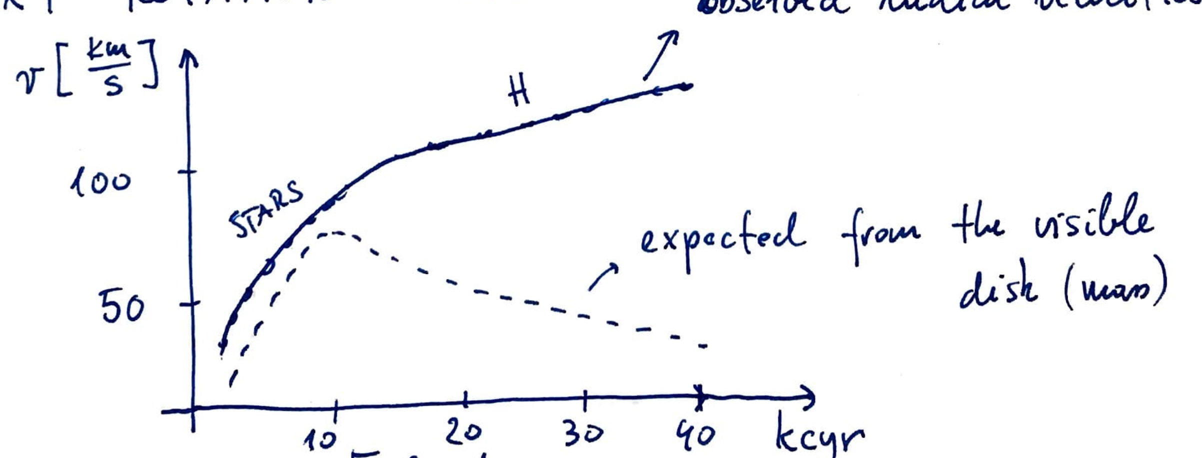
# 6.) DARK MATTER PHENOMENOLOGY

- Evidences for DM / astro parameters
- Properties & constraints
- Candidates, requirements
- Production mechanisms
- Detection prospects (direct, indirect, colliders)

## i) EVIDENCES: [HISTORY, see Hooper & Bertone: 1605.04909]

Up to this point, the only observational evidence for DM comes from gravitational physics and astrophysical observations.

GALAXY ROTATION CURVES



Typical size of galaxies $\sim 10^4$ cyr $\sim$ 10 - 100 kpc

Typical velocities are $\sim 100$ km/s

For gravitationally bound systems, the relation between the encapsulated (spherical) mass and velocities can be derived with Newtonian gravity


Gravitational force

$$\frac{\cancel{m} M G}{R^{\cancel{2}}} = \cancel{m} \frac{v_c^2}{\cancel{R}} \, , \quad v = \sqrt{\frac{MG}{R}}$$

Clearly if we would like $v$ to stay constant, $M(R) \propto R$, otherwise the velocity would drop as $R^{-1/2}$, as in the plot.

Since $\rho \sim \frac{M}{R^3}$ having $M \propto R$ requires $\rho \propto \frac{1}{R^2}$ and it should extend beyond the stellar disk, eventually shutting off, otherwise $M(R \to \infty) = \infty$

# Mass profiles from simulations

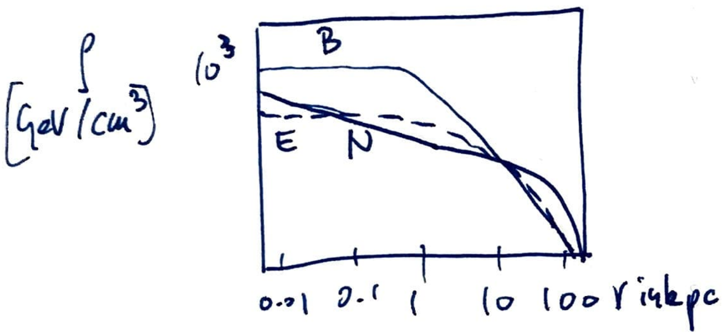
NFW (CUSP) : $\rho_{NFW} = \dfrac{\rho_0}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$ , $r_s \sim 20\,\mathrm{kpc}$

