \frac{\alpha_s}{\pi}\right)$$

$\frac{\alpha_s}{\pi} \sim \frac{0.18}{\pi} \sim 0.06 \sim 6\%$

$\sum_i |V_{ij}|^2 = 1$, $\sum_{ij} |V_{ij}|^2 = N_g = 3$ (not top, though; $V_{tb} \sim 1$)

- In total $\Gamma_{W^+ \to q\bar{q}'} = \frac{\alpha_W}{4} \underbrace{\sum_{u,c,d,s} |V_{ij}|^2}_{2} M_W \left(1 + \frac{\alpha_s}{\pi}\right)$

$$= \frac{\alpha_W}{2} M_W \left(1 + \frac{\alpha_s}{\pi}\right)$$

- What is α_W ?

Remember : $\alpha(M_Z) = \frac{1}{127} = \frac{e^2}{4\pi}$, $s_W^2 = 0{,}23$

Moreover, we had $e = g\, s_W^2$ & $e = g'\, c_W^2$

Putting it together, we have :

$$\alpha_W = \frac{\alpha}{s_W^2} = \frac{1}{29.8} \quad , \quad \alpha' = \frac{\alpha}{c_W^2} = \frac{1}{99} .$$

The weak interaction coupling is actually <u>stronger</u> than the QED one. The processes are more rare because the rates ~~are~~ at low energies are suppressed by $M_{W,Z}$.

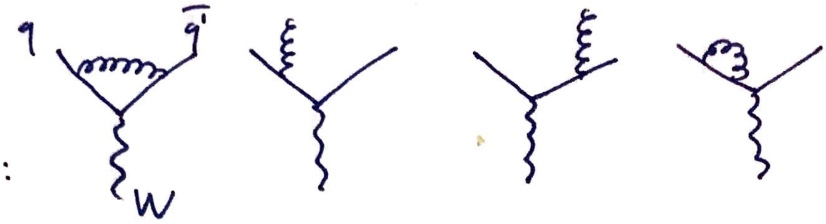
In fact : $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2} = \frac{1}{2 v^2}$, $v = 246$ GeV.

- with g & g' (or s_W) we have

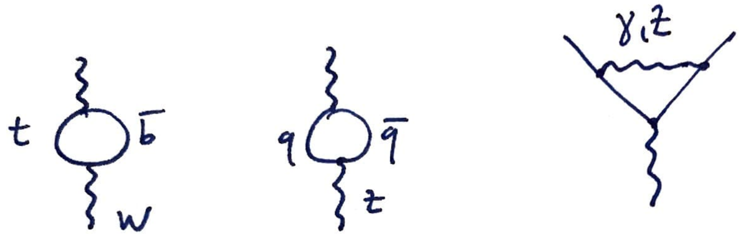
$M_W = 80.2$ GeV

$M_Z = 91.2$ GeV

• Similarly to QCD corrections

QCD: 

There are also EW radiative corrections



5% level for M_W/M_Z

• Summing it all up (quarks & leptons), we have

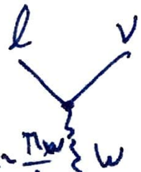
$$\Gamma_W^{tot} = \frac{\alpha_W}{12} M_W \Big(\underbrace{3}_{\mu, e, \tau} + \underbrace{2}_{\text{2 light generations}} \cdot \underbrace{3.1}_{N_c \times \text{QCD 1 loop}} \Big) = 2.1 \text{ GeV}$$

• We also get the Br's

$$\frac{\Gamma_e}{\Gamma_{tot}} = Br_e \sim 11\%$$, so 33% to leptons

$$\frac{\Gamma_{qq}}{\Gamma_{tot}} = Br_{jj} \sim 34\%$$ 67% to dijets

• What we see at colliders



charged lepton $p_T \sim \frac{M_W}{2}$

ν = missing energy $\sim \frac{M_W}{2}$

dijets (+QCD emission) w $M_{jj}^{inv} \sim M_W$

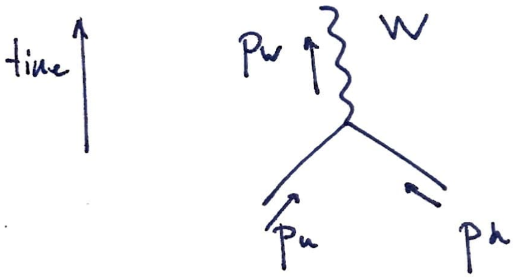
- Let us focus on the production part, the

$pp \to W, Z$ or $2 \to 1$ creation (like $\pi\pi \to \rho$)

