5.) THE ORIGIN OF PARTICLE SPECIES

•) Today the universe is out of equilibrium for photons and matter. However, the blackbody spectrum of the CMB with $T_\gamma \sim 3K$ tells us that the universe was in thermal eq.

•) Extrapolating to the past, different interactions will come in equilibrium because the universe becomes smaller and $\Gamma \gtrsim H$.

•) With the Boltzmann equation at hand, we can quantify these moments for

*) Neutrinos : decoupling, N_{eff}, $\Omega_\nu h^2$

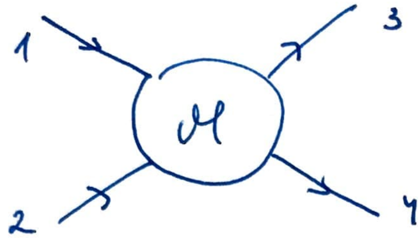
*) Recombination : the moment photons start to free-stream

*) Dark matter production

*) Baryogenesis

*) Big Bang Nucleosynthesis (by L. Ubaldi)

•) All of the above are out-of-equilibrium processes governed by the Boltzmann equation in an expanding background.



$$\frac{dn_1}{dt} + 3Hn_1 = a^{-3}\frac{d}{dt}\left(a^3 n_1\right) = \int_{p_1} C[f_1]$$

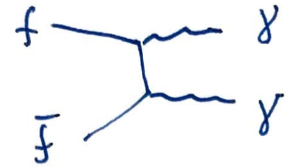
$$= \prod_{i=1}^{4} \int_{\Pi_i} (2\pi)^4 \delta^{(4)}(\Sigma p_i)\, |\overline{M}|^2 \left[f_3 f_4 (1 \pm f_1)(1 \pm f_2) - f_1 f_2 (1 \pm f_3)(1 \pm f_4) \right]$$

•) When momenta / energies are exchanged often enough, the distribution functions depend only on (E, T, μ) and we have either f_{FD} or f_{BE}.

$$f_{F/B} = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

•) When chemical equilibrium is reached, we also have $\sum_{init} \mu_i = \sum_{final} \mu_i$ and chemical potentials from conserved charges (E.M., baryon #, lepton #) are preserved.

·) Furthermore, remember that for bosons $\mu_B = 0$ and $\mu_f = -\mu_{\bar{f}}$.



$$\mu_f + \mu_{\bar{f}} = 2\mu_\gamma = 0 \qquad\qquad \begin{array}{c} \mu_f = \mu_f + \mu_\gamma \\ \Downarrow \\ \mu_\gamma = 0 \end{array}$$

·) Finally, when we are interested in the part of the distribution where $T < E - \mu$, the FD/BE distinction is not relevant $f \to e^{-(E-\mu)/T} = f_{MB}$ and becomes a universal Maxwell-Boltzmann.

In this regime $e^{-(E-\mu)/T} \ll 1$ and we can ignore the blocking terms and simplify [...] to get

$$f_3 f_4 - f_1 f_2 \text{ , all MB-type.}$$

energy conservation
↓
$E_1 + E_2 = E_3 + E_4$

$$= e^{-(E_3+E_4-\mu_3-\mu_4)/T} - e^{-((E_1+E_2)+(\mu_1+\mu_2))/T}$$

$$= e^{-(E_1+E_2)/T}\left(e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T}\right)$$

This quantity vanishes in chemical equilibrium

$$\mu_1 + \mu_2 = \mu_3 + \mu_4 \Rightarrow [\ldots] = 0$$

·) In the Maxwell-Boltzmann limit, the relation between number density and the chemical potential is very simple

$$n = g\int_p f = g e^{\mu/T}\int e^{-E/T} = e^{\mu/T} n^{(0)}$$

·) The zero chemical potential number densities are simple to evaluate in the massless $m=0$ and massive limits.

i) $m=0$, $E=p$ (photons, neutrinos, radiation)

$$n^{(0)}_{m=0} = g\int_0^\infty \frac{4\pi\,dp}{(2\pi)^3}\, p^2 e^{-p/T} = \frac{gT^3}{2\pi^2}\int_0^\infty x^2 e^{-x}\,dx = \frac{gT^3}{\pi^2}$$