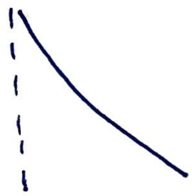
$\rho_0 \sim 0.3\,\mathrm{GeV\,cm^{-3}}$

$\rho_{cr} \sim 1\,\mathrm{GeV}/m^3$

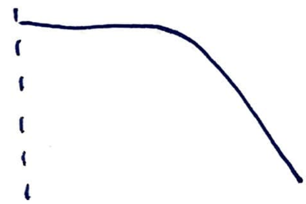
Einasto (CUSP) : $\rho_{Ein} = \rho_0\, e^{-\frac{2}{\gamma}\left(\left(\frac{r}{r_s}\right)^\gamma - 1\right)}$ $\quad \gamma = 0.17$

Burkert (CORE) : $\rho_{Burk} = \dfrac{\rho_0}{\left(1+\frac{r}{r_s}\right)\left(1+\left(\frac{r}{r_s}\right)^2\right)}$



cuspy



cored

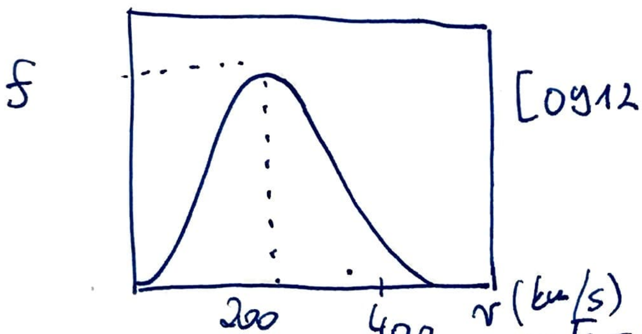
## Velocity profile

•) for a virialized system one expects a Maxwell Boltzmann distribution

$$f(v_t) = \frac{v_t}{N_t}\, e^{-\left(v_t^2/\bar{v}_t^2\right)^{\alpha_t}}$$

$\alpha_t \sim 0.7$

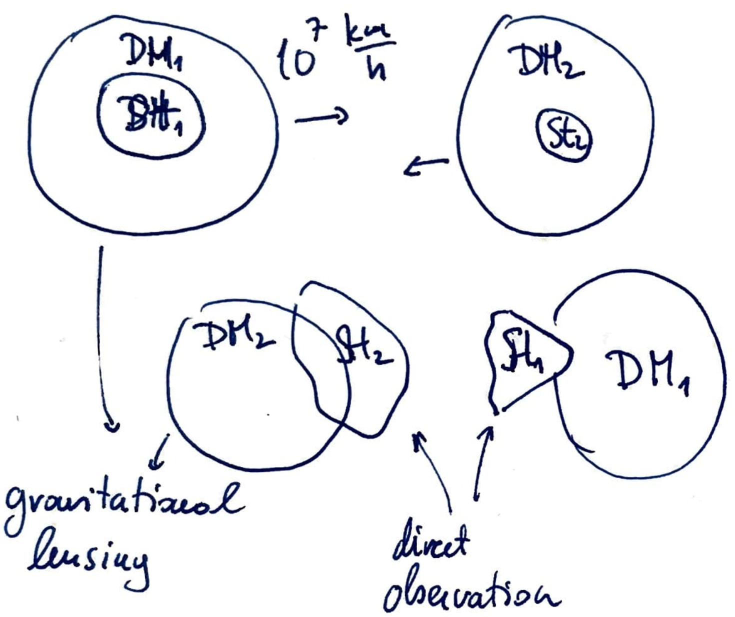
$\bar{v} \sim 120\,\frac{km}{s}$




[0912.2358]

# Bullet cluster

·) A colliding pair of galaxy clusters observed by the Chandra X-ray [1E 0657-56] at $z = 0.3$ gives a very compelling evidence for DM.




