5. THE QUARK MODEL

- Hadrons are particles that interact strongly.

They are separated into $s = 0, 1$ mesons $q\bar{q}'$ because, as you know for $s = 1/2$ we have two spins, therefore $2 \times 2 = \underset{2\cdot 0+1}{1} + \underset{2\cdot 1+1}{3}$ or $\frac{1}{2} \times \frac{1}{2} = 0 + 1$.

- We can also bind three quarks into a baryon, for example uud = proton, udd = neutron, etc.

			SPIN
HADRONS	MESONS	$q\bar{q}$	0, 1
	BARYONS	qqq	1/2, 3/2

5.1. DISCOVERY OF HADRONS

Lightest states are the $\pi^\pm, \pi^0$ mesons with

$$m_{\pi^+} = 139 \text{ MeV}, \quad m_{\pi^0} = 135 \text{ MeV}$$

EX: Check the PDG to find

$$Br(\pi^+ \to \mu^+ \nu_\mu) \cong 100\%$$

$$Br(\pi^+ \to e^+ \nu_\mu) \cong 10^{-4} = 0.01\%$$

and $Br(\pi^0 \to \gamma\gamma) \simeq 100\%$

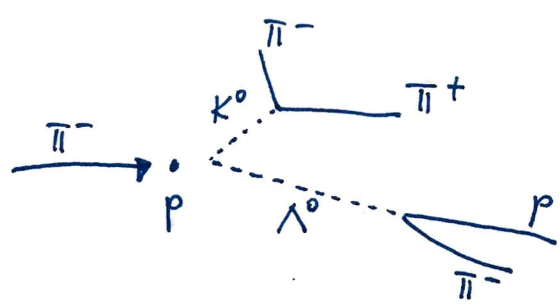
- Bottom-line π's get produced (pair-wise) in strong interactions & produce muons & photons from π^+ & π^0 decays.

- The π-mesons were understood in mid 30-s to 40s in cosmic rays when the resulting muons were seen to be produced by π^+ with a short decay path.

$$\text{Yukawa}: \quad (\partial^2 + m_\pi^2)\pi = 0 \qquad V(r) = \frac{g_s^2}{4\pi r} \underbrace{e^{-m_\pi r}}_{\text{exponential Yukawa potential}}$$

- In 50s and 60s more particles were discovered, mainly due to advances in accelerators & chamber detectors.

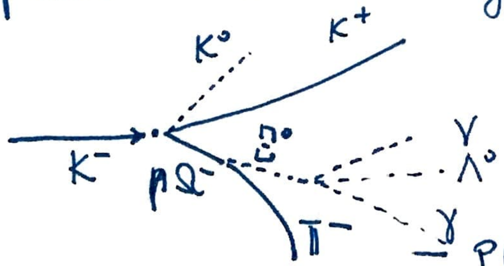
EXAMPLES : 1)



KAONS ~500 MeV

Λ^0

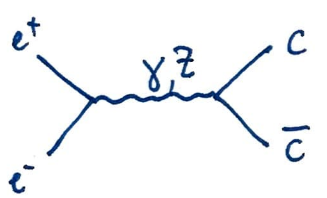
2) Ω^- see an interesting BBC documentary on this prediction & discovery w. Gell-Mann & Feynman.

