

FREEZE-IN

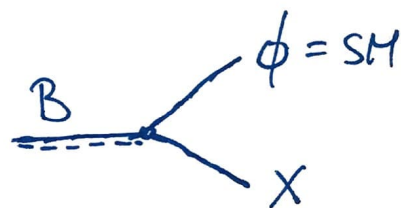
[0911.1120]

-) The freeze-in mechanism considers a very weakly coupled particle X , which is coupled to the heat bath B via λ . Suppose we have a "heavy" particle in the bath B , which then decays to X & the SM ϕ .

TOY
MODEL

e.g.:
DECAY
ONLY

$$\lambda \bar{X} B \phi$$



$$\Gamma_B = \frac{\lambda^2}{8\pi} m_B$$

-) Naive estimate: $\frac{dn}{n} \sim \Gamma dt$, $H \propto T^2 \sim \frac{1}{t}$ for rad. dom.

$$Y_X = \frac{N_X}{S} \approx \gamma_B \Gamma_B^0 \cdot t \approx \frac{m_B}{T} \Gamma_B^0 \frac{1}{H} \approx \frac{m_B \Gamma_B^0}{T^3} \approx \frac{(m_B \lambda)^2}{T^3}$$

-) The yield of X depends on m_B & λ and shuts off at $T \rightarrow \infty$, dominated by low T .

•) A bit more rigorously:

$$a^{-3} \frac{d(n_x a^3)}{dt} = \int_{\pi_{x \neq x}} |\mathcal{M}|^2 f_B$$

we neglected the
"inverse decay" due
to the lack of X in
the plasma and
($1 \pm f_x$) \rightarrow 1, ($1 \pm f_{sm}$) \rightarrow 1

$$\approx 2g_B \int_{\pi_B} \Gamma_B m_B f_B$$

$$= 2g_B \int \frac{d^3 p_B}{(2\pi)^3} \chi_{E_B} m_B \Gamma_B f_B \quad (\text{dropping the index, all B})$$

$$= g_B \int_0^\infty \frac{4\pi^2}{8\pi^3} p^2 dp \frac{e^{-E/T}}{E} m \Gamma$$

$$p dp = E dE$$

$$= \frac{g m \Gamma}{2\pi^2} \int_m^\infty \sqrt{E^2 - m^2} e^{-E/T} dE$$

$$\frac{E}{m} = xy, \quad x = \frac{m}{T}$$

$$\left[dE = T dx \right. \\ \left. = \frac{g m \Gamma}{2\pi^2} \int_{\frac{m}{T}}^\infty \sqrt{x^2 - \left(\frac{m}{T}\right)^2} e^{-x} dx T^2 \right]$$

$$* K_n(x) = \frac{\sqrt{\pi}}{(n-\frac{1}{2})!} \left(\frac{x}{2}\right)^n \\ \int_1^\infty e^{-xy} (y^2-1)^{n-1/2} dy$$

$$= \frac{g m \Gamma}{2\pi^2} \int_1^\infty m \sqrt{y^2-1} e^{-xy} dy m$$

$$= \frac{g m^3 \Gamma}{2\pi^2} \frac{K_1(x)}{x} = \frac{g m^2 \Gamma}{2\pi^2} T K_1\left(\frac{m}{T}\right)$$

•) We thus have :

$$a^{-3} \frac{d(n_x a^3)}{dt} = \frac{g m^2 \Gamma}{2\pi^2} T K_1\left(\frac{m}{T}\right)$$

Introducing the usual yield : $Y = \frac{n}{s}$, $s = \frac{2\pi^2}{45} g_{\text{rs}} T^3$

and

$$a^{-3} \frac{d(Y_x s a^3)}{dt} = s \frac{dY_x}{dt} = s \frac{dY_x}{dT} \cdot \frac{dT}{dt}$$

radiation : $H = \frac{1}{2t}$, ($H^2 \propto a^{-4}$, $a da = dt$, $a \propto \sqrt{t}$, $\frac{da}{a} = \frac{1}{2} \frac{dt}{t}$)

$$H \propto T^2 = \frac{1}{2t} \Rightarrow -\ln t = 2 \ln T$$

$$-\frac{dt}{t} = 2 \frac{dT}{T} \Rightarrow \frac{dT}{dt} = -HT$$

$$s \frac{dY_x}{dT} (-HT) = \frac{g m^2 \Gamma}{2\pi^2} T K_1\left(\frac{m}{T}\right)$$

$$Y_x = \frac{g m^2 \Gamma}{2\pi^2} \int_{T_{\min}}^{T_{\max}} \frac{K_1\left(\frac{m}{T}\right)}{s H} dT, \quad s = \frac{2\pi^2}{45} g_{\text{rs}} T^3$$

$$= \frac{45}{1.7 \cdot 4\pi^4} \frac{g \Gamma M_{\text{pe}}}{m^2 g_{\text{rs}} \sqrt{g_x}} \int_{x_{\min}}^{x_{\max}} K_1(x) x^3 dx$$

$$\approx \frac{135}{1.7 \cdot 8\pi^4} \frac{g_{\text{B}}}{g_{\text{rs}} \sqrt{g_x}} \left(\frac{M_{\text{pe}} \Gamma_{\text{B}}}{m^2} \right) \int_0^{\infty} K_1 x^3 dx = \frac{3\pi}{2}$$

•) The final abundance is

~~entropy density~~
entropy density today

$$\Omega_x h^2 = \frac{\rho_x}{\rho_{cr}} = \frac{m_x n_x}{\rho_{cr}} = \frac{m_x Y_x S_0}{\rho_{cr}}$$

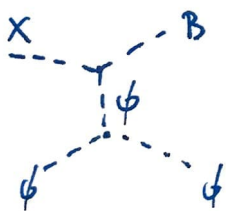
$$= \frac{10^{27}}{g_* s \sqrt{g_*}} \left(\frac{m_x \Gamma_B}{m_B^2} \right) \frac{3 H_0^2}{8\pi G}$$

•) To get the observed $\Omega_{DM} \sim 0.25$

$$\lambda \sim 10^{-13} \left(\frac{m_B}{m_x} \right)^{1/2} \left(\frac{g_*(M_B)}{100} \right)^{3/4} \left(\frac{g_B}{10^2} \right)^{-1/2}$$

•) In actual scenarios, one has to be careful

• to include other interactions and scatterings



+ ... that may affect the picture.

Thermal freeze-in of sterile neutrinos

-) A well-motivated scenario for providing a DM candidate and neutrino masses, is the sterile neutrino ν_s . ν_s is a neutral fermion, which is not coupled to the SM by gauge interactions, only via Yukawa.

$$L = L_{SM} + y_D \bar{L} \Phi \nu_s + M_s \underbrace{\nu_s^T C \nu_s} \quad L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \\ \phi = \begin{pmatrix} 0 \\ \nu \end{pmatrix}$$

in contrast to charged fermions, we can write down the Majorana mass term for ν_s . This is a Lorentz invariant term, which can be $M_s \gg M_w$.

$$\Rightarrow L_{mass} \ni (\bar{\nu}_L \quad \bar{\nu}_s^c) \begin{pmatrix} 0 & M_D \\ M_D & M_s \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_s^* \end{pmatrix}$$

(6x6 for 3 generations)

From the 2x2 mass matrix, we get

$M_s \sim M_s \dots$ the mass of ν_s

$M_\nu \sim -m_D^T M_s^{-1} m_D \dots$ masses of light neutrinos.