* Stars in the central disk collide & deform $St_1$ & $St_2$

* DM halos $DM_1$ & $DM_2$ simply pass through

·) The earliest example / proof for DM came from comparing the virialized mass with the observed

$$M(R)\langle v^2 \rangle \cong 3\, \underbrace{v_{\parallel}^2}\, M = \frac{GM^2}{R}$$

visible one

observations of $v_{\parallel}$ & stars $\Rightarrow M_{grav} \sim 10^{15} M_\odot$

$$M_{vis} \sim 10^{14} M_\odot < M_{grav}.$$

# DM PROPERTIES

From the particle's physics point of view, (or even more generally), the DM candidate has to satisfy certain criteria:

i) MASS RANGE

We saw the typical sizes and masses of DM halos to be $M \sim 10^{12} M_\odot$ , $R \sim 100\,\text{kpc}$

and for Dwarf Spheroidals, thought to be dominated by DM:

$$M \sim 10^7 M_\odot \quad , \quad R \sim \text{kpc}$$

From these inputs, we can derive a lower limit on $m_{DM}$, depending on statistical properties.

BOSONS : If DM has spin 0,1, then it can be efficiently packed (think BE condensation) in a small volume. However, DM still has to be confined to a certain volume $\lesssim R^3$, therefore the de Broglie wavelength cannot be too large. If it were, structures would be erased on those scales and galaxies would puff up.

$$\lambda_{DB} = \frac{1}{p} = \frac{1}{mv} = \frac{1}{m}\sqrt{\frac{R}{GM}} < R$$

$$v^2 = \frac{GM}{R} \qquad m_{DM} > \frac{1}{\sqrt{GMR}} \sim \frac{M_{Pl}}{\sqrt{\underset{10^7 M_\odot}{M}\;\underset{\text{kpc}}{R}}}$$

$$\Rightarrow \quad m_{DM} \gtrsim 10^{-21}\,\text{eV} \xleftarrow{\text{dwSph}}$$

•) Such candidates are referred to as fuzzy DM.

## FERMIONS

For fermions the bound is somewhat different and more stringent. It comes from the Pauli exclusion principle, which manifests itself in the FD distribution function, namely $f_{FD} < 1$ even when $E-\mu \ll T$, as opposed to BE: $f_{FD/BE} = \dfrac{1}{e^{(E-\mu)/T} \underset{\text{grows!}}{\pm 1}}$

•) The number density is thus limited: we cannot "pack" an arbitrary # of fermions in a small volume. This limits the total DM halo mass,

since $\rho = m_{DM}\, n$ & $M_{halo} = m_{DM} \int d^3x\, n = m_{DM} \int_{p,x} \overset{1}{\overset{\vee}{f}}$

inserting $d^3x \sim V \sim R^3$, $d^3p \sim (mv)^3$, $v^2 = \frac{GM}{R}$

$$M < m_{DM} R^3 m_{DM}^3 \left(\frac{GM}{R}\right)^{3/2}$$

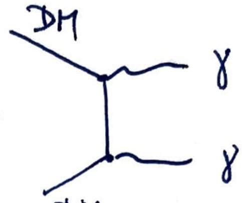
$$m_{DM}^4 > M\,(MGR)^{-3/2}$$

or $$m_{DM} > (G^3 R^3 M)^{-1/8} \sim \underline{0.5\ \text{keV}}$$

•) Stricter lower bounds may apply to thermal light DM from (Lyman-$\alpha$) the absorption of light passing through the intergalactic H-gas.

## •) EM charge of DM : $Q_{DM}^{max}$

If DM were charged it would shine and couple to photons via e.g.

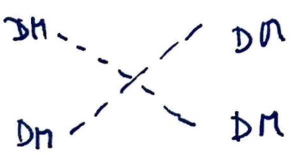



This would affect the plasma at DECOUPLING times (remember recombination) and thus

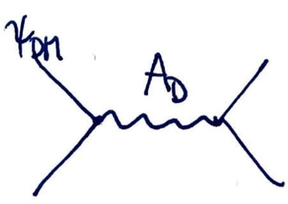
$$Q_{DM} \lesssim 3.5 \cdot 10^{-7} \left(\frac{m_{DM}}{GeV}\right)^{0.6} \quad [PDG\ '21]$$

## •) SELF-INTERACTION

Even though DM has to have a small charge and, as we will see, weak couplings to the SM, it can have relatively strong DM-DM coupling = self-interaction