$m>T$, $E=\sqrt{m^2+p^2} \simeq m\left(1+\frac{p^2}{2m^2}\right) \simeq m + \frac{p^2}{2m}$

$$n^{(0)}_{m>T} = g\int_0^\infty \frac{p^2\,dp}{2\pi^2}\, e^{-m/T} e^{-p^2/2mT} \quad , \quad x = \frac{p^2}{2mT}$$

$$dx = \frac{p\,dp}{mT}$$

$$= \frac{g}{2\pi^2} e^{-m/T}\int_0^\infty mT\,dx\,\sqrt{2mTx}\,e^{-x}$$

$$= \frac{g}{\sqrt{2}\pi^2} e^{-m/T}(mT)^{3/2}\underbrace{\int_0^\infty x^{1/2}e^{-x}\,dx}_{=\Gamma\left(\frac{3}{2}\right)=\frac{\sqrt{\pi}}{2}} = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}.$$

•) In summary:

$$n^{(0)} = g_s \begin{cases} \frac{1}{\pi^2} T^3 & , \quad m \ll T \\ \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} & , \quad m \gg T \end{cases}$$

We can now use the $n^{(0)}$ expressions to normalize n

$$\frac{n}{n^{(0)}} = e^{\mu/T} \Rightarrow [\dots] = e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T}$$

$$= \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} .$$

This already simplifies the Boltzmann equation.

•) Finally, it is useful to introduce the thermally averaged cross-section $\langle \sigma v \rangle$.

$$\langle \sigma v \rangle = \frac{1}{n_1^{(0)} n_2^{(0)}} \prod_{i=1}^{4} \int_{\Pi_i} (2\pi)^4 \delta^{(4)}(\Sigma p) \overline{|\mathcal{M}|^2} \underbrace{e^{-(E_1+E_2)/T}}_{f_1 f_2} .$$

$$= \frac{1}{n_1^{(0)} n_2^{(0)}} \int_{p_{1,2}} f_1 f_2 \int_{\Pi_{3,4}} (2\pi)^4 \delta^{(4)}(\Sigma p) \frac{\overline{|\mathcal{M}|^2}}{4 E_1 E_2}$$

From this expression we can understand why it's called the thermally averaged cross-section. Remember that the usual cross-section is defined as (see for example the PDG's "Kinematics" review)

$$\sigma = \int_{\Pi_{3,4}} (2\pi)^4 \delta^{(4)}\left(\sum p_i\right) \frac{\overline{|\mathcal{M}|}^2}{4E_1E_2 v_{12}} = \int_{\Pi_{3,4}} d\sigma$$

where : $$v_{12} = \frac{\sqrt{(p_1p_2)^2 - (m_1m_2)^2}}{E_1E_2} \xrightarrow{m_{1,2}\to 0} 1$$

$$\xrightarrow{v_{1,2} \ll 1} \frac{\cancel{m_1m_2}\sqrt{(1+v_1v_2)^2-1}}{\cancel{m_1m_2}} \simeq \sqrt{2v_1v_2}$$

•) Combining this with the previous result, we have

$$\langle \sigma v \rangle = \frac{1}{n_1^{(0)} n_2^{(0)}} \int_{p_{1,2}} f_1 f_2 \underbrace{\int_{\Pi_{3,4}} d\sigma\, v_{12}}$$

This is a thermal average of

$$1 = \frac{1}{n_1^{(0)} n_2^{(0)}} \int_{p_1} f_1 \int_{p_2} f_2 \qquad \text{(normalization)}$$

•) Thermal averaging will pick the momenta close to the peak of distribution, defined by T.

That's why for $m=0$ $\quad p \longrightarrow T$

$m \gg T$ $\quad p \longrightarrow m \quad$ & $e^{-m/T}$

as we shall see.