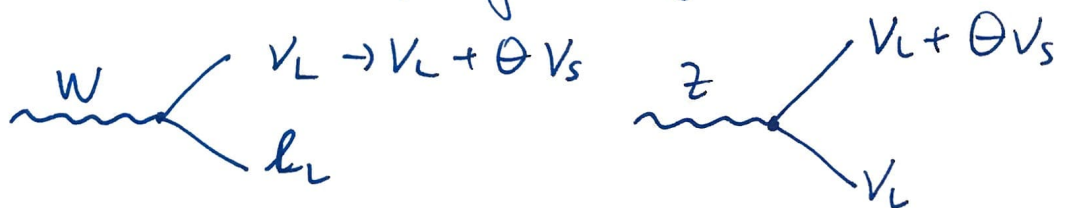
-) The important point is that ν_s is very weakly coupled and is not thermalized in the early universe. After spontaneous breaking of $SU(2)_L$, when $\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$ we have to rotate ν_L, ν_s into the mass basis:
- $$\begin{aligned} \nu_L^{(m)} &\sim \nu_L + \Theta \nu_s \\ \nu_s^{(m)} &\sim \nu_s - \Theta \nu_L \end{aligned} ; \quad \Theta \sim \frac{m_D}{M_S}$$

The mixing angle is very small $\Theta \sim \sqrt{\frac{m_D^2}{M_S M_S}} \sim \sqrt{\frac{m_\nu}{M_S}}$

e.g. for $m_\nu \sim 10^{-4} \text{ keV}$ and $M_S \sim \text{keV} \sim 10^{-6} \text{ GeV}$

$$\Theta \sim \left(\frac{10^{-4}}{10^3} \right)^{1/2} \approx 10^{-4} = \text{tiny}$$

-) Now we have small coupling to gauge bosons as well:

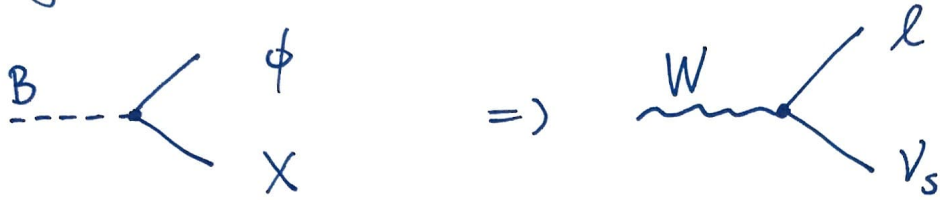


i.e. $\propto \frac{g_L}{\sqrt{2}} \cdot \Theta \bar{l}_L \gamma^\mu \nu_s W_{L\mu}$

This leads to: $\langle \sigma v \rangle n_s \sim G_F T^2 \Theta^2 T^3 \sim G_F T^5 \Theta^2$

-) As in the freeze-out case above, the Y_{ν_s} starts to accumulate slowly as the temperature drops.

Using the same formalism as above



$$\Omega_{\nu_s} h^2 = \frac{10^{27}}{g_{*S} \sqrt{g_{*L}} \frac{M_s}{M_w}} \frac{M_s \Gamma_{W \rightarrow \nu_s e}}{M_w^2}$$

$$\Gamma_{W \rightarrow \nu_s e} = \frac{\alpha_w}{4} \theta^2 M_w, \quad g_{*L}(M_w) \sim 100$$

$$\Omega_{\nu_s} h^2 = 0.1 \left(\frac{M_{\nu_s}}{\text{keV}} \right) \left(\frac{\theta}{10^{-7}} \right)^2$$

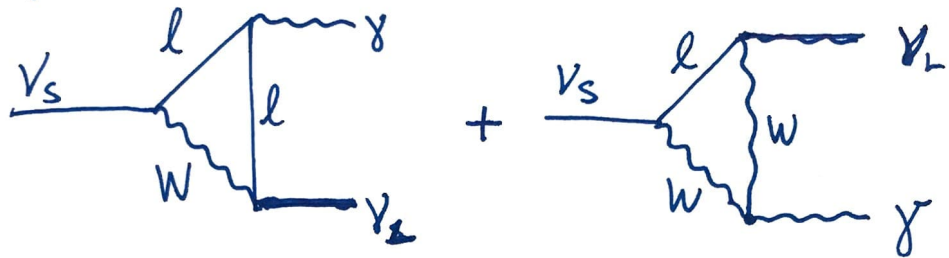
- There are important corrections in this scenario, such as ν -oscillations, T -dependent potentials

$$\theta_0 \rightarrow \theta(T) \approx \frac{\theta_0}{1 + \left(\frac{T}{T_0}\right)^6}, \quad T_0 \approx 100 \text{ MeV}$$

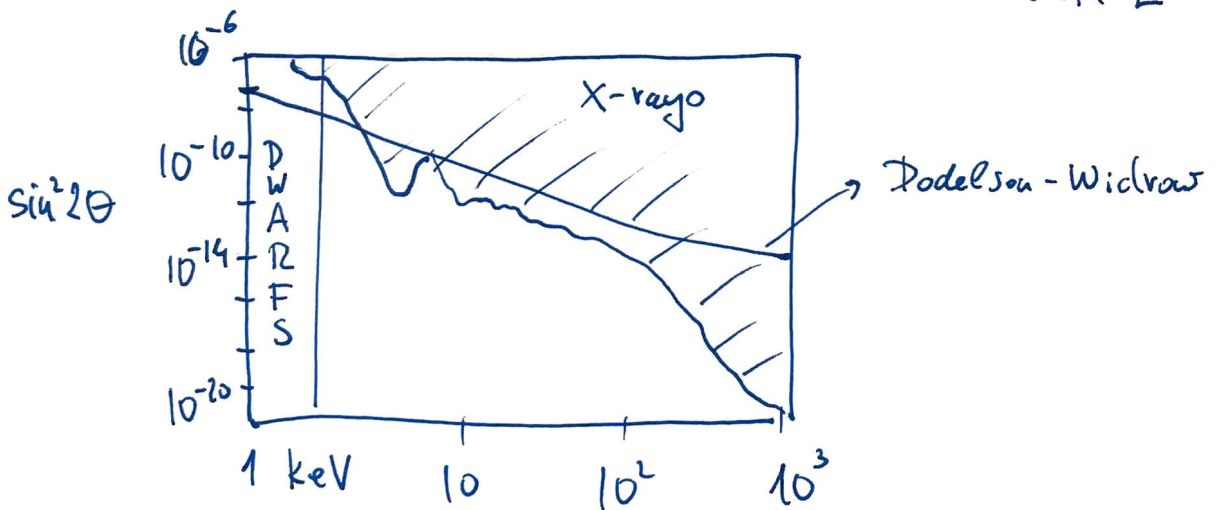
Resonant production, refined estimate

$$\Omega_{\nu_s} h^2 = 0.1 \left(\frac{\theta^2}{3 \cdot 10^{-9}} \right) \left(\frac{M_{\nu_s}}{3 \text{ keV}} \right)^{1.8}$$

-) This is referred to as the Dodelson-Widrow mechanism. It is strongly constrained by indirect searches. Because of the mixing, ν_s couples to the SM and is destabilized at one loop level: $\nu_s \rightarrow \gamma \nu_L$, $E_\gamma \sim m_{\nu_s}/2$



$$\tau = \frac{1}{\Gamma} = 10^{30} \text{ s} \left(\frac{10^{-7}}{s^2} \right) \left(\frac{\text{keV}}{m_{\nu_s}} \right)^5 \quad \Downarrow \quad \begin{array}{l} \text{X-ray} \\ \text{with } E \sim \text{keV} \end{array}$$

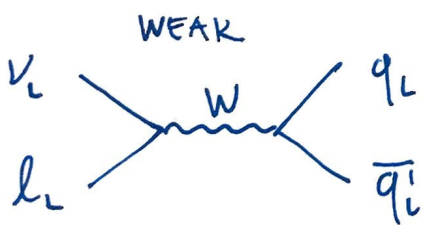


-) Presence of primordial lepton asymmetry or additional interactions open up the allowed parameter space [1310.04901]

Thermal over-production and entropy dilution

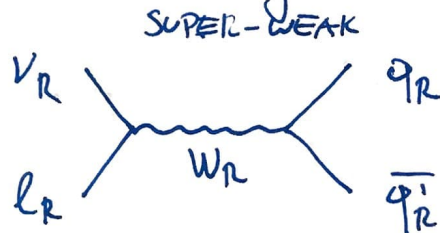
-) Let us re-consider the freeze-out of a massive but relativistic species. This is very similar to the SM neutrinos decoupling but via BSM (weaker) interactions. A somewhat generic example is a W_R with $M_{W_R} \gg M_W \sim \text{few TeV}$ that couples to L_R, Q_R . This is motivated by parity restoration at high scales and necessarily contains the V_R , which we take $M_{V_R} \sim \text{keV}$.