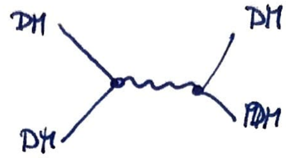



or




from, e.g. the bullet cluster $\frac{\sigma_{DM-DM}}{M_{DM}} \lesssim 1.2 \frac{cm^2}{g} \lesssim 0.8 \frac{barn}{GeV}$

nucleons : $\frac{\sigma_{p-n}}{M_n} \sim 10 \frac{barn}{GeV}$

Note that for the usual WIMP 


$(\hbar c)^2 = 1 \sim 0.4$ mbarn $\mathrm{GeV}^2$, so $\sigma_{\text{wimp-wimp}} \sim \frac{\alpha_W^2}{M_W^2} \sim 10^{-8}\,\mathrm{GeV}^{-2}$
$\sim 10^{-8}$ mbarn

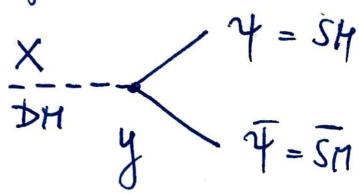
Thus we can safely neglect DM-DM interactions.

## DM candidates

Most obvious requirements

i) COSMOLOGICAL STABILITY : $\tau_{DM} \gtrsim t_u \sim 15$ byrs

This translates to $\Gamma_{DM} \lesssim \frac{1}{1.5 \cdot 10^{10}\,\text{yrs}} \lesssim 10^{-41}\,\mathrm{GeV}$

Suppose DM is a scalar, coupled to two light fermions. $[\Gamma] = 1$ and 

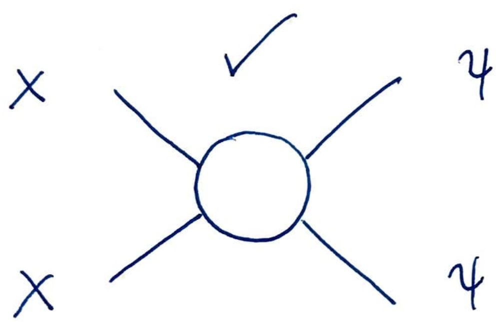

$$\Gamma_X = \frac{m_X}{8\pi} y^2 \lesssim \left(\frac{m_X}{\mathrm{GeV}}\right)\left(\frac{y}{10^{-20}}\right)^2 \Rightarrow$$ for a GeV-scale candidate, the couplings to the SM are $< 10^{-20}$ in order to suppress, stabilize such decay modes.

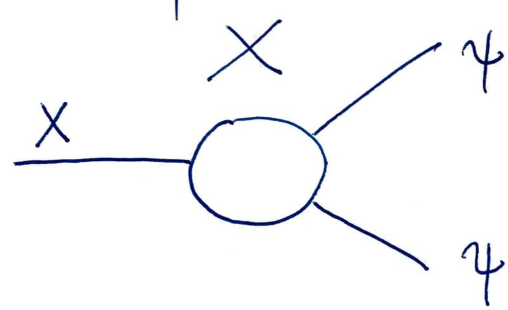
(* 3BODY  $\Gamma_\mu \sim \frac{\alpha_2^2}{M_W^4} M_\mu^5 \frac{1}{192\pi^3}$ so again, need to suppress for DM

•) Multi body decays may be slower due to heavy mediators and $(8\pi)^2$ phase space factors.

•) In principle one could just set those couplings to be small by hand. However in atypical cases, models possess additional particles (like the SM, MSSM, ...) and interactions that can destabilize at higher loop levels. This may be suppressed by <u>SYMMETRIES</u>.

e.g.: $\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{DM}$ $\qquad Z_2$ : $\underset{\psi}{\overset{}{SM}} \to SM$ , $DM \to -DM$




$Z_2 = 1 \cdot 1 = 1$ $\qquad Z_2 = 1 \cdot 1$ $\qquad Z_2 = -1$ $\qquad Z_2 = 1 \cdot 1 = 1$

ANNIHILATION OK ✓ $\qquad$ DECAYS FORBIDDEN ✗

e.g. SM + scalar singlet S with $Z_2$

+ $SU(2)_L$ doublet $Z_2$ = INERT HIGGS

(or higher multiplets)