- Historically, $q\bar{q} \to W, Z$, $q\bar{q}' \to \ell^+\ell^-$ are refered to as the Drell-Yann processes

$$\hat{\sigma}(u\bar{d} \to W^+) = \frac{1}{2\hat{s}} \cdot \int d\Pi_1 \, \overline{|\mathcal{M}|^2}$$

this is now at parton level, hence the hats



$p_W^2 = M_W^2$ in the CM frame

$\hat{s} = M_W^2$

- Working out the phase space (remember $\pi\pi \to \rho$ in the "Toolz" chapter 7, see also Ch 3)

$$d\Pi_1 = \frac{d^3 p_W}{(2\pi)^3 \, 2E_W} (2\pi)^4 \, \delta^{(4)}(p_u + p_{\bar{d}} - p_W)$$

$$= 2\pi \, \delta(\hat{s} - M_W^2)$$

- This simplifies the PDF integration (remember $\hat{s} = x_1 x_2 s$) and eliminates one $\int dx_{1,2}$ integration variable.

• We also need the $|\mathcal{M}|^2$

$$\frac{1}{2\cdot 2}\,\frac{1}{3\cdot 3}\sum_{\text{sp,col}}|\mathcal{M}|^2 = \frac{1}{36}\cdot 3\cdot \frac{g^2}{2} M_W^2\cdot 2 = \frac{g^2}{12}M_W^2$$

two incoming $s=1/2$ fermions, the averaging factor is $(2s_1+1)(2s_2+1)$

the initial colors can be different

$$\hat\sigma(u\bar d\to W^+) = \frac{1}{\cancel{2}\hat s}\,\cancel{2}\pi\,\delta(\hat s - M_W^2)\,\frac{\overset{4\pi\alpha_w}{\overset{\shortparallel}{g^2}}}{12}\cancel{M_W^2}$$

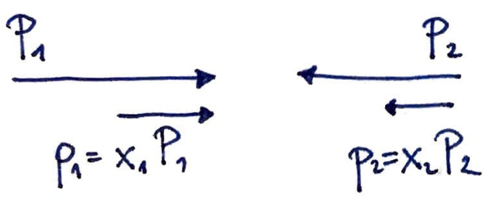
$$= \frac{\pi^2\alpha_w}{3}\,\delta(\hat s - M_W^2)$$

• This has to be integrated over the PDFs, e.g.

@ Tevatron $p\bar p$

$$\sigma(p\bar p\to W^+) = \int dx_{1,2}\left(f_u(x_1)f_{\bar d}(x_2) + f_u(x_2)f_{\bar d}(x_1)\right)\times$$
$$\hat\sigma(u(x_1P)\,d(x_2P)) + \hat\sigma(u(x_2P)\,d(x_1P))$$

• Let's do the kinematics



$$p_1 = (x_1E, 0, 0, x_1E)$$

$$p_2 = (x_2E, 0, 0, -x_2E)$$

$$p_W = ((x_1+x_2)E, 0, 0, (x_1-x_2)E)$$

- Alternatively, we can use rapidity

$$p_W = M(\cosh y, 0, 0, \sinh y) \qquad p_W^2 = M^2(\cosh^2 y - \sinh^2 y)$$
$$= M^2$$

$$= E(x_1+x_2, 0, 0, x_1-x_2)$$

↪ sum / substract : $\underbrace{2E}_{\overset{\shortparallel}{E_{CM} = \sqrt{s}}} x_1 = M \frac{e^y + \cancel{e^{-y}} + e^y - \cancel{e^{-y}}}{2} = M e^y$

$$\Rightarrow x_{1,2} = \frac{M}{\sqrt{s}} e^{\pm y}, \quad dx_1 dx_2 \to J\, dM\, dy$$

Jacobian :

$$\frac{dx_{1,2}}{dM,Y} = \begin{array}{cc} dx_1/dM & dx_2/dM \\ \shortparallel & \shortparallel \end{array} \begin{vmatrix} e^y/\sqrt{s} & e^{-y}/\sqrt{s} \\ e^y M/\sqrt{s} & -e^{-y}M/\sqrt{s} \end{vmatrix} = \frac{2M}{s}$$

so the PDF integration turns into $\frac{1}{dM^2/dM = 2M}\delta(M-M_w)$

$$\int dx_1 dx_2\, \delta(\overset{M^2}{\overset{\shortparallel}{p_W^2}} - M_W^2) = \frac{2M}{s} dM\, dy\, \overset{\shortparallel}{\delta(M^2-M_W^2)}$$
$$= \frac{dy}{s}$$