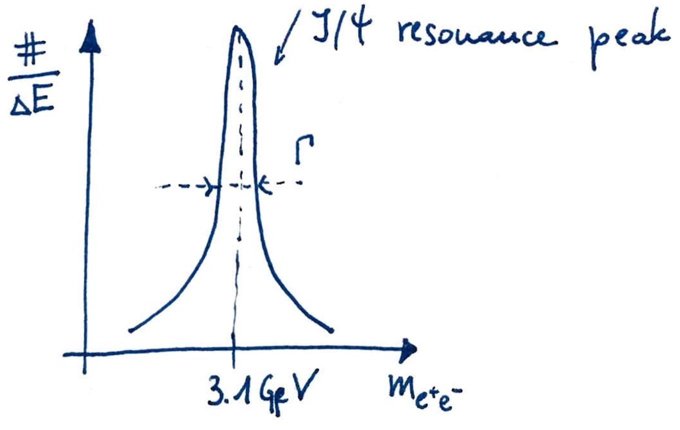


5.2. J/ψ or $c\bar{c}$: CHARMONIUM

- A very simple and precise way of producing & studying particles is by e^+e^- annihilation, where the energy (c.m.) of the two beams can be tuned to the mass of a resonance. This enhances the cross-sections (we'll derive this later on) and allows to study angular distributions, polarization, precise width determination, etc

FIRST SUCH EXAMPLE was the charmonium @ ~ 3 GeV





- SEE THE PDG plots for resonances
- some history 60s e^+e^- (SLAC) measure e^+e^- annihil. up to 2 GeV, e.g. $\eta' = 1.45$ GeV & $\Gamma_{\eta'} = 400$ MeV

'74 SPEAR @ SLAC finds $m_{J/\psi} = 3.1$ GeV.

- Very narrow $\Gamma_{J/\psi} = 0.1$ MeV ; Br (hadrons) = 88 %
  Br (e^+e^-) = 6 %

- hadronic colliders $(p, \bar{p})$ can also be useful, e.g.

J/ψ was found in 1974 by Ting et.al. in the

$$pp \to e^+e^- + X \qquad \text{reaction}$$

not observed

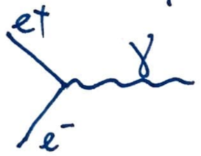
- $\tau_{J/\psi} = \frac{1}{\Gamma}$, since $\Gamma_{J/\psi} \ll \Lambda_{QCD} \Rightarrow J/\psi \sim$ long-lived

- a few weeks later SPEAR finds the ψ' @ 3,7 GeV

- Further on at $E \sim 9,6$ GeV the Υ resonances appear.

'77 @ Fermilab by Lederman in $pp \to \mu^+\mu^- + X$

QUANTUM NUMBERS from e^+e^- annihilation



$$= \langle 0 | j^\mu(x) | e^+e^- \rangle$$

- QED current has the same quantum numbers as the photon: $S=1$, $P=-1$, $C=-1$. This means that the resonances, which are most efficiently produced in this process have $J^{PC} = 1^{--}$ (ψ, Υ)

- Different quantum numbers are produced by radiative returns when e.g. $\psi' \to X + \gamma$