•) With these definitions and simplifications, we get

$$a^{-3}\,\frac{d(n_1 a^3)}{dt} = n_1^{(0)} n_2^{(0)} \langle \sigma v\rangle \left(\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right)$$

$\downarrow$ expansion $\qquad\qquad\qquad$ $\downarrow$ interaction

$\wr$

$$H \cancel{n_1} = \cancel{n_1}\, \underbrace{n_2 \langle \sigma v \rangle}_{\Gamma}$$

Γ ... interaction rate for n

Note that for a fixed $\langle \sigma v\rangle$, H grows with T and eventually the $(\ldots)$ will have to go to zero, such that

$$\frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} = \frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}}$$

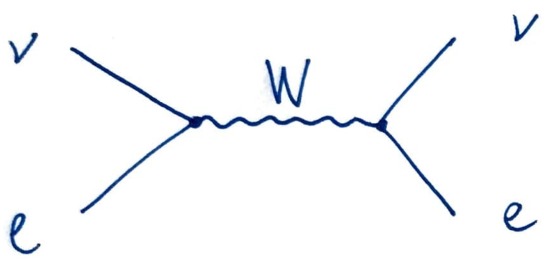
(so-called Saha equation)

This is how the chemical equilibrium is established: $e^{(\mu_1+\mu_2)/T} = e^{(\mu_3+\mu_4)/T}$

or : $\mu_1 + \mu_2 = \mu_3 + \mu_4.$

Neutrino Decoupling

·) Perhaps the most straightforward physical application of decoupling is to consider the weak interactions and light neutrinos.



$$\mathcal{A} \propto g^2 \frac{p^2}{s - M_W^2}$$

For momenta below M_W, $T \lesssim 80\,\text{GeV}$ we thus get $\mathcal{A} \propto \frac{1}{M_W^2}$, $\langle \sigma v \rangle \propto G_F^2 T^2$ by dimensional analysis.

Now : $\Gamma \simeq n \langle \sigma v \rangle \simeq T^3 G_F^2 T^2 \simeq G_F^2 T^5$

In the radiation-dominated universe

$$H = \sqrt{\frac{8\pi G}{3} \rho_\gamma} = \sqrt{\frac{8\pi}{3 M_{Pl}^2} \frac{\pi^2}{15}}\, T^2$$

@ $T = T_{dec}$: $\quad G_F^2 T^{\cancel{5}\,3} = \sqrt{\frac{8\pi^3}{45}} \frac{1}{M_{Pl}} \cancel{T^2}$

•) We thus get the neutrino decoupling T

$$T^3 \sim \sqrt{\frac{8\pi^3}{45}\,\frac{G_N}{G_F^4}}$$

$$T_{dec} \sim \left(\frac{M_W^{84}}{M_{Pl}^{\cancel{2}}}\right)^{\overset{2}{\cancel{1}}/6} \sim \left(\frac{10^8}{10^{18}}\right)^{1/3} \text{GeV}$$

$$\simeq 10^{-20/6}\ \text{GeV} = \text{few } \underline{\underline{\text{MeV}}}$$

- A more precise calculation with Z diagrams and numerical treatment of the Boltzmann equation gives $T_{\nu\,dec} \simeq 1.5$ MeV, in agreement with the naïve estimate from dim-analysis.

- This tells us that neutrinos froze-out while being very relativistic with $p_\nu \sim \text{MeV} \gg m_\nu \lesssim 0.1\text{eV}$ by more than 7 orders of magnitude.

After decoupling, neutrinos don't interact anymore but simply free-stream.

Their distribution is still Fermi-Dirac

$$f_\nu(T) = \frac{1}{e^{p/T}+1} \quad \text{and} \quad n_\nu \propto a^{-3}.$$

Its temperature T_ν tracks the photon T_γ until $\sim 2m_e$ when e^+e^- annihilation heats up the photons and $\frac{T_\nu}{T_\gamma} \sim \left(\frac{4}{11}\right)^{1/3}$ as we discussed (due to entropy conservation).

◦) Additional (dark) radiation = N_{eff}

$$\rho_{rad} = \rho_\gamma + \underbrace{\rho_{extra}}_{\text{additional light species, extra neutrinos}}$$

$$N_{eff} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{\rho_{rad}-\rho_\gamma}{\rho_\gamma}\right)$$

$$\xrightarrow{\text{in the SM}} 3\left(\frac{11}{4}\right)^{4/3}\left(\frac{T_\nu}{T_\gamma}\right)^4 \xrightarrow[\text{dec.}]{\text{inst.}} 3$$