- and the distribution over rapidity is given by

$$\frac{d\sigma}{dy} = \frac{\pi^2 \alpha_W}{3s}\left(f_u(x_1) f_{\bar{d}}(x_2) + f_u(x_2) f_{\bar{d}}(x_1)\right)$$

• Moving on to the Z boson, we have similar formulae. However, we have to account for both left and right (or axial and vectorial) couplings to all the fermions.

$$e \to \frac{g}{c_W}, \qquad Q \to Q_Z$$

• $$\Gamma(Z \to f\bar{f}) = \frac{1}{2M_Z}\,\frac{1}{8\pi}\left(\frac{g}{c_W}\right)^2 M_Z^2\,\frac{2}{3}\,Q_Z^2 = \frac{\alpha_W}{6c_W^2} M_Z Q_Z^2$$

	ν	e	u	d
Q_{ZL}	$\frac{1}{2}$	$-\frac{1}{2}+s_W^2$	$\frac{1}{2}-\frac{2}{3}s_W^2$	$-\frac{1}{2}+\frac{1}{3}s_W^2$
Q_{ZR}	$\cancel{\ }\,0$	s_W^2	$-\frac{2}{3}s_W^2$	$\frac{1}{3}s_W^2$

• $$\Gamma_Z^{tot} = \frac{\alpha_W}{6c_W^2} M_Z \Big(3 \times \underbrace{0.25}_{\nu_e} + 3 \times \underbrace{0.126}_{\ell} + \underbrace{2}_{u,d}\,\underbrace{(3.1)}_{\substack{QCD \\ N_c = 3\times(1+\frac{\alpha_s}{\pi})}}\,\overbrace{0.144}^{Q_{ZL}^2+Q_{ZR}^2}$$

$$+ \underbrace{2.98}_{\substack{d,s,b \\ \&\text{ top correction}}} \times \underbrace{(3.1)}_{QCD} \times 0.185 \Big)$$

• We get the Br's

	ν	e	u	d
Br	6.7%	3.3%	12	15.3

$\times$ generations

17.4. PRECISION TESTS OF THE SM

LEP : e^+e^- collider $\sqrt{s} = 90 \ldots 200\,GeV$ in the 90's

data collected by DELPHI, OPAL give very precise measurements of the Z width.

$$e^+e^- \to Z \to \ell^+\ell^-$$
$$\nu\bar{\nu} = \not{E}$$
$$\text{hadrons}$$

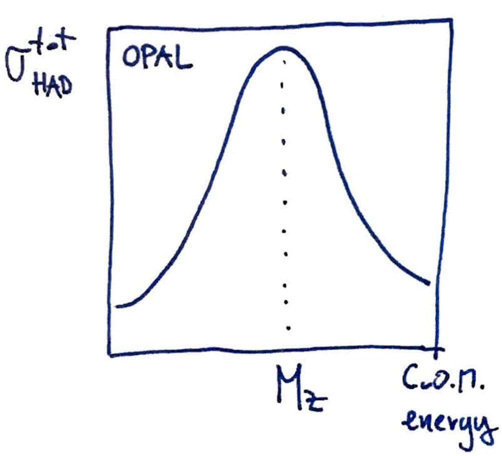
- The measurements test the total rates

$$\Gamma \propto S_f = Q_{ZL}^2 + Q_{ZR}^2$$

and also asymmetries

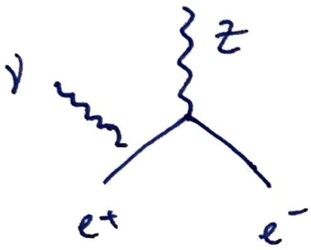
$$A_f = \frac{Q_{ZL}^2 - Q_{ZR}^2}{S_f}$$

- We get very precise shape plots

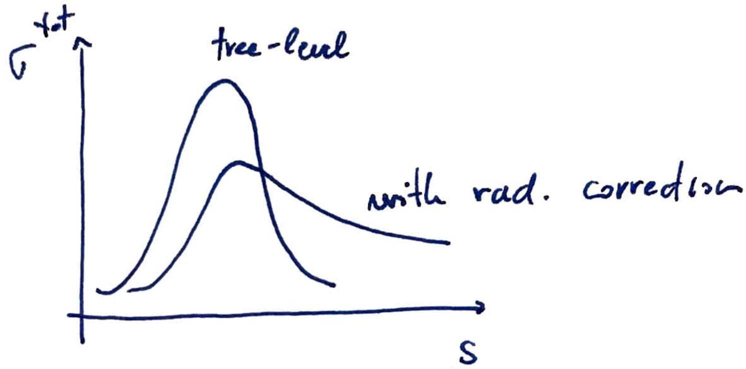


$$\propto \left| \frac{1}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2$$