$\hookrightarrow J/\psi + \gamma$

<u>Now</u> : $\psi' \to \chi + \gamma$ → photon w. $s=1$ $P=-1$, $C=-1$

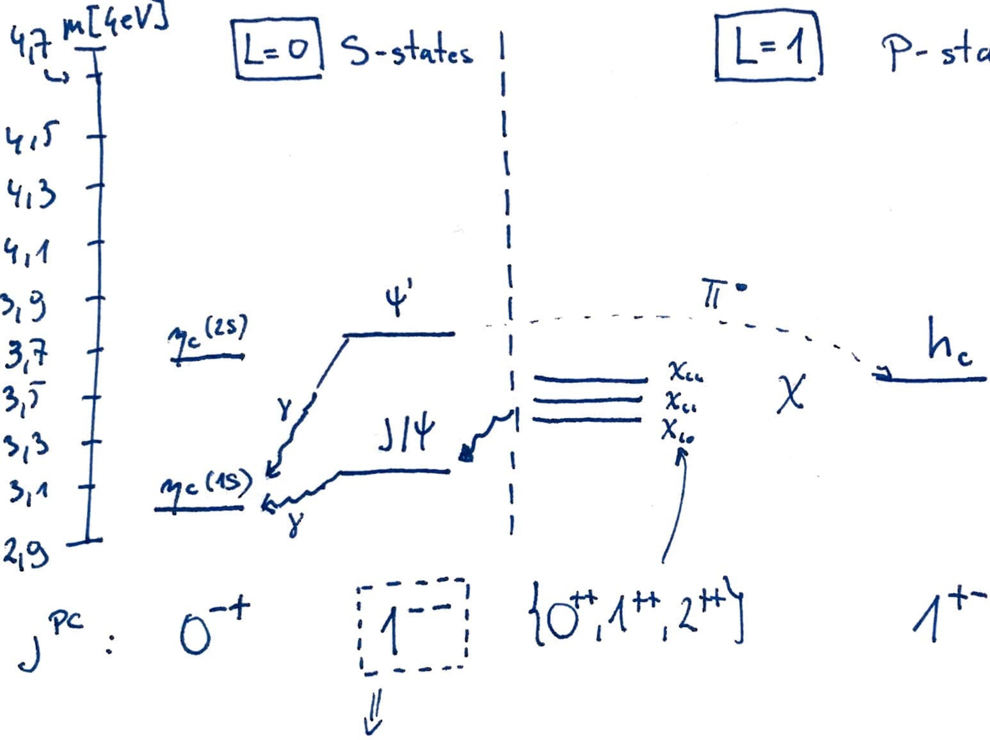
$\sim E1$ $\Delta L = 1$ transitions

$\Rightarrow$ χ s (actually three of them) have $C=1$

χ_{c0} has $J^{PC} = 0^{++}$

$m_{\chi_{c0}} = 3{,}4$ GeV , $\Gamma = 10$ MeV

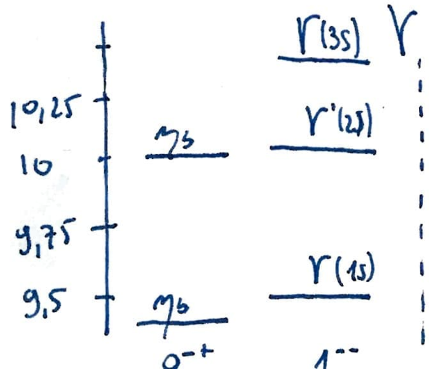
• see the "Charmonium system" on PDG



SIMILAR FOR BOTTOMONIUM but for $m \in (9{,}5 - 11)$ GeV

• All the ψ & Υ states, produced via the $P=-1$, $C=-1$ current $\langle 0 | \vec{j} | e^+ e^- \rangle$ $\Rightarrow$ $J^{PC} = 1^{--}$

• We identify $\psi \sim c\bar{c}$, $m_c \sim m_\psi / 2 \sim 1.8$ GeV



$\Upsilon \sim b\bar{b}$, $m_b \sim \frac{m_\Upsilon}{2} \sim 5$ GeV

5.3. THE LIGHT MESONS

- Let's focus on the *light* mesons $\propto 100$ MeV

- the lightest family are π - mesons w $m_\pi \sim 140$ MeV

  turns out $J^P = 0^-$ $\Rightarrow$ pseudoscalars w. $P = -1$.

  the $\pi^\pm$ go into $C = \pm 1$ and $\pi^0 \to \overset{C=-1}{\gamma\gamma}$ $\Rightarrow$ $C(\pi^0) = 1$

- The entire $J^P = 0^-$ family consists of 9 states

PSEUDO-SCALARS $\boxed{0^-}$

η'	958	[MeV]	~ 960
η	548		~ 550
K^- $\bar{K}^0$ K^0 K^+	498		~ 500
π^- π^0 π^+	140		~ 140

$C(\pi^0, \eta, \eta') = 1$ ($\gamma\gamma$ decays)

- On top of the pseudoscalar, made from $\frac{1}{2} \times \frac{1}{2} = 0 + 1$, the two fermions can create a $J = 1$ bound state. These are the somewhat heavier vector mesons.