•) A more precise calculation of N_{eff} includes the additional EW interactions, T-corrections & ν-oscillations

$$N_{eff}^{SM} = 3.044 \qquad \left[2001.04466\right]$$

for BSM

Neutrino number density

•) To calculate the present-day n_ν of the CνB, first remember

$$n_\gamma = 2 \int_p \frac{1}{e^{p/T}-1} = \frac{2\zeta(3)}{\pi^2} T_\gamma^3$$

Neutrinos are not kept in equilibrium after freeze-out, so their $f_\nu = f_{FD}(p,T)$

$$\forall \text{ generation}: \quad n_{\nu_i} = 2\int_p \frac{1}{e^{p/T}+1} = \frac{2\zeta(3)}{\pi^2}\left(1-\frac{1}{4}\right)T_\nu^3$$

$$= \frac{3\zeta(3)}{2\pi^2} T_\nu^3$$

$$T_\nu^3 = \frac{4}{11} T_\gamma^3 \quad \text{from } \Delta S = 0$$

$$\Rightarrow n_\nu = 3 \cdot \frac{3}{4}\left(\frac{T_\nu}{T_\gamma}\right)^3 n_\gamma = \frac{9}{11} n_\gamma \simeq 340\,\text{cm}^{-3}$$

$$n_\gamma = 410\,\text{cm}^{-3}$$

Neutrino energy density

·) The energ density of neutrinos is fascinating, because it depends precisely on whether ν's should be considered as radiation or matter.

For a single species, we have

$$\rho_{\nu_i} = 2 \int_p \sqrt{p^2 + m_{\nu_i}^2}\, f_\nu$$

$$= \begin{cases} \frac{7\pi^2}{120} T_\nu^4 \,, & m_\nu \sim 0 \\ m_{\nu_i} n_{\nu_i} \,, & m_\nu \gg p\,(T_\nu) \end{cases}$$

·) With <u>massless</u> neutrinos, we would get $(N_g = 3)$

$$\rho_\nu^{m=0} = \frac{21\pi^2}{120}\left(\frac{T_\nu}{T_\gamma}\right)^4 \frac{15}{\pi^2}\rho_\gamma = \underbrace{\frac{21}{8}\left(\frac{4}{11}\right)^{4/3}}_{0.7} \rho_\gamma$$

$$\Omega_\nu h^2 = 1.7 \cdot 10^{-5} < \Omega_\gamma h^2$$

·) With <u>massive</u> neutrinos, the formula becomes

$$\rho_\nu^{m_\nu} = \sum m_{\nu_i} n_{\nu_i} \Rightarrow \Omega_\nu h^2 \simeq \frac{\sum m_\nu}{93.14\,\mathrm{eV}}.$$

•) Interestingly, none of the two options can be established with certainty, m_ν is 'right in the middle'.

From neutrino oscillations, we have

$$\Delta m^2_{\nu_{12}} \sim 10^{-3}\,eV^2 \,, \quad \Delta m^2_{\nu_{23}} \sim 10^{-4}\,eV^2$$

This means that neutrinos cannot be "massless" today when: $T_\nu^{CVB} = \left(\frac{4}{11}\right)^{1/3} 2.7\,K \cong 2\cdot 10^{-4}\,eV$.

Even if the lightest neutrino were $m_{\nu_{min}} = 0$, we get that

$$\sum_i m_{\nu_i} \sim \underset{\overset{\parallel}{0}}{m_{\nu_1}} + \underbrace{m_{\nu_2} + m_{\nu_3}}_{\substack{2! \\ 10^{-3/2}\,eV}} \sim \underline{0.03\,eV}.$$

•) However, neutrinos also cannot be too massive, there are direct laboratory bounds from

i) endpoint β-decay kinematics (KATRIN) $m_\nu < 0.9\,eV$ [1909.06048]

ii) neutrinoless double beta decay KAMLAND-ZEN $0\nu 2\beta$ (only if Majorana) $m_\nu^{ee} < 0.17\,eV$

iii) CMB + bckg. evol. + LSS [PDG '21] $\rightsquigarrow$ $\sum m_{\nu_i} < 0.11\,eV$