•) A very popular (now vintage) theoretical setup is

SUPERSYMMERY $\qquad \mathcal{L}_{SM} + \mathcal{L}_{SUSY-SM} = \mathcal{L}_{MSSM}$

| SM | SUSY |
| --- | --- |
| $\gamma, Z, W, g$ | $\tilde{\gamma}, \tilde{Z}, \tilde{W}, \tilde{g}$ |
| $u, d, s, c, b, t$ | $\tilde{q}, \tilde{\ell},$ $\quad \tilde{h}$ |
| $e, \mu, \tau, \nu$ $\quad h$ | |

fermions
" photino, Zino
wino, gluino

squarks, sleptons
" bosons
higgsinos = fermions

R-parity : $(-1)^{3(B-L)+2S}$

$\Downarrow$ acts as an effective $Z_2$ symmetry.

protects from P-decay, stabilizes the lightest susy particle
$\Rightarrow$ natural WIMP. But no susy @ colliders, also $m_\nu = 0$.

•) Another type of DM candidate is the <u>axion</u>, a very light $m_a < 0.1\,eV$ pseudoscalar particle, ~~des~~ It is a remnant of a special symmetry, which explains why strong interactions do not violate CP.

•) Right-handed neutrinos. $m_\nu \neq 0 \lesssim 0.1\,eV$ from neutrino oscillations. Introducing $\nu_s$... sterile neutrino provides both $m_\nu$ and a viable DM candidate.

·) There is a range of other options ( super-light $10^{-21}$ eV scalars - fuzzy DM, gravitinos, Kaluza-Klein states from extra dimensions).

·) Nevertheless (or precisely because), there is no unique universally accepted, canonical theory of DM that would have predicted its existence.

## DM PRODUCTION

Here we will study three popular mechanisms for DM (or even more generally) production in the early universe. This is not an exhaustive list, there are other ways and variants in the literature.

### 1) Thermal FREEZE-OUT

This is the original and perhaps still the most popular mechanism for DM production. We assume a stable particle X with $\tau_X > t_U$ and a weak coupling to the SM,

# DARK MATTER FREEZE-OUT

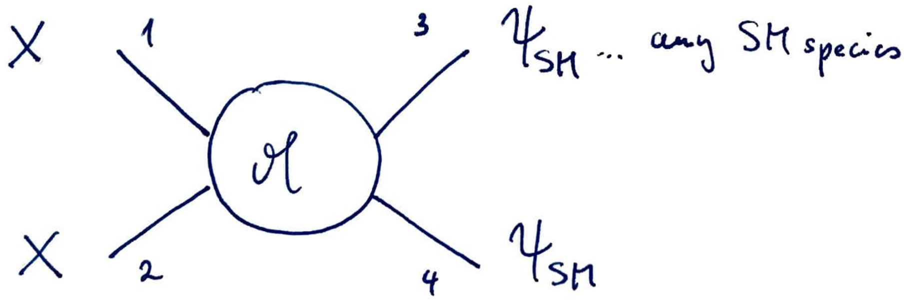
•) Perhaps the most popular mechanism for producing DM in the early universe is the DM freeze-out.

•) Let us assume a cosmologically stable and EM. neutral particle with $m_X \in [\lesssim \text{GeV} - 100\,\text{TeV}]$.

(We will discuss the DM pheno and two other production mechanisms in the next chapter).

•) A $T \simeq 100$ GeV, the SM is in full equilibrium with $g_* \sim 100$ : $e, \mu, \tau, q_i, g, W, Z, h, t \dots$

Likewise, if X interacts (somewhat weakly) with the SM, it too thermalizes.




•) Just like neutrinos (or ionized QED interactions), the rate of annihilation – scattering eventually shuts down.

- Our task here will be to estimate / calculate the moment of X decoupling and the remaining yield of DM.