VECTOR MESONS $\boxed{1^-}$

ϕ^0	1020
K^{*-} $\bar{K}^{*0}$ K^{*0} K^{*+}	892
ω^0	781
ρ^- ρ^0 ρ^+	770

$C(\rho^0, \omega, \phi) = -1$ ($\pi^0\gamma$ decays)

- The $K^{(*)}$ mesons are somewhat special, they are not produced singly, only in pairs, or together with particular excited states of the proton.

• For example, we have: $\pi^- p \to n K^+ K^-$

$\to \Lambda^0 K^0 \quad (\cancel{n K^0})$

excited state of p.

This strange behaviour is explained by postulating the <u>strangeness</u> quantum number, which is preserved by strong (& EM) interactions.

By convention $S(K^0, K^+, K^{*0}, K^{*+}) = -1$

strangeness $S(\bar{K}^0, K^-, \bar{K}^{*0}, K^{*-}) = +1$

and $S(\Lambda^0) = +1$, such that $\Lambda^0 K^0$ has

$S = 1 - 1 = 0$ ✓

• We can interpret all these states with three "light" quarks: u, d, s with spin 1/2

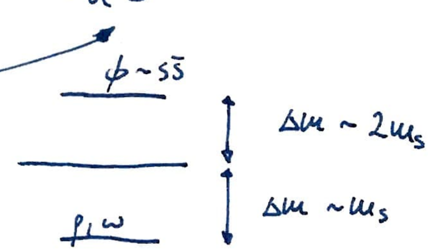
$\pi^+ \sim u\bar{d}$, $\pi^- \sim \bar{u}d$ } when u & d combine to spin 0

$\rho^+ = u\bar{d}$, $\rho^- = \bar{u}d$ } when they go to spin 1

$K^+ \sim u\bar{s}$ | $K^{*+} \sim u\bar{s}$

$K^- \sim \bar{u}s$ | $K^{*-} \sim \bar{u}s$

s quark has strangeness +1



- From here, $Q_u - Q_d = 1 = Q_u - Q_s$

- We saw that $m_s \sim 120$ MeV (really more like 90 MeV)

but since $\underbrace{m_{\pi^\pm} \sim m_{\pi^0} \;\&\; m_{K^\pm} \sim m_{K^0}}_{\text{apart from E.M. radiative effects}}$ it must be that

$m_u \sim m_d$

$Q=0$

- If so, then, up to mass differences, there would be an approximate symmetry on u, d, s exchange

$$u \leftrightarrow d \;,\quad u \leftrightarrow s \;,\quad d \leftrightarrow s$$

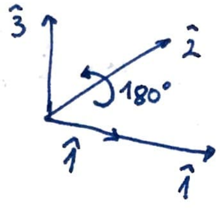
- In fact Heisenberg suggested that they p & n form a doublet, meaning $\begin{pmatrix} u \\ d \end{pmatrix}$ transforms as an doublet under SU(2). This is a global approximate symmetry, called ISOSPIN.

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow e^{-i\alpha^a \frac{\sigma^a}{2}} \begin{pmatrix} u \\ d \end{pmatrix} \;\&\; \underbrace{\begin{pmatrix} u \\ d \\ s \end{pmatrix}}_{\text{TRIPLET under SU(3)}} \rightarrow e^{-i\alpha^a \frac{\lambda^a}{2}} \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

even <u>more</u> approximate SU(3).

G - PARITY : instead of C or P, it's useful to define G-parity, which is a combination of C & SU(2) isospin, and is therefore (approximately) conserved.