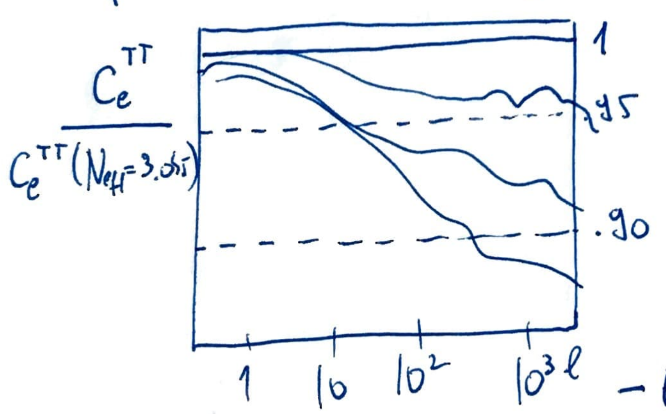
·) Neff constraints

In ΛCDM, we only have γ & ν behaving as radiation (at least for part of the time). However in many BSM models, one encounters light species, fermions (sterile neutrinos, gravitinos, neutralinos, ...) or scalars (axions, pseudo-Goldstones), vector bosons (like dark photons), which may be coupled to the photon plasma and behave as additional rel. degrees of freedom.

Neutrinos become relativistic when $\langle p \rangle \sim m_{\nu_i}$, i.e.

$$\langle p \rangle = \frac{\int pf}{\int f} = \frac{7\pi^4}{180\zeta(3)} T \simeq 3.15\, T_\nu \sim m_{\nu_i}$$

With additional N_{eff}, both CMB and LSS power spectra are changed



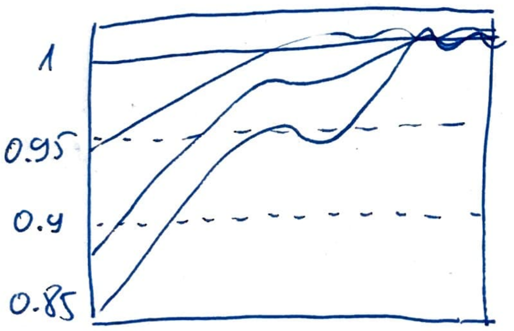
The main effect of N_{eff} is to <u>suppress</u> the spectrum at large $\ell \sim \mathcal{O}(100)$ multipoles, i.e. on small $\Theta \sim \frac{\ell}{\pi} \sim 1°$ scales.

•) On scales of $\ell \sim 1000$ the $\delta C_\ell^{CMB} \lesssim 1\%$ and we thus get tight constraints

		N_{eff}
[PDG '21]	CMB :	2.92 ± 0.36
	CMB & LSS :	2.99 ± 0.34

•) $\sum m_\nu$ constraints

Similarly to N_{eff}, $\sum m_{\nu_i}$ enters in CMB & LSS, however with a slightly different way by mostly distorting the low ℓ multipoles



[PDG '21]	$\sum m_\nu$ [eV]
CMB	$< 0.54 - 0.26$
CMB & LSS	$< 0.44 - 0.2$

These values are getting close to the upper limit on $\sum m_\nu$ from ν oscillations above

$$\sum m_\nu^{max} \sim 3\sqrt{\Delta m_0^2} \sim 3 \cdot 10^{-3/2}\,eV \sim 0.09\,eV \overset{0.1\,eV}{\lesssim}$$

Cosmic neutrino background CνB

·) Before concluding, let's just recapitulate that ΛCDM predicts the existence of the cosmic neutrino background with $E_\nu \sim \langle p_\nu \rangle \sim 3T_\nu \sim 10^{-3}\,eV$.

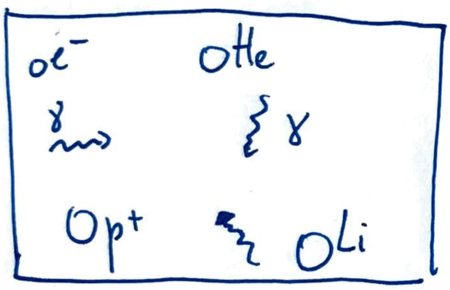
The flux can be calculated [1910.11878] and is actually quite large $\phi_\nu^{max} \sim 10^{16} / eV\,cm^2 s$.

The challenging part is that it peaks at $\sim 10^{-3}\,eV$, and the cross-sections at these energies are very small.