- Boltzmann eq. for X is

$$a^{-3}\frac{d}{dt}(n_X a^3) = \langle\sigma v\rangle \overbrace{n_1^{(0)} n_2^{(0)}}^{n_X^{(0)2}} \Big[ \underbrace{\frac{n_4^2}{n_4^{(0)2}}}_{=1} - \frac{n_X^2}{n_X^{(0)2}} \Big]$$

$$= \langle\sigma v\rangle \big( n_X^{(0)2} - n_X^2 \big)$$

creation from the SM

pair annihilation

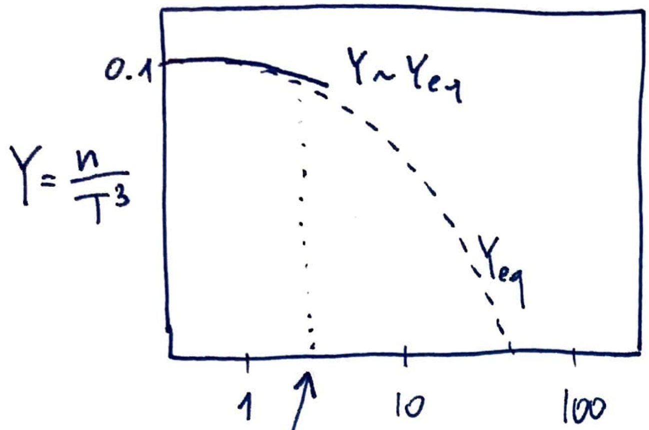
Define: $Y_* = \frac{n_X}{T^3}$ (or more generally $Y = \frac{n}{s}$ if $g_*$ varies)

$$a^{-3}\frac{d}{dt}\big(Y_*(aT)^3\big) = T^3 \frac{dY_X}{dt} = T^6 \langle\sigma v\rangle \big(Y_{EQ}^2 - Y^2\big)$$

$$\underbrace{\frac{dY_X}{dt} = T^3 \langle\sigma v\rangle \big(Y_{eq}^2 - Y^2\big)}$$

- In early times DM thermalizes, $\Gamma \gg H$ and we have to set $Y = Y_{eq}$ for $T \gg m_X$

- To track time, we go to T and rescale by: $\underline{x = \frac{m_X}{T}}$.




$$Y_{eq} = n_x^{(0)} T^{-3} = \frac{1}{\pi^2}\int_0^\infty \frac{y^2\,dy}{e^{\sqrt{x^2+y^2}}\pm 1}$$

•) At this point $T \lesssim 2m_X$, $x\sim 2$, and pair creation (inverse scattering of $\psi\psi\to XX$) becomes inefficient because $Y\sim Y_{eq}$ is becoming Boltzmann suppressed.

•) Without expansion DM would simply annihilate away but because $H\sim T^2$ we will hit $\Gamma\simeq H$ and interaction rate ceases to be effective $\Rightarrow$ FREEZE-OUT.

Let's proceed to calculate. First, we change variables from $t\to T\to x$ in a radiation dominated universe.

$$x \propto T^{-1} \propto a$$

$$\ln x = \ln a + c_0 \quad \Big/ \frac{d}{dt}$$

$$\frac{\dot{x}}{x} = \frac{\dot{a}}{a} = H \Rightarrow \frac{dx}{dt} = Hx$$

•) In radiation dominance $H \propto T^2 \propto x^{-2}$

thus $\frac{H(x)}{H(x=1)} = \frac{1}{x^2} \Rightarrow H = \frac{H(m_x)}{x^2}$

$\Downarrow$

$T = m$

Putting these two together, we have

$$\frac{dY_x}{dt} = \frac{dY_x}{dx}\frac{dx}{dt} = \frac{dY_x}{dx} H(x)\cdot x = \frac{dY_x}{dx}\frac{H(m_x)}{x^{\cancel{2}}}\cancel{\cdot x}$$

$$= T^3 \langle\sigma v\rangle (Y_{eq}^2 - Y^2) = \frac{m_x^3}{x^3}\langle\sigma v\rangle (Y_{eq}^2 - Y^2)$$

or :

$$\boxed{Y' = -\frac{\lambda}{x^2}(Y^2 - Y_{eq}^2)}, \quad \lambda = \frac{m_x^3 \langle\sigma v\rangle}{H(m_x)}$$

•) This is a typical freeze-out equation for a single species annihilating to the SM. The $\lambda$ parameter characterizes the strength of annihilation, the larger it is, the more DM annihilates and less will remain.

•) For many cases $\langle\sigma v\rangle \sim$ const., but we will ~~see~~ refer to works that deal with general cases in the end.

•) <u>Freeze-out temperature</u> can be estimated from the $\Gamma = H$ moment.