$C|\pi^{\pm}\rangle = |\pi^{\mp}\rangle$ & $C|\pi^{0}\rangle = |\pi^{0}\rangle$

Now take $R_2(\pi)$, a 

rotation around the $\hat{2}$ axis of isospin rotations.

$$G = C e^{i\pi\frac{\sigma_2}{2}} = C e^{i\pi I_2} = C R_2(\pi)$$

$$R_2(\pi)(\underbrace{\hat{1}+i\hat{2}}_{\pi^+}) = -\hat{1} + i\hat{2} = -(\underbrace{\hat{1}-i\hat{2}}_{\pi^-}) \quad , \quad R_2(\pi)\hat{3} = -\hat{3}$$

$$\pi^{\pm} = \frac{1}{\sqrt{2}}(\pi^1 \pm i\pi^2)$$

$$\Rightarrow G|\pi^+\rangle = C(R_2(\pi)|\pi^+\rangle) = -C|\pi^-\rangle = -|\pi^+\rangle$$

$\Rightarrow G|\pi^{\pm}\rangle = -|\pi^{\pm}\rangle$, $G|\pi^0\rangle = -|\pi^0\rangle$ so -1 for all the pions. For example $J/\psi = c\bar{c}$ (or $\Upsilon = b\bar{b}$) both had $I = 0$ (no u, d, s quarks $\Rightarrow$ isospin $= 0$) and thus has $G = C = -1$. Therefore, in order to conserve G-parity, they always decay to an <u>odd</u> number of πs.

$$R_2(\overset{x}{\cancel{\pi}}) = e^{i\frac{\overset{x}{\cancel{\pi}}\sigma_2}{2}} = 1 + i\frac{\overset{x}{\cancel{\pi}}\sigma_2}{2} + \frac{1}{2}\left(\frac{i\overset{x}{\cancel{\pi}}\sigma_2}{2}\right)^2 + \dots \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + \frac{1}{2}\left(\frac{x}{2}\right)^2 + \frac{1}{4}\left(\frac{x}{2}\right)^4 + \dots, & \frac{x}{2} + \frac{1}{3!}\left(\frac{ix}{2}\right)^3(-i) \\ -\frac{x}{2} & \end{pmatrix} \qquad \sigma_2^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\frac{x}{2} & , & \sin\frac{x}{2} \\ -\sin\frac{x}{2} & , & \cos\frac{x}{2} \end{pmatrix} \xrightarrow{x \to \pi} \begin{pmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$R_2(\pi)\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} d \\ -u \end{pmatrix} \qquad \pi^+ = u\bar{d}, \quad \pi^- = \bar{u}d$$

$$\pi^+ \xrightarrow{R_2(\pi)} d(-\bar{u}) = -d\bar{u}$$

$$= -\pi^-$$

$$U = e^{i\frac{\vec{\sigma}\cdot\vec{\theta}}{2}} = \cos\theta/2 + \frac{i\,\vec{\sigma}\cdot\vec{\theta}}{\theta}\sin\frac{\theta}{2}, \quad \theta = \sqrt{\theta_1^2+\theta_2^2+\theta_3^2}$$

5.4. THE HEAVY MESONS

- The same way we formed 0^-, 1^- mesons from u, d, s, we can form their heavier counterparts from c & b quarks.

$\boxed{0^-}$

$D_s^+ = c\bar{s}$ — 1968

D^0 — $D^+ = c\bar{d}$ — 1869

$\boxed{1^-}$

D_s^{+*} — 2112

$D^{0*} = c\bar{u}$ — D^{+*} — 2010

$\psi(3S) \to D_x \bar{D}_x$

And similarly for the b-quark

$\boxed{0^-}$

$B_s^0 = b\bar{s}$ — 5367

B^- ($b\bar{u}$) — B^0 ($b\bar{d}$) — 5279

$\boxed{1^-}$

B_s^{0*} — 5415

B^{*-} — B^{*0} — 5325

$\Upsilon(4S) \to B_x \bar{B}_x$, $Q(u,c) = Q(d,s,b) + 1$

- Finally we have the top quark $m_t = 173\,\mathrm{GeV}$ with a width of $\Gamma_t = 1{,}2\,\mathrm{GeV}$.

SUMMARY :

u	c	t	6 flavors,
d	s	b	3 families.