·) Current proposals like PTOLEMY aim to use the zero-energy threshold inverse beta decay to capture the cosmic neutrinos on Tritium.

Recombination

•) Another example of how the Boltzmann equation works, is to consider the post-BBN era of recombination. This is the moment when atoms get created and photons begin to free-stream, the universe becomes transparent and "dark".



@ $T < MeV$ we have created in BBN the light nuclei, such as H, He and traces of Li.

•) The energetic photons at $\sim MeV$ will scatter by Compot scattering on e^- in the plasma and keep in equilibrium. When their energies drop $E_\gamma \sim 3\,T_\gamma$ below the Hydrogen binding energy $E_{Ry} = \frac{1}{2}\alpha^2 m_e = 13.6eV$ the atoms are no longer destroyed and e^- can combine with p^+, He^+, ...

•) Before going into the Boltzmann equations for recombination, let's remember.

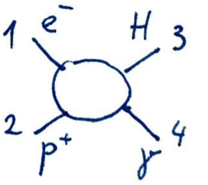
i) Charge is unbroken in the SM, therefore the net charge of the universe is zero. At the beginning of recombination we only have n_e and n_p, whose total charge cancels

e.m. charge density →

$$Q_{tot} = 0 = Q_e n_e + Q_p n_p = n_p - n_e = 0.$$

ii) The net baryon number is non-zero (we will come back to how this may be explained by Baryogenesis) and is given by $n_B = \eta n_\gamma$.

In this case we will look at



$$1 + 2 \longrightarrow 3 + 4$$
$$e^- + p^+ \longrightarrow H + \gamma$$
$$B : 0 + 1 = 1 + 0$$, so the total
$$Q_e : -1 + 1 = 0 + 0$$

baryon number is : $\dfrac{B^{tot}}{V} = B_p n_p + B_H n_H$
$$= n_p + n_H = n_B.$$

•) The Boltzmann equation is (for $1 = e^-$)

$$a^{-3}\frac{d(n_e a^3)}{dt} = n_e^{(0)} n_p^{(0)} \langle \sigma v \rangle \left[\frac{n_H}{n_H^{(0)}} - \frac{n_e^2}{n_e^{(0)} n_p^{(0)}} \right]$$

where we already used $n_p = n_e$ and $n_\gamma = n_\gamma^{(0)}$ $(\mu_\gamma = 0)$. Let us define the electron fraction $X_e = \frac{n_e}{n_e + n_H}$ $\left(= \frac{n_p}{\underbrace{n_p + n_H}_{= n_B}} \right)$

which measures the amount of remaining ionization.

•) When we are in eq. $[\dots] = 0$ and

$$\frac{n_e^2}{n_H} = \frac{n_e^{(0)} n_p^{(0)}}{n_H^{(0)}} \quad , \quad n^{(0)} \propto \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$$= \left(\frac{m_e T}{2\pi}\right)^{3/2} \underbrace{\left(\frac{m_p}{m_H}\right)^{3/2}}_{= 1} e^{-\overbrace{(m_e + m_p - m_H)}^{\varepsilon_0 = E_{Ry}}/T}$$

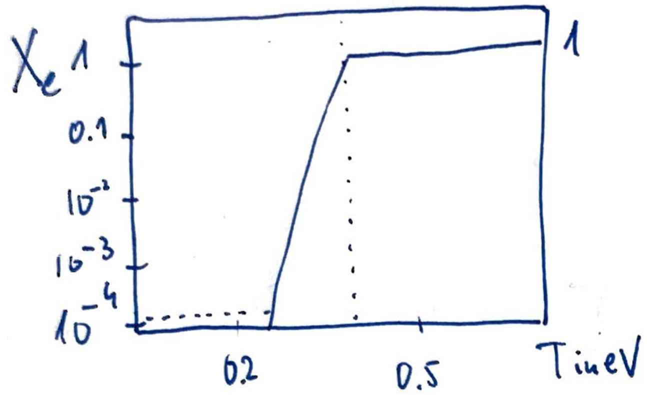
•) From here, we can already estimate X_e by

$$n_e^2 = X_e^2 n_B^2 \quad , \quad n_H = n_B - n_e = n_B(1 - X_e)$$

$$\frac{n_e^2}{n_H} = \frac{X_e^2}{1 - X_e} n_B = \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\varepsilon_0/T}$$