-) Relativistic thermal freeze-out goes as in the SM.



$$\Gamma \sim G_F^2 T^5 = \frac{\sqrt{g_*} T^4}{M_{\text{Pe}}}$$

$$\Rightarrow T_{\text{dec}} \cong g_*^{1/6} \left(\frac{M_W^4}{M_{\text{Pe}}} \right)^{1/3}$$



$$G_F \propto \frac{1}{M_W^2} \rightarrow G_F^1 \propto \frac{1}{M_{W_R}^2}$$

$$G_F^2 \left(\frac{M_W}{M_{W_R}} \right)^4 T^3 = \frac{\sqrt{g_*}}{M_{\text{Pe}}}$$

$$T_f \sim 0.4 \text{ GeV} \left(\frac{M_{W_R}}{5 \text{ TeV}} \right)^{4/3}$$

•) With suppressed interactions, the freeze-out happens earlier $T_f \sim 0.4 \text{ GeV}$, $g_*(T_f) \sim 70$.

Earlier decoupling means higher temperatures

$T_f \sim \text{GeV}$ and ~~too~~ less Boltzmann suppression.

•) For $m_{\nu_R} \sim \text{keV} \ll T_f \sim 0.4 \text{ GeV}$, the number density is given by the relativistic formula

for $m \ll T$

$$n_{\nu_R} \sim g \frac{3}{2\pi^2} \zeta(3) T_f^3$$

$$Y_{\nu_R} = \frac{n_{\nu_R}}{s} = \frac{135 \zeta(3)}{4\pi^4 g_*(T_f)} \quad , \quad s = \frac{2\pi^2}{45} g_{\text{rel}} T^3$$

•) As for the usual SM neutrinos, the ν_R simply cools down and the energy density is

$$\rho_{\nu_R} = M_{\nu_R} n_{\nu_R} = M_{\nu_R} Y_{\nu_R} s$$

where $s_{\text{today}} = \frac{2\pi^2}{45} \left(2 + 2 \cdot 3 \cdot \frac{7}{8} \left(\frac{T_\nu}{T_\gamma} \right)^3 \right) T_\gamma^3 = \frac{2\pi^2}{45} \frac{43}{11} T_\gamma^3$

$$2 + \frac{21}{4} \frac{4}{11} = \frac{43}{11} \approx 3000 \text{ cm}^{-3}$$

- E - 6 - 31 - [PDG '21]

•) The final energy density parameter is then given by

$$\Omega_{\nu_e} \approx 3.3 \left(\frac{m_{\nu_n}}{\text{keV}} \right) \left(\frac{70}{g_*(T_f)} \right)$$

OVER-ABUNDANT by

$$\frac{\Omega_{\nu_n}}{\Omega_{DM}} \sim \frac{3}{0.25} \sim 12.$$

phase-space bound

•) This is quite similar to the SM with

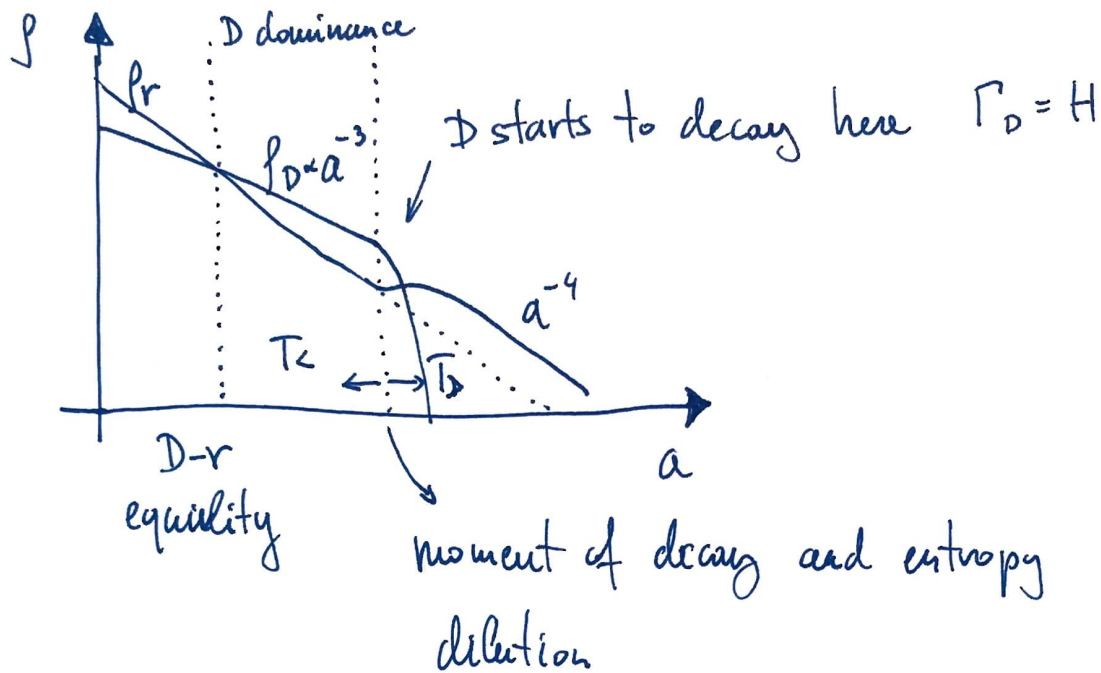
$$\Omega_{\nu_i} \approx \frac{\sum m_{\nu_i}}{93 \text{ eV}}$$

ENTROPY DILUTION BY LATE DECAYS [book by Kolb & Turner]

[Scherrer-Turner '85]

•) The issue of over-abundance (similar to the monopole problem) can be addressed by entropy production. This may happen when a massive species is long-lived enough to dominate the $f_m > f_r$ and then decays to final states that thermalize and go into f_r .

•) Let us refer to the massive species as the dilutor D with mass m_D & decay rate Γ_D .



•) Before the decay, D is matter and dominates

$$H^2 = \frac{8\pi G}{3} \rho_m = \frac{8\pi G}{3} Y_D m_D \rho = \frac{8\pi}{3H_{pe}^2} Y_D m_D \frac{2\pi^2}{45} g_{*S} T^3$$

$$= \Gamma_D^2 \Rightarrow T_< \sim \left(\frac{\Gamma_D^2 M_{pe}^2}{Y_D m_D g_{*S}} \right)^{1/3}$$

•) After D decays into radiation, its $\rho_m \rightarrow \rho_r$ and the universe is again radiation-dominated.

$$\Gamma_D = H \approx 1.7 \sqrt{g_*} \frac{T_>^2}{M_{pe}} \quad \text{or} \quad T_> = \sqrt{\frac{\Gamma_D M_{pe}}{g_*^{1/2}}}$$

•) The ratio between these two is given by

$$\frac{T_{>}}{T_{<}} \approx \frac{\sqrt{\Gamma M_{pe}}}{(\Gamma M_{pe})^{2/3}} \frac{(Y_D \omega_D g_*)^{1/3}}{g_*^{1/4}} = \left(\frac{g_*^{1/4} Y_D \omega_D}{\sqrt{\Gamma M_{pe}}} \right)^{1/3}$$

Because the entropy goes as T^3 , the dilution factor is given by

$$\frac{S_{>}}{S_{<}} = \frac{g_*^{1/4} Y_D \omega_D}{\sqrt{\Gamma M_{pe}}}$$

•) Of course, this entropy is associated only to the SM particles (radiation), we assumed that D does not decay to ν_R . Thus the energy density $f_{\nu_R} = \omega_{\nu_R} N_{\nu_R}$ is diluted by precisely this factor. The takeaway message is that late decays heat up the radiation and effectively dilute the n of any particles that don't get produced by their decays.

•) SUMMARY: The amount of dilution depends on

* $Y_D m_D$: the more abundant Y_D is (e.g. if produced relativistically) the more it dilutes. Also, the higher its mass, the more of it carries and heats up radiation more.

* $\propto \Gamma_D^{-1/2}$ or $\tau_D^{1/2}$: the longer it lives, the more it dominates and the more effective the energy transfer = higher dilution.