- To compare with the data, one has to include radiative corrections, similar to QCD



- final and initial state radiation



- QED, EW and QCD corrections needed

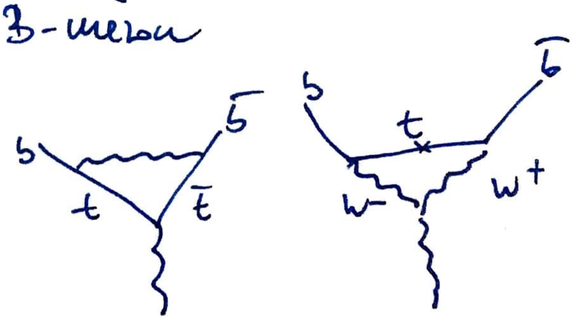
$$\Rightarrow \quad \sin^2 2\theta_W = (2 c_W s_W)^2 = \frac{4\pi\alpha}{\sqrt{2}\, G_F M_Z^2} \Rightarrow s_W^2 = 0.231079$$

- <u>Neutrinos</u> $\quad \Gamma_{Z\to\nu_i\bar{\nu}_i} = 170\ \text{MeV} \quad \forall i = 1, 2, \dots n_\nu$

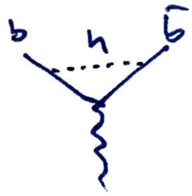
$\Rightarrow$ fit to $\Gamma_Z \Rightarrow n_\nu = 2.984 \pm 0.01$

- <u>Hadrons</u>, esp. B mesons ... vertexing finds a short-lived B-meson

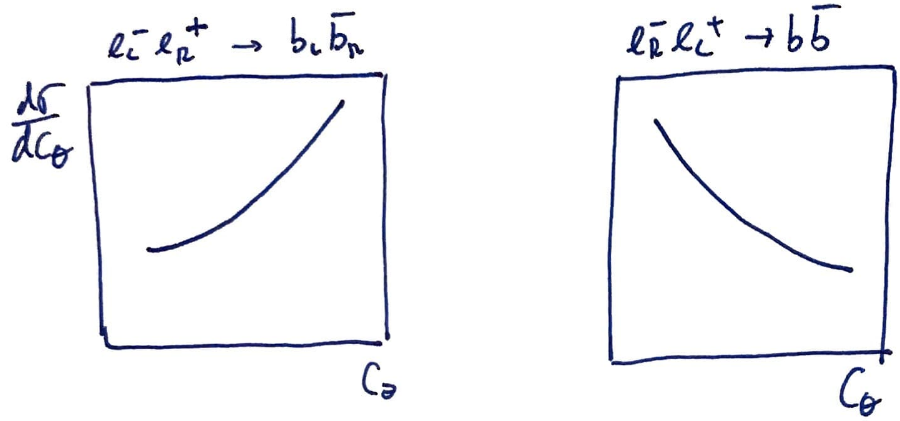
$$R_b = \frac{Br(Z\to b\bar{b})}{Br(Z\to hadr)}$$



- Indirect <u>Higgs</u>



- With e^+e^- linear colliders one can polarize the beams with lasers $e_L^- e_R^+$ or $e_R^- e_L^+$ and measure the resulting $\frac{d\sigma}{dc_\theta}$

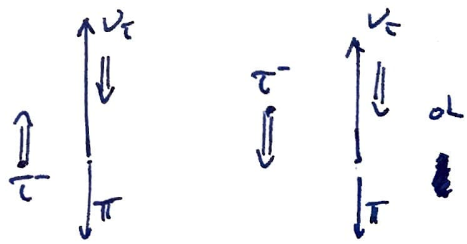


- needs to distinguish b & $\bar{b}$ jets
- sensitive to loop corrections
$\sim 2\sigma$ discrepancy

- <u>Leptonic asymmetries</u> in τ decays

$\tau^- \to \nu_{L\tau}\, \pi^-$ only via $W \to \tau_L$
spin 0

$Z \to \tau^+ \tau^-$



or → neutrino goes opposite to the τ spin.

$M_Z/2 > m_\tau \Rightarrow$ boosted τ's $\gamma > 1$

$\Rightarrow \tau_R^-$ gives an energetic π^- and less energ. ν

τ_L^- gives more momentum to ν_τ and less to π^-