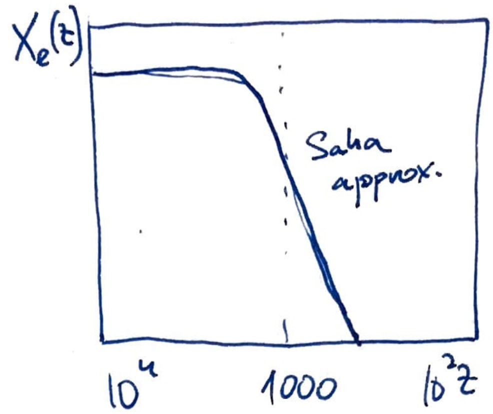
$$\text{or:} \quad \frac{X_e^2}{1 - X_e} = \frac{1}{\eta n_\gamma}\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\varepsilon_0/T} \quad ; \quad \eta = 6 \cdot 10^{-10}, \quad n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3$$

$\rightsquigarrow$ solving the quadratic equation for X_e, gives



$T \to z$

$\Longrightarrow$



$$\frac{T}{T_\gamma} = \frac{1}{a} = z+1$$

•) note that for $T > \mathcal{E}_0$

$$\text{RHS} \cong \frac{1}{10^{-9} T^3} (m_e T)^{3/2} e^{-0} \simeq 10^9 \left(\frac{m_e}{T}\right)^{3/2}$$

$$\sim 10^9 \left(\frac{10^5 \cancel{eV}}{10 \cancel{eV}}\right)^{3/2} \sim 10^9\, 10^6 \sim 10^{15}\text{, which}$$

firmly sets $\dfrac{X_e^2}{1-X_e} = 10^{15} \Rightarrow X_e = 1$. <u>all ionized!</u>

•) Let us go back to the Boltzmann equation and note that $n_B a^3 = \text{const.}$ (Baryon number is conserved in the SM when $T \sim 10\,\text{eV}$), $n_e = X_e n_B$

$$a^{-3} \frac{d(n_e a^3)}{dt} = \cancel{a^{-3}} n_B \cancel{a^3} \frac{dX_e}{dt} = \langle \sigma v \rangle \left(n_H \frac{n_e^{(0)} n_p^{(0)}}{n_H^{(0)}} - \underbrace{n_e^2}_{= X_e^2 n_B^2} \right)$$

$$= \langle \sigma v \rangle \left((1-X_e) n_B \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\mathcal{E}_0/T} - X_e^2 n_B^2 \right)$$

•) We get : $\frac{dX_e}{dt} = \underbrace{(1-X_e)\langle\sigma v\rangle\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-\varepsilon_0/T}}_{\beta} - X_e^2 n_B \underbrace{\langle\sigma v\rangle}_{\alpha^{(2)}}$

$$= (1-X_e)\beta - X_e^2 n_B \alpha^{(2)}$$

•) $(1-X_e)\beta$ → <u>Ionization</u> rate shuts off due to $e^{-\varepsilon_0/T}$

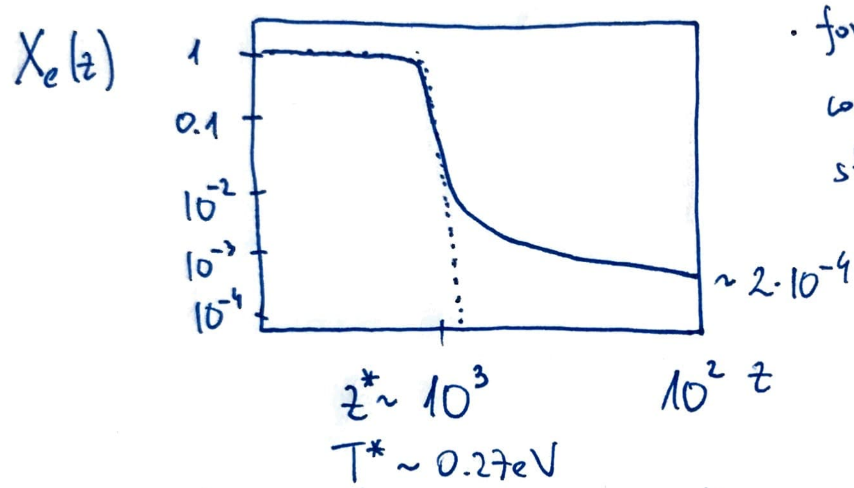
$X_e^2 n_B \alpha^{(2)}$ → <u>Recombination</u> rate, this term depletes the ions, becomes negligible for small X_e; X_e^2.