Thermal overproduction

Entropy dilution

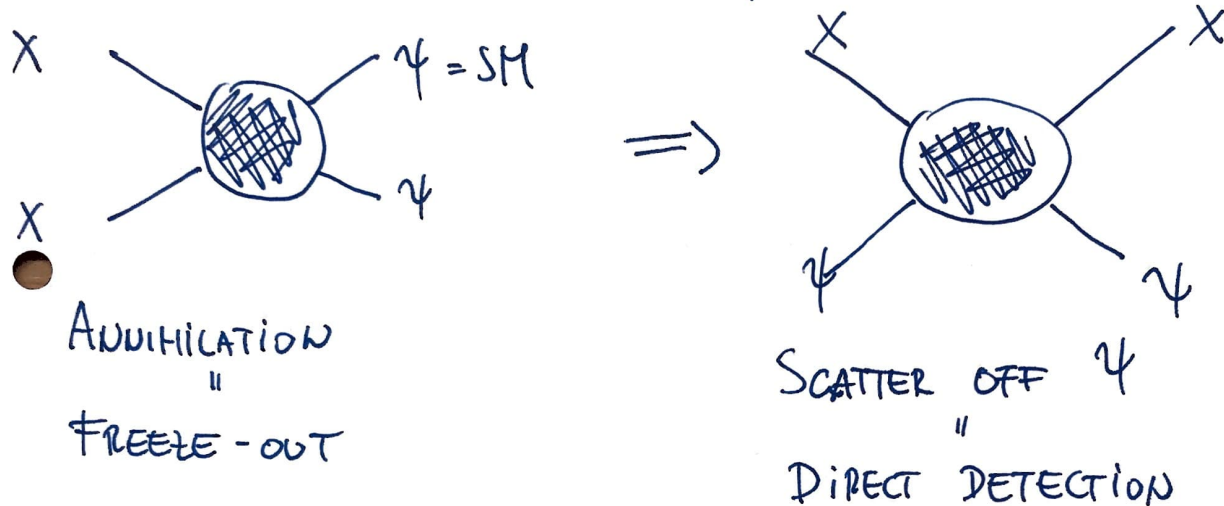
$$\Omega_{\nu_e} \geq 3 \left(\frac{m_{\nu_e}}{\text{keV}} \right) \implies \Omega_{\nu_e} \sim 0.23 \left(\frac{m_{\nu_e}}{\text{keV}} \right) \left(\frac{2 \text{ GeV}}{m_D} \right) \left(\frac{1 \text{ s}}{\tau_D} \right)^{1/2}$$

•) We will see, however, that the lifetime of D cannot be arbitrarily long $\tau_D \leq 1 \text{ sec}$, otherwise it spoils the successful prediction of BBN in the SM.

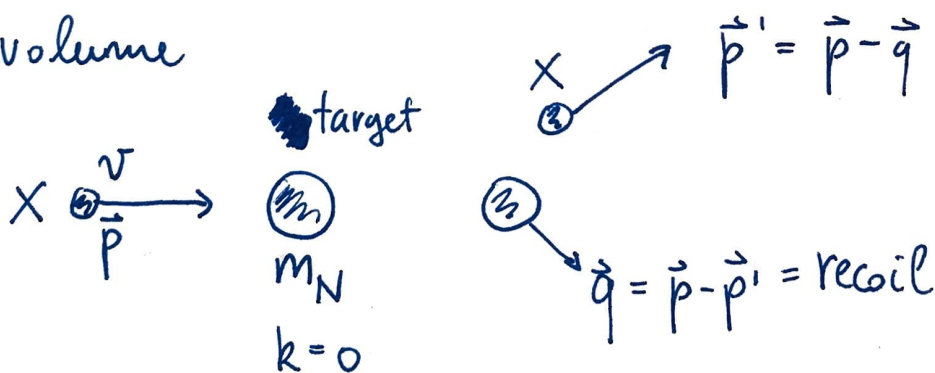
DIRECT DETECTION OF DARK MATTER

[1904.07915]

-) A typical WIMP scenario with $\langle \sigma v \rangle \sim 10^{-8} \text{ GeV}^{-2}$ is an ideal candidate for direct detection.



-) The incoming DM with $v \sim 200 \text{ km/s}$ or $v/c \sim 10^{-3}$ can scatter off a nucleus sitting at rest in a target volume



-) The whole process is non-relativistic and thus

$$E_i = \frac{p^2}{2m_X}, \quad E_f = \frac{(\vec{p} - \vec{q})^2}{2m_X} + \frac{q^2}{2m_N}$$

$$p = m_X v$$

•) Energy conservation will give us the max. recoil

$$E_f = E_{fi} \Rightarrow \frac{p^2 - 2pq c_\theta + q^2}{2m_x} + \frac{q^2}{2m_N} = \frac{p^2}{2m_x}$$

$$\Rightarrow \frac{pq c_\theta}{m_x} = \frac{q^2 (m_x + m_N)}{2m_x m_N} = \frac{q^2}{2\mu}$$

Now since $|c_\theta| < 1$

reduced DM-mass

$$q_{\max} = \frac{2p\mu}{m_x} \approx 2v\mu \approx 10^{-3} \times (10-100) \text{ GeV} \quad m_N \sim \text{Amp}$$

Maximum momentum transfer between $X = \text{DM} \& \text{N}$.

•) From here, we get the largest recoil energy $E_{R\max}$

$$E_{R\max} = \frac{q_{\max}^2}{2m_N} = \frac{2v^2 \mu^2}{m_N} \sim 10^{-6} \frac{m_x^2 m_N}{(m_x + m_N)^2} \sim (10-100) \text{ keV}$$

$$\text{and: } v_{\min} = \sqrt{\frac{E_R m_N}{2\mu^2}}$$

for a fixed measured E_R , this is the smallest v that

DM can have to trigger a signal. Remember that

v follows a Maxwell distribution and has suppressed high v tails $\propto e^{-(v/v_0)^2}$, $v_0 \sim 220 \frac{\text{km}}{\text{s}}$.

•) Furthermore, note that the momentum transfer

$$q \sim (10-100) \text{ keV} \Rightarrow q^{-1} \sim \lambda \sim (1-10) \text{ fm},$$

which implies we are scattering DM on an entire nucleus, not individual p, n with $m_p \sim 1 \text{ GeV}$.

•) The final number that we wish to measure is the rate of scattering / time / E_2 : $\frac{dR}{dE_2}$. To calculate

that, one has to go through a series of theories,

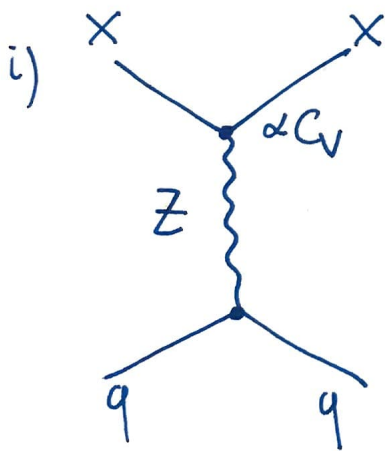
none of which we can analyze in depth here

(assuming QFT is not an option). So we will

• sketch the procedure in enough detail to appreciate the progress of direct detection exp.