For a massive species the number density is $\propto (mT)^{3/2} e^{-m/T}$.
The thermally averaged $\langle \sigma v \rangle$, we take to be of EW scale $\langle \sigma v \rangle \sim \alpha_w^2 / M_w^2$ and also $m_{DM} \gtrsim M_W$

Then : $$\Gamma = n_x^{(0)} \langle \sigma v \rangle \simeq \left( \frac{m_x T}{2\pi} \right)^{3/2} e^{-m_x/T} \alpha_w^2 m_x^{-2}$$

$$\simeq 10^{-4} m_x x^{-3/2} e^{-x}$$

Hubble rate in radiation domination is given by

$$H \cong \sqrt{\frac{8\pi g_*(m_x)}{3}} \frac{T^2}{M_{Pe}} \simeq 10^2 \frac{m_x^2}{M_{Pe}} x^{-2}$$

Freeze-out happens when

$$\frac{\Gamma}{H} = 1 = 10^{-6} \frac{\cancel{m_x} M_{Pe}}{m_x^{\cancel{2}}} x^{+1/2} e^{-x}$$

or $$e^x x^{-1/2} = 10^{-6} \frac{M_{Pe}}{m_x} \simeq 10^{10} \qquad \boxed{x_f \sim 20}$$

•) With the freeze-out $T_f$ at hand, we can proceed to calculate the <u>final abundance of DM.</u>

$$@\, x \sim 1 \qquad \frac{dY}{dx} \sim Y \ll \lambda \left( Y^2 - Y_{eq}^2 \right)$$

remember: $\lambda \simeq \dfrac{m_x^3 \langle \sigma v \rangle}{H(m_x)} \simeq \dfrac{m_x^3 \alpha_w^2 m_x^{-2} M_{Pl}}{10^2 m_x^2} \sim 10^{-4} \dfrac{M_{Pl}}{m_x} \sim 10^{12}$

in any case $\lambda \gg 1$ so $Y^* = Y_{eq}$ when $x \lesssim 1$.

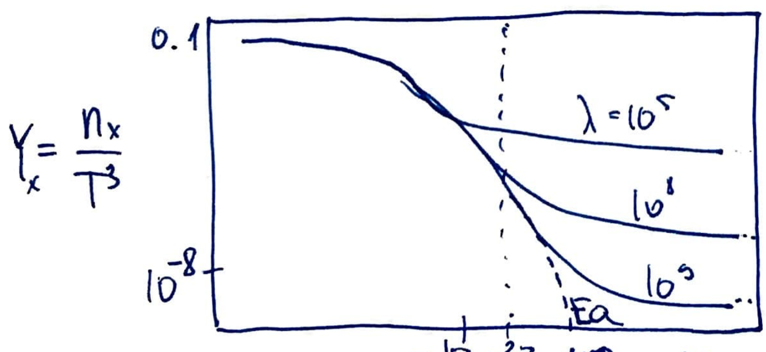
•) When $x$ goes up $Y_{eq} \to 0$ but $Y$ stays $\gg Y_{eq}$ because we are nearing $\Gamma \sim H$, so we have

$$Y' = \frac{dY}{dx} \cong -\frac{\lambda}{x^2} Y^2 \Rightarrow \frac{dY}{Y^2} = -\lambda \frac{dx}{x^2}$$

$$\Rightarrow Y^{-1} \Big|_{x_f}^{x_\infty} \simeq -\frac{\lambda}{x} \Big|_{x_f}^{x \to \infty} \Rightarrow Y_\infty^{-1} = Y_f^{-1} - \underbrace{\frac{\lambda}{\infty}}_{=0} + \frac{\lambda}{x_f}$$

$$Y_f > Y_\infty$$

$$\Rightarrow \boxed{Y_\infty \simeq \frac{x_f}{\lambda}}, \quad x_f \sim 20$$




• this estimate agrees well with the numerics, see the plot

$$Y_\infty \simeq \frac{x_f}{\lambda}$$

•) DARK MATTER energy density parameter $\Omega_{DM}$ can now be calculated / estimated fairly easily. We just have to follow the expansion of the universe from $T_1 \gtrsim T_f$ down to $T_\gamma^{CMB} = T_0$ or today, and calculate $\Omega_{DM} = \frac{\rho_{DM}}{\rho_{cr}} = \frac{m_\chi n_\chi}{\rho_{cr}}$