•) When T drops and t evolves, we go from $X_e = 1$ in equilibrium to small X_e (following Saha initially) until the rates drop and $\frac{dX_e}{dt} \sim 0$, the fraction stabilizes and $X_e \sim$ const.

•) There is a complication (see the book by Peebles for more details on this) due to the fact that recombination has to occur to a higher level of H

$$\alpha^{(2)} \sim 10 \frac{\alpha^2}{m_e^2}\left(\frac{\varepsilon_0}{T}\right)^{1/2} \ln\left(\frac{\varepsilon_0}{T}\right)$$

•) Numerical integration of $\dot{X}_e = (1-X_e)\beta - X_e^2 \alpha^{(2)}$ gives the frozen ionization fraction $X_e(\infty)$, which enters the CMB predictions.



· for the numerical code with excited states, He inclusion, see CLASS [1104.2933]

•) We will see how to estimate the freeze-out abundances in DM cases analytically as well.

•) After recombination photons don't interact much and universe enters the dark ages until the first stars turn on: reionization period @ $z \sim 6$, $T \sim meV$

Photon Decoupling

- Recombination and photon decoupling are two seperate processes that happen to occur at roughly the same time.

- Photons scatter off e^- via Compton scattering with:

$$\Gamma = n_e \sigma_T = X_e n_B \sigma_T = H$$

The Thomson $\sigma_T = \frac{8\pi}{3}\left(\frac{\alpha}{m_e}\right)^2 \sim 2 \cdot 10^3 \, GeV^{-2}$

$$\Omega_B a^{-3} = \frac{n_B m_p}{\rho_{cr}} \Rightarrow n_B = \frac{\Omega_B a^{-3}}{m_p} \rho_{cr}$$

This gives us the rate

$$\Gamma = X_e \frac{\Omega_B a^{-3}}{GeV} \underbrace{\rho_{cr} \frac{8\pi}{3}}_{\frac{H_0^2}{G}} \frac{\alpha^2}{m_e^2}$$

- We are in mixed matter-radiation for which

$$\frac{H}{H_0} = \sqrt{\Omega_m}\, a^{-\frac{3}{2}} \sqrt{1 + \frac{a_{eq}}{a}}$$

•) Putting it all together in $\Gamma = H$

$$1 = \frac{\Gamma}{H} = \frac{n_e \sigma_T}{H} = 123 \, X_e \left(\frac{\omega_b}{0.02}\right)\left(\frac{0.1}{\omega_m}\right)^{1/2}\left(\frac{1+z}{1000}\right)^{3/2}\left(1 + \frac{1+z}{3360}\frac{0.1}{\Omega_m}\right)^{-1/2}$$

The most ~~drasticallyto~~ variating variable here is X_e which drops steeply at $z \sim 1000$. So $X_e \sim 10^{-2}$ essentially satisfies $\Gamma \sim H$ and decoupling of photons coincides with recombination.

•) Note that Compton scattering would go out-of equilibrium even if $X_e = 1$, i.e. for a completely ionized universe. This ~~will~~ would happen much later for smaller redshifts so $\left(1 + \frac{1+z}{10^3} \to 1\right)$

$$\Rightarrow \left(10^2 \left(\frac{\omega_b}{0.02}\right)\left(\frac{0.1}{\omega_m}\right)^{1/2}\right)^{2/3} \frac{1+z}{1000} = 1$$

and $$1 + z_{dec}^{\gamma} \simeq \underline{\underline{40}} \left(\frac{0.02}{\omega_b}\right)^{2/3}\left(\frac{\omega_m}{0.1}\right)^{1/3}.$$

Thus QED goes out of eq. even in a fully ionized universe due to the expansion.