DIRECT DETECTION THEORY LADDER

- i) Start with \mathcal{L}_{SM+DM} ; q 's, l 's, W, Z, X (QCD)
- ii) Match the theory to $\mathcal{L}_{p,n}$ = theory for non-relativistic neutrons, protons $q \sim 1$ -to 10 keV $\ll m_p$
- iii) Calculate the amplitude for X - n couplings
- iv) Compute the final X -nucleus N cross-section by averaging over the local DM velocity distribution.



strength of the DM-X, $g_2^2 C_V$ or $C_V c c^2$

$$\frac{C_V}{M_Z^2} \bar{X} \gamma^\mu X \bar{q} \gamma^\mu q \dots \text{vector coupling}$$

$$\frac{C_A}{M_Z^2} \bar{X} \gamma^\mu \gamma^5 X \bar{q} \gamma^\mu \gamma^5 q \dots \text{axial coupling}$$

spin-independent
spin-dependent

ii) This is now matched to n, p :

$$\langle n(k') | \bar{q} \gamma^\mu q | n(k) \rangle = \bar{u}_n(k') (F_1^n(q^2) \gamma^\mu + \frac{i}{2m_n} F_2^n(q^2) \sigma^{\mu\nu} q_\nu) u_n(k)$$

$$u_n \rightarrow \sqrt{m_n} \begin{pmatrix} \xi \\ \zeta \end{pmatrix}$$

$$\bar{u}_n \gamma^0 u_n \rightarrow$$

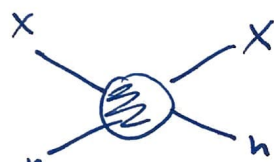
i) The $F_{12}(q^2)$ are the nucleon form-factors, just like the $g-2$ in QED, for example. These can be computed in the heavy baryon XPT.

iii) Taking the NR limit for both X ($v \sim 10^{-3}$) and for the n, p , the 2 diagrams gives

$$\mathcal{M} = \frac{C_V}{M_Z^2} m_X m_n \underbrace{\left\{ \begin{array}{c} \psi_X^\dagger \psi_X \\ \psi_n^\dagger \psi_n \end{array} \right\}}_{\text{spinors for } X, n} \Rightarrow \mathcal{O}(1)$$

$$|\overline{\mathcal{M}}|^2 = \left(\frac{3C_V}{M_Z^2} \right)^2 (4m_X m_n)^2$$

↳ this is now integrated over the final 2-body phase-space of X and n



$$d\Gamma_n = \frac{1}{4m_X m_n v} \int_{\Pi_{X,n}} |\overline{\mathcal{M}}|^2 (2\pi)^4 \delta^{(4)}(\sum p) \quad , \quad \int_{\Pi_X} = \int \frac{d^3 p_X}{2E_X (2\pi)^3} \Big|_{m_X}$$

$$\Gamma_n = \frac{\mu^2}{\pi} \left(\frac{3C_V}{M_Z^2} \right)^2$$

iv) Finally, we go from nucleus n, p to nuclei

$N = \text{Xe, Ar, Si, Ge, ...}$ with $A \sim 70-100$.

•) The differential σ is then given by
with $d|q|^2 = 2\mu_N dE_R$

$$\frac{d\sigma_N}{dE_R} = \frac{\sigma_N M_N}{2(\mu v)^2} F^2(q) \Theta(v - v_{min}) \quad \text{known form-factor (Helium)}, \quad F^2 \sim A^2$$

$$\Downarrow$$

$$\frac{dR}{dE_R} = N_T n_x \int \frac{d\sigma_N}{dE_R} v f(v) d^3v$$

$$\parallel$$

$$= N_T \frac{\rho_x}{m_x} \frac{\sigma_N M_N}{2\mu^2} F^2(q) \underbrace{\int \frac{f}{v} \Theta(v - v_{min}) d^3v}_{g(v_{min})}$$

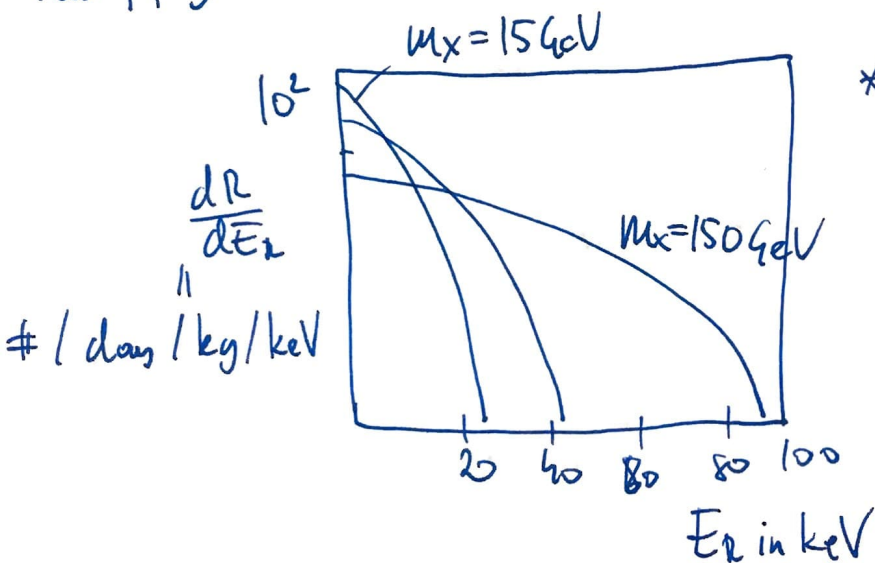
of nuclei / man

~ Avogadro x mass number

•) $g(v_{min}) \sim e^{-m_N E_R / 2\mu^2 v^2}$, $v_0 \sim 220 \text{ km/s}$

•) Helium f.f. \uparrow

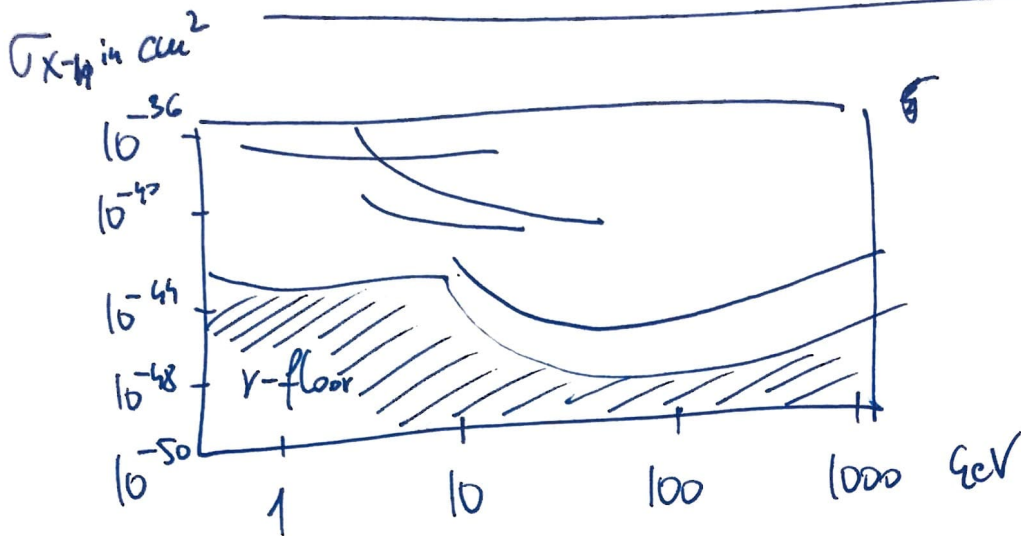
$$F(q) = \frac{3}{qr_n} J_1(qr_n) e^{-(qs)^2/2}, \quad r_n \sim f_n A^{1/3}, \quad s \sim f_n$$



* the rate is exponentially suppressed for higher E_R
 $g(v_{min}) \propto e^{-E_R}$

* better for low M_x , $v_{min} \propto \frac{1}{M_x}$

SUMMARY OF DIRECT DETECTION EXP.



* At low m_χ , the recoil is too small: $E_R \ll \text{keV}$ to be detectable by searches with $E_R^{\text{thr}} \sim \text{few keV}$.

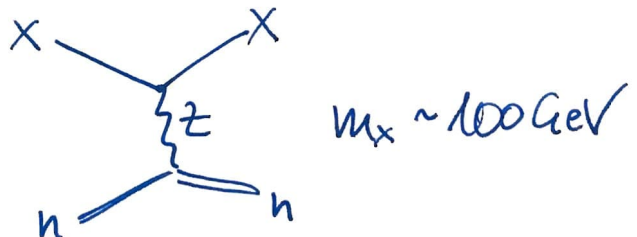
$$\propto \mu^2 \propto m_\chi^2$$

x) At high m_χ , the number density n_χ goes down as m_χ^{-1} , $f_\chi = m_\chi n_\chi = \text{fixed by } \Omega_{\text{DM}}$.

x) How strong are these limits?

taking $C_V \sim g_2 \sim 0.65$ or $O(1)$, we have

$$\sigma_n^V \approx 10^{-37} \text{ cm}^2$$



$$m_\chi \sim 100 \text{ GeV}$$

This is way excluded \Rightarrow needs $C_V \ll 1$, however this affects the freeze-out and WIMP "miracle" starts to fade.