5.5. THE BARYONS

- With three quarks we can form spin $1/2$ & $3/2$ fermions, collectively called <u>baryons</u>. The lightest most stable ones are the p^+ and n (proton & neutron).
- The neutron is <u>slightly</u> heavier and decays <u>weakly</u> to the proton, e^-, $\bar{\nu}_e$ with $\tau_n = 881\,s$ (14,7 min).
- The proton is <u>stable</u>. This is explained by another global (accidental in the SM) symmetry, called the <u>baryon</u> number B.

$$B(p) = 1 = B(n)$$
$$B(\bar{p}) = B(\bar{n}) = -1$$

- This forbids $p \to e^+ \pi^0$ or $p \to \nu K^+, \ldots$

$$\tau > 8.10^{33}\,yr \,, \qquad \tau > 6.7 \times 10^{32}\,yr$$

$s = \frac{1}{2}$

OCTET

Ξ^-	Ξ^0		1315
Σ^-	Σ^0	Σ^+	1192
	Λ^0		1116
n	p		938

$s = \frac{3}{2}$

DECOUPLET

	Ω^-			1672
	Ξ^{*-}	Ξ^{*0}		1532
Σ^{*-}	Σ^{*0}	Σ^{*+}		1385
Δ^-	Δ^0	Δ^+	Δ^{++}	1232

Parity +1 by convention

- Once we fixed $P=+1$ for the proton, the fact that Δ^0 has $P=+1$ and the π has $P=-1$ means that $\Delta^0 \to p\pi^-$ decay goes through the P-channel with $L=1$ and not the $L=0$ S-channel.

- According to ISOSPIN $\begin{pmatrix} u \\ d \end{pmatrix}$ form a doublet w. $\pm\frac{1}{2}$.

Thus $I\left(N=\begin{pmatrix} p \\ n \end{pmatrix}\right)=\frac{1}{2}$, $I(\Delta)=\frac{3}{2}$

$$I(\Sigma)=I(\Sigma^*)=1, \quad I(\Xi)=I(\Xi^*)=\frac{1}{2}$$

The EIGHTFOLD WAY (Gell-Mann in 60s)

- The octet and decuplet can be understood as irreducible representations of SU(3), where the quarks are assigned as triplets and anti-triplets of $SU(3)_c$.

$$3_i \to (U3)_i \quad i=1,2,3, \quad U=e^{i\frac{\lambda_a}{2}\theta_a}$$

$$\overset{\shortparallel}{q_i'} = \left(e^{i\frac{\lambda_a}{2}\theta_a}\right)_{ij} q_j \qquad a=1,\dots,8 \quad (n^2-1)$$

$$\bar{3}: \quad \bar{q}_i = (\bar{q}\,U^+)_i = \bar{q}_j (U^+)_{ji}$$

- Larger representations of SU(n) are formed as tensors. Symmetric and anti-symmetric transform separately.

- Let's look at $A_{ab} \to U_{ac} U_{bd} A_{cd}$, which is a 3×3 matrix with 9 components $\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \times & \times & \cdot \end{pmatrix} = 6$ symmetric

so $3 \otimes 3 = 6_A \oplus 9_s$ $\begin{pmatrix} 0 & \cdot & \cdot \\ \times & 0 & \cdot \\ \times & \times & 0 \end{pmatrix} = 3$ antisymmetric

- The baryons are made of 3 quarks, so representations with $\underline{3}$ indices are relevant: $A_{abc} \to U_{ad} U_{be} U_{cf} A_{def}$

totally symmetric $\dfrac{\cancel{3}\cdot \overset{2}{\cancel{4}} \cdot 5}{\cancel{3!}} = 10$

Restricting to $\underbrace{i = 1,2}_{\text{only spin } \frac{1}{2}}$: $\dfrac{\cancel{2}\cdot\cancel{3}\cdot 4}{\cancel{3!}} = \boxed{4}^{\boxed{*}}$ $\underbrace{\cancel{2}\cdot \frac{3}{\cancel{2}} + 1 = 4}_{\text{the } s = \frac{3}{2} \text{ rep. of } SU(2)}$