@ $T_f$ : $n_{\chi f} = Y_\infty T_1^3$ , $T_1 \gtrsim T_f$

today : $n_\chi = n_{\chi f} \left( \frac{a_1}{a_0} \right)^3 = Y_\infty T_0^3 \left( \frac{a_1 T_1}{a_0 T_0} \right)^3$

$$= Y_\infty T_0^3 \frac{g_{*s}(T_0)}{g_{*s}(T_1)} \simeq \frac{1}{30} Y_\infty T_0^3$$

•) This gives us

$$\Omega_{DM} = \frac{m_\chi}{\rho_{cr}} \overbrace{Y_\infty}^{\simeq \frac{x_f}{\lambda} \sim \frac{x_f H(m_\chi)}{m_\chi^3 \langle \sigma v \rangle}} T_0^3 \frac{1}{30}$$

$$H^2(m_\chi) = \frac{8\pi G}{3} \rho_\gamma(m_\chi) = \frac{8\pi G}{3} \frac{\pi^2}{30} g_*(m_\chi) m_\chi^4$$

DM mass cancels out

$$\Omega_{DM} = \frac{\cancel{m_\chi}}{\rho_{cr}} \frac{x_f}{\cancel{m_\chi^3} \langle \sigma v \rangle} \left( \frac{4\pi G}{45} g_*(m_\chi) \right)^{1/2} \cancel{m_\chi^2} T_0^3 \frac{1}{30}$$

$$\Omega_{DM} = \left(\frac{4\pi^3 g_*}{45 M_{Pl}^2}\right)^{1/2} \frac{x_f T_0^3}{30 f_{cr} \langle\sigma v\rangle}$$

$$\Omega_{DM} h^2 \simeq 0.1 \left(\frac{x_f}{20}\right)\left(\frac{g_*}{100}\right)^{1/2} \frac{10^{-26}\,\frac{cm^3}{s}}{\langle\sigma v\rangle}$$

•) Note that the DM mass basically disappeared and the $\langle\sigma v\rangle$ is of typical EW size

$$10^{-26}\,\frac{cm^3}{s} = 10^{-26} \cdot 0.3 \cdot 10^{-6}\,(10^{13}\,fm)^2 = 3 \cdot 10^{-7}\,fm^2$$

$$= 3 \cdot 10^{-7} \cdot 4 \cdot 10^{-2}\,GeV^{-2} \sim 10^{-8}\,GeV^{-2} \sim \underline{\underline{\frac{\alpha_W^2}{M_W^2}}}.$$

This is why some people refer to this scenario as the WIMP miracle. For the typical EW $\langle\sigma v\rangle$, the DM abundance comes out just right.

•) For a more detailed treatment of $\langle\sigma v\rangle$, see the seminal Gondolo & Gelmini '93 paper.

•) For exceptions (co-annihil., mass thr., up-scatter) [Griest, Seckel '91]

•) Refined analytics for $x_f$ and $Y_\infty$ [1204.3622]

# $\underline{\text{FREEZE-OUT SUMMARY}}$

i) Assume DM in equilibrium at high $T$, early $t$.

ii) Instead of complete annihilation, expansion prevents the equilibrium's Boltzmann suppression and $Y_\chi$ gets stuck near $x_f \sim 20$, i.e. at $T_f \sim \frac{m_\chi}{20}$.

iii) The final abundance is independent of $m_{DM}$ and scales as $\frac{x_f}{\lambda}$ so $\propto \frac{x_f}{\langle \sigma v \rangle}$.

Estimate procedure

1) Boltzmann eq. for MB statistics, SM in eq.

$$n_\chi = Y_\chi s \,, \qquad x = \frac{m_\chi}{T}$$

2) Get the $T_f$ via $\Gamma = H \Big|_{x_f} \Rightarrow x_f \sim 20$

3) At $x_f$ $\; Y \gg (Y_{eq} \sim 0) \Rightarrow Y_\infty \simeq \frac{x_f}{\lambda}$

4) Rescale $n_{\infty} \to n_\chi$ today $\Rightarrow \Omega_{DM} = \dfrac{m_\chi n_\chi (T_0)}{\rho_{cr}}$.