INDIRECT DETECTION

[1710.05137]

-) The idea behind indirect detection is to look for visible products of DM decay or annihilation.

The final states are SM particles such as $\gamma, \nu, e, W, Z, p, n, \dots$).

- The upshot for such searches is that the DM energy density is HUGE $\Omega_{DM} \approx 5\Omega_B$ so there is plenty of DM to produce a signal.

The downsides are:

- i) DM might be very weakly coupled (perhaps even only gravitationally)
- ii) There are many astrophysical backgrounds and uncertainties (like pulsars), which make it very hard to claim a signal from DM only.

Neutral final states propagate basically unimpeded (almost) like γ 's and ν 's. $\Rightarrow (\theta, \varphi, r)$

- E - 6 - 43 - from the \uparrow redshift

-) Charged final states (e^\pm, p^\pm , other cosmic) get deflected by ~~astro~~ intergalactic B-fields. This makes the analysis more involved (there are tools for this, like GALPROP) and harder to pinpoint the origin of the DM signal.

ESTIMATES of number of events @ earth

i) from DECAYS

ii) from ANNIHILATIONS

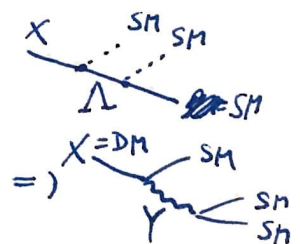
i) As we saw in the case of ν_s , DM may decay

A very generic estimate with $O(1)$ couplings can be made on dimensional grounds $[\Gamma] = 1$.

$$\Gamma = \begin{cases} \frac{M_X^3}{\Lambda^2} \sim 1 \text{ s}^{-1} & \text{where } \Lambda \sim M_{\text{GUT}} \sim 10^{16} \text{ GeV} \\ \frac{M_X^5}{\Lambda^4} \sim 10^{26} \text{ s}^{-1} \end{cases}$$

(a): This comes from a $d=5$ operator, e.g.

(b): From $d=6$ operator, like μ decay



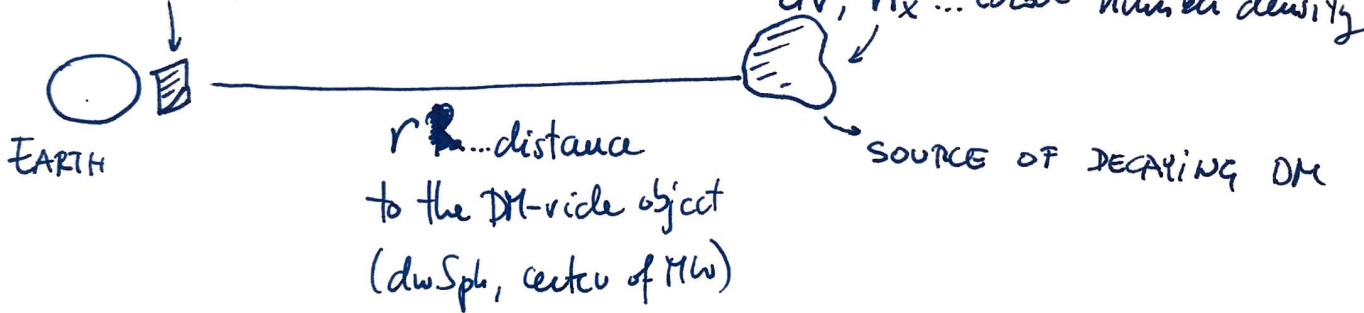
a) Is too fast \Rightarrow DM unstable

b) Is long enough: $\tau_{DM} \sim 10^{26} \text{ s} = 10^{19} \text{ yrs} \gtrsim 10 \text{ byrs}$
 $\sim t_0$

Clearly, it is very much feasible that DM is stable on cosmological scales and starts decaying at the present age.

How many events can we expect to observe?

detector, surface $A \sim \text{m}^2$



$$\frac{dN}{dt} \approx \underbrace{\frac{n_x dV}{\tau_x}}_{\text{source volume}} \underbrace{\frac{A}{4\pi r^2}}_{\text{geometric acceptance}}, \quad dV = r^2 dr d\Omega$$

$\underbrace{\frac{n_x dV}{\tau_x}}_{\text{source volume}}$
 $\underbrace{\frac{A}{4\pi r^2}}_{\text{geometric acceptance}}$

$$= A n_x \left(\frac{d\Omega}{4\pi} \right) \frac{dr}{\tau_x} \quad (\text{note } r^2 \text{ cancels away})$$

The diagram shows a circle labeled 'EARTH' inside a larger dashed circle. A line from the center of the dashed circle to the Earth is labeled '1 kpc'. The density ρ_0 is indicated near the dashed circle.

$$\rho_0 \sim 0.4 \text{ GeV}/\text{cm}^3 \quad \Rightarrow \quad \frac{dN}{dt} = \frac{A n_x r}{\tau_x} = \frac{A \rho_0 r}{M_x \tau_x}$$

$r \lesssim 1 \text{ kpc}$ for

etc $E \sim M_{\text{pl}}$ to propagate
 $- E - 6 - 45 -$

\Rightarrow a uniform local sphere with $r \sim 1 \text{ kpc}$ around the earth gives us:

$$\frac{dN_{\text{dec}}}{dt} = 10^{-4} \text{ s}^{-1} \left(\frac{A}{\text{m}^2} \right) \left(\frac{\rho_0}{0.4 \frac{\text{GeV}}{\text{cm}^3}} \right) \left(\frac{r}{\text{kpc}} \right) \left(\frac{\text{TeV}}{m_x} \right) \left(\frac{10^{26} \text{ s}}{\tau_x} \right)$$

This implies $\sim 10^3$ events per year.

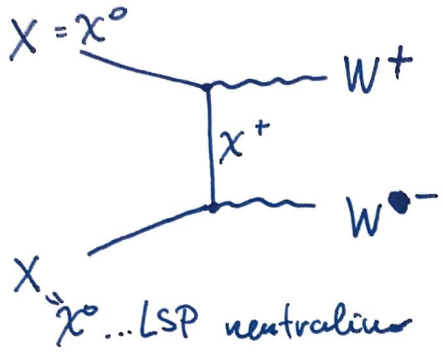
•) The case of sterile neutrinos decay also gave a similar lifetime but $m_x \sim \text{keV}$

$$\tau_x \sim 10^{30} \text{ s} \left(\frac{10^{-7}}{\theta^2} \right) \left(\frac{\text{keV}}{m_x} \right)^5$$

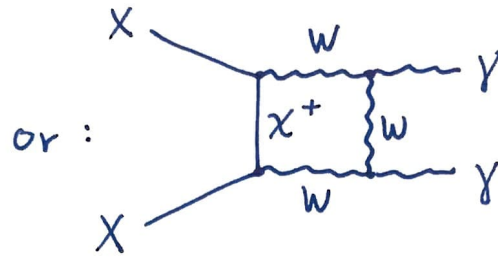
•) Even though the rate is higher, the total energy that goes into the signal is $\propto m_x N$ and thus stays roughly constant, $m_x N \propto \frac{A \rho_0 r}{\tau_x}$.

•) We'll see that constraints are also quite insensitive to the details of $f(r)$, so we get similar results for sources with common M .

ii, Moving on to annihilations, let's consider



Tree-level, allowed
by R-parity



Loop-level suppressed,
DM does not couple to γ
at tree-level.