- The task at hand is to combine flavor and spin into well-defined representation : $\underbrace{(u, d, s) \times (\uparrow, \downarrow)}_{\text{six possibilities}}$

$$|\Delta^{++}(S_z = +\tfrac{3}{2})\rangle = |u\uparrow, u\uparrow, u\uparrow\rangle$$

$$|\Omega^-(S_z = \tfrac{3}{2})\rangle = |s\uparrow, s\uparrow, s\uparrow\rangle$$

- Three index symmetric : $\dfrac{\cancel{6}\cdot 7 \cdot 8}{\cancel{3!}} = 56 = \underbrace{(10 \cdot \boxed{4}_{\boxed{*}})}_{\text{DECUPLET}} + \underbrace{(8 \cdot 2)}_{\text{OCTET}}$

- As we emphasized a couple of times, the diagonal operators (like I_3) ennumerate states and the raising/lowering operators build all the states. Let's see how this works: Start from $|\Delta^{++}, s_3 = \frac{3}{2}\rangle = |u\uparrow u\uparrow u\uparrow\rangle$

$$I^- |\Delta^{++}\rangle = |\Delta^+\rangle = \frac{1}{\sqrt{3}}\left(|d\uparrow u\uparrow u\uparrow\rangle + |u\uparrow d\uparrow u\uparrow\rangle + |u\uparrow u\uparrow d\uparrow\rangle\right)$$

$$S^- |\Delta^+, s_3 = \tfrac{3}{2}\rangle = |\Delta^+ \; s_3 = \tfrac{1}{2}\rangle = \frac{1}{\sqrt{9}}\left(|d\downarrow u\uparrow u\uparrow\rangle + |d\uparrow u\downarrow u\uparrow\rangle + |d\uparrow u\uparrow u\downarrow\rangle + \dots\right)$$

$$= |\Delta^+, Q = 1, St = 0, s_3 = \tfrac{1}{2}, s = \tfrac{3}{2}\rangle$$

- From the $|\Delta^+\rangle$ state we can find the $s = \frac{1}{2}$ state in the OCTET by finding $\langle p|\Delta^+\rangle = 0$, i.e. the proton wave function needs to be orthogonal to Δ^+ and with $S = 1/2$

$$|p, Q = 1, Str = 0, S = \tfrac{1}{2}, S_3 = +\tfrac{1}{2}\rangle = \frac{1}{\sqrt{18}}\left(2|u\uparrow u\uparrow d\downarrow\rangle \mp |u\uparrow u\downarrow d\uparrow\rangle - |u\downarrow u\uparrow d\uparrow\rangle + \dots\right)$$

- The total charge is $2Q_u + Q_d = 1$, $Q_d = Q_u - 1$

$$3Q_u = 2 \Rightarrow Q_u = \tfrac{2}{3}, \; Q_d = -\tfrac{1}{3}$$

We do not observe single quarks in nature, they are allways confined to hadrons.

Need for the colour quantum number.

We found the trend: $\Delta^{++} = u\!\uparrow u\!\uparrow u\!\uparrow$ or $\Omega^- = s\!\uparrow s\!\uparrow s\!\uparrow$, which are completely symmetric wrt exchange of u and s fermions. However, according to the spin-statistic theorem, these are fermions that need to be described by a totally anti-symmetric wave function. To this end we have $SU(3)_c$ or QCD.

$$q_a = (q_r, q_g, q_b) = 3 \text{ of color}$$

$$\bar{q}^a \delta^a_b q^b = \bar{q}^a q_a \quad \ldots \text{singlet}$$

$$\varepsilon^{abc} q_a q_b q_c \quad \ldots \text{again a singlet of } SU(3)_c.$$