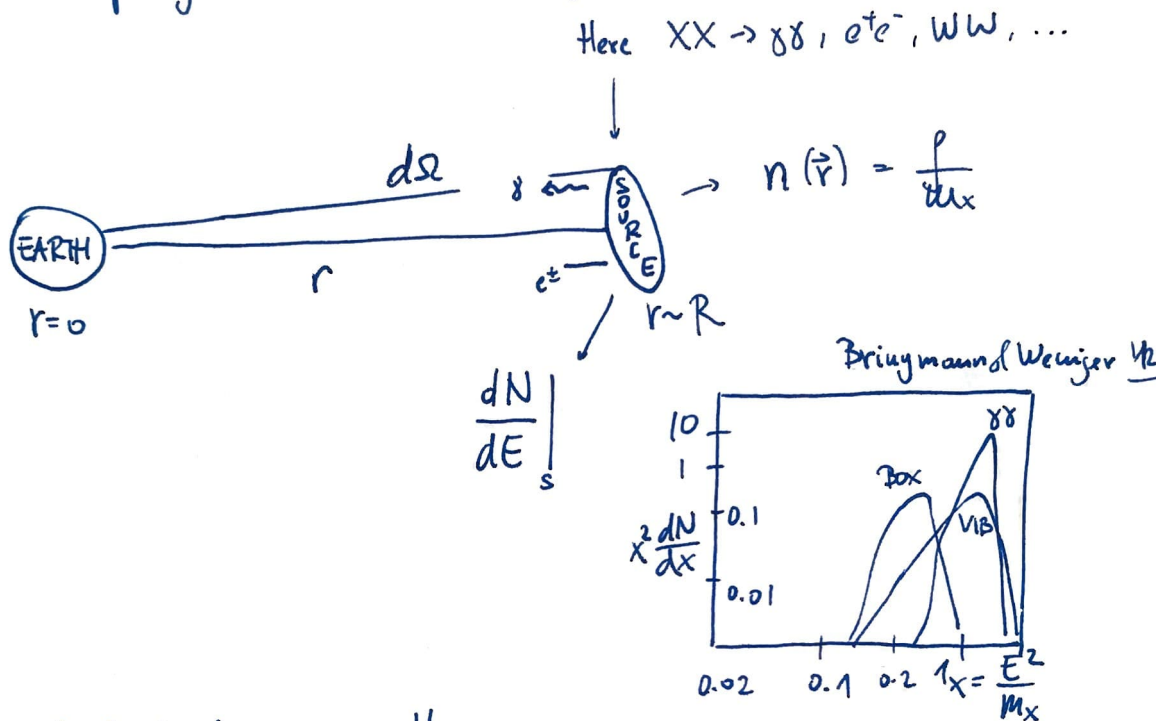
•) For $m_X \geq \text{TeV}$ (or for light final states like γ 's)
the $\langle \sigma v \rangle \approx \frac{\alpha_{DM}^2}{m_X^2}$, which may be the same
process that sets the thermal freeze-out abundance.

•) $\frac{dN}{dt dV} \equiv \underbrace{\sigma}_{\text{x-section}} \underbrace{n_X v}_{\text{incident flux}} \underbrace{n_X}_{\text{density of the target}} \quad \left(\frac{n_X}{\rho_X} \right)_{\text{for decays}}$

$$\begin{aligned} \Rightarrow \frac{dN_{\text{ann}}}{dt} &= A \langle \sigma v \rangle \left(\frac{\rho_0}{m_X} \right)^2 (1 \text{ kpc}) \\ &\sim A \cdot 10^{-26} \frac{\text{cm}^3}{\text{s}} \left(\frac{0.4 \text{ GeV}}{m_X} \right)^2 (1 \text{ kpc}) \text{ cm}^{-6} \\ &= 5 \cdot 10^{-8} \text{ s}^{-1} \left(\frac{A}{\text{m}^2} \right) \left(\frac{\rho_0}{0.4 \text{ GeV}} \right)^2 \left(\frac{v}{\text{kpc}} \right) \left(\frac{\text{TeV}}{m_X} \right)^2 \left(\frac{\langle \sigma v \rangle}{10^{-26} \text{ cm}^3 \text{ s}^{-1}} \right) \\ &\sim 1 \text{ event / year.} \end{aligned}$$

J-FACTORS

-) A very common practice in the field is to factor out the astrophysical source dependence into a so-called J-factor.



-) The rate distribution is then:

$$\frac{dN}{dE dt dV} = \frac{dN}{dE} \Big|_s \left(\frac{A}{4\pi r^2} \right) \begin{cases} \langle \sigma v \rangle \frac{n_x^2}{2} & , \text{annihilations} \\ \frac{n_x}{\tau_x} & , \text{decays} \end{cases}$$

$$= \frac{dN}{dE} \Big|_s \left(\frac{A}{4\pi r^2} \right) \begin{cases} \frac{\langle \sigma v \rangle}{2m_x^2} \int_0^\infty \int_{8\pi}^{\text{Jann}} p^2(r) dr d\Omega \\ \frac{1}{m_x \tau_x} \int_0^\infty \int_{8\pi}^{\text{Jann}} p(r) dr d\Omega \sim \frac{M}{m_x \tau_x R^2} \\ \sim \frac{1}{R^2} \int p dV \sim \frac{M}{R^2} \end{cases}$$

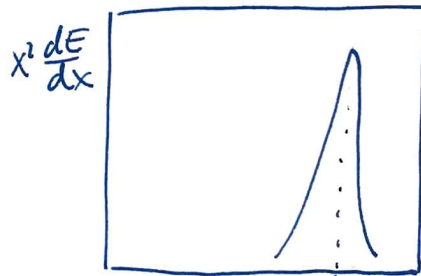
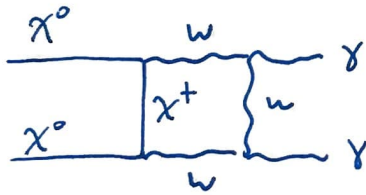
•) We thus have for annihilation:

With NFW $\rho = \frac{\rho_0}{(\frac{r}{R_s})(1+\frac{r}{R_s})^2}$
 $\int_{Dw} \rho^2 dV$

$$\frac{1}{A} \frac{dN_{ann}}{dE dt} = \frac{\langle \sigma v \rangle}{M_x^2} \left(\frac{dN}{dE} \right)_s J_{ann}, \quad J_{ann} = \begin{cases} 10^{17-20} \frac{\text{GeV}^2}{\text{cm}^2} \\ 10^{22} \frac{\text{GeV}^2}{\text{cm}^2} \end{cases}$$

The J-factors assumed the NFW profiles for dwarf & MW center (1°). Local overdensities, like clumps with more DM may enhance by orders of mag.

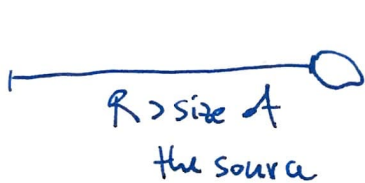
•) Finally the energy spectra at the source $\left(\frac{dN}{dE} \right)_s$ depend on the microscopic process.



$\frac{1}{2}x \sim 1 \quad x = \frac{E}{m_x}$

$$\left. \frac{dN}{dE} \right|_s \approx 2 \delta(E - m_x)$$

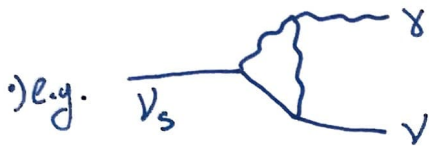
•) For decays, the J-factor is simple:



$$J \sim \frac{M}{V} \int (\rho - R)$$

$$\int \rho dr d\Omega \sim \frac{1}{R^2} \int \rho dV = \frac{M}{R^2}$$

$$\bullet) \frac{1}{A} \frac{dN}{dE dt} = \frac{1}{4\pi} \frac{1}{u_x c_x} \left(\frac{dN}{dE} \right)_s \frac{M}{R^2}$$



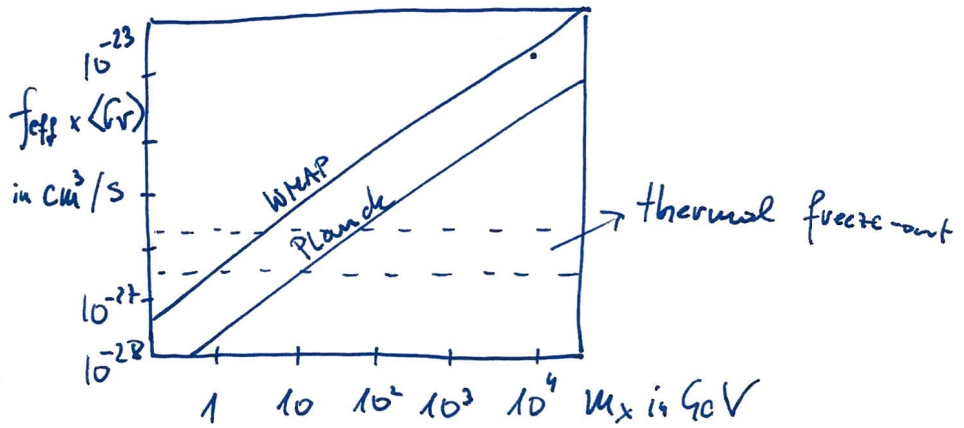
$$\frac{dN}{dE} \sim \delta(E - u_x/2)$$

*) For distant sources, the redshift $d_L(z)$ and γ absorption are relevant, too.

CURRENT LIMITS

-) The exact limit depends on the final state and the exposure of the experiment.

a) Early universe
WMAP & Planck
bound from
CMB effects.

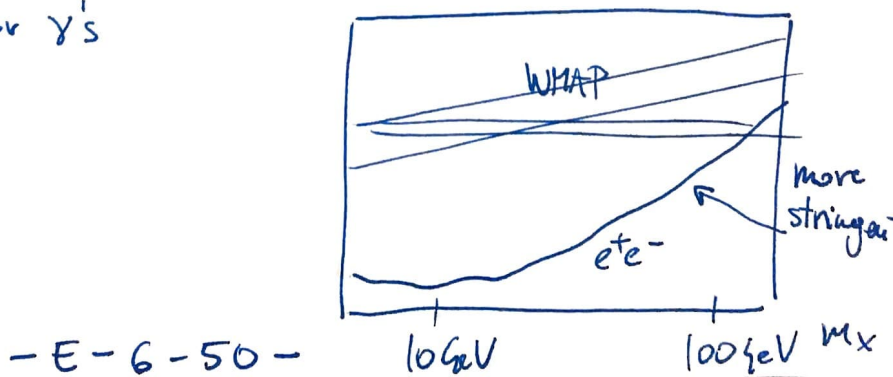


• @ $z_{\text{ann}} \sim 600, z_{\text{dec}} \sim 300$

•) f_{eff} parametrizes the final state transmission to CMB

$$f_{\text{eff}} \equiv \begin{cases} 0.4 & \text{for } e, \mu, \text{top} \\ 0.2 & \text{the rest, except } \gamma\text{'s} \\ \sim 0 & \text{for } \gamma\text{'s} \end{cases}$$

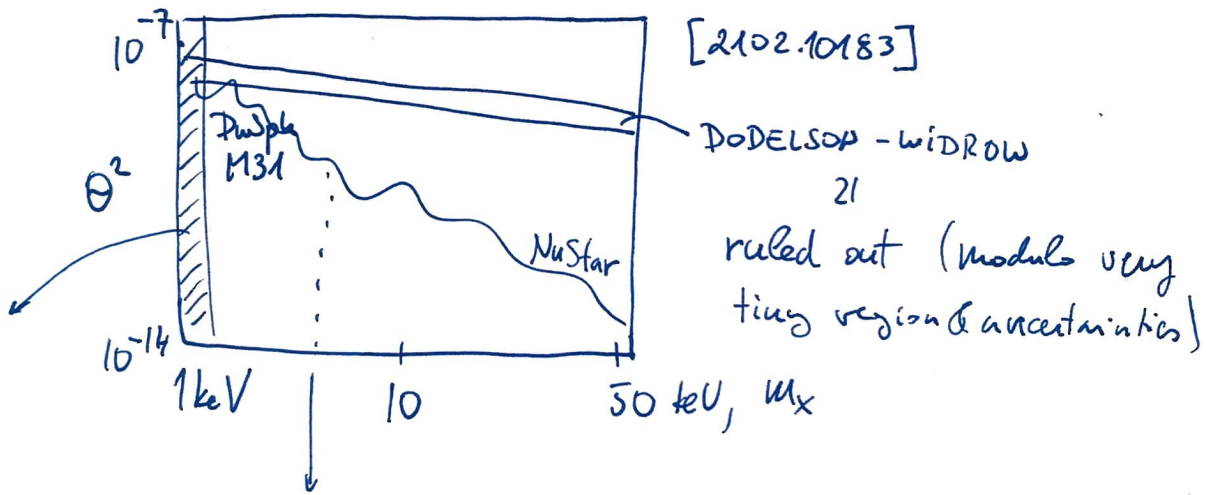
b) Charged leptons
(galaxy propagation)
AMS '02 exp.



-) for the γ final state, the $\langle \sigma v \rangle$ is typically loop suppressed by at least $\frac{\alpha^2}{4\pi}$ (or more if there are heavy mediators in the loop)

FERMI : $\langle \sigma v \rangle_{XX \rightarrow \gamma\gamma} \leq 10^{-28} \frac{\text{cm}^3}{\text{s}} @ m_X = 10 \text{ GeV}$

-) There are dedicated searches for X-rays, looking for photon monochromatic lines.



there was an exciting excess @ 3.5 keV, gone.

SUMMARY ON INDIRECT DETECTION

•) Indirect detection experiments are looking for visible final state products of DM decay or annihilation.

•) The rate of incoming particles depends on

$\langle \sigma v \rangle n_x^2$ for annihilation

$\frac{n_x}{\tau_x}$ for decays

•) The astrophysical uncertainties are stored in the J-factors, known for DWSph, MW ℓ^2 .

•) Depending on the final state (like e^+e^-), the

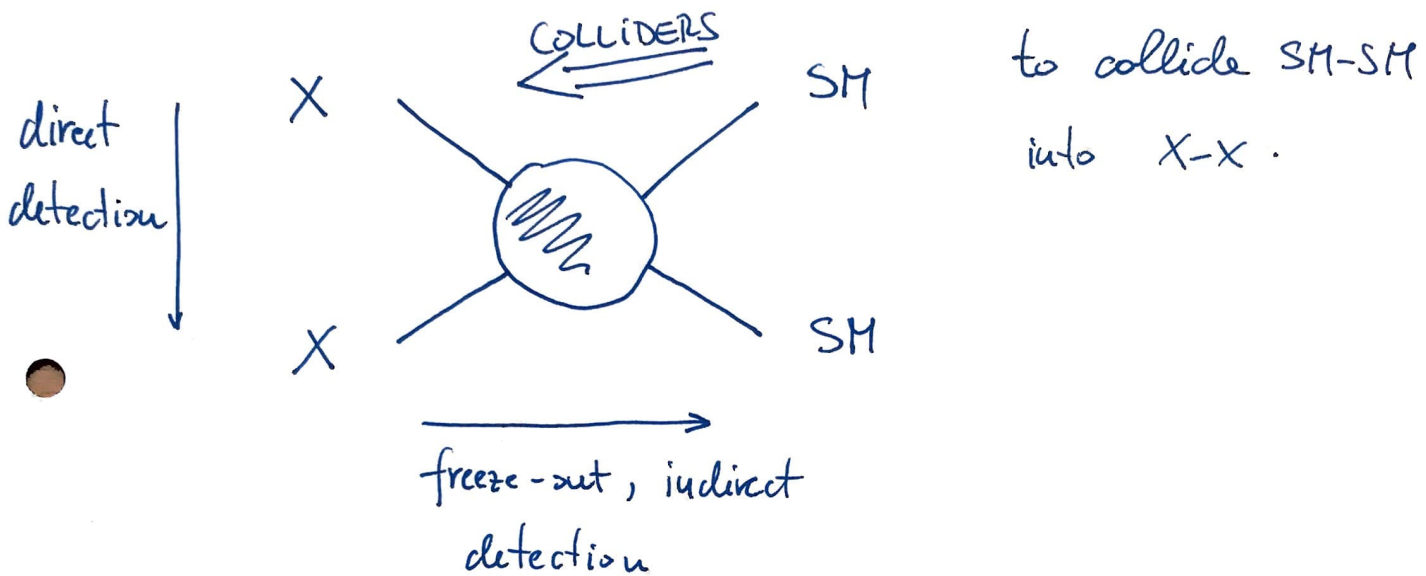
limits may be very stringent and even below

the $\langle \sigma v \rangle$ for thermal freeze-out or below

the Θ^2 for DW freeze-in.

COLLIDER SEARCHES FOR DM

•) We saw the 2-2 illustration for freeze-out, direct & indirect detection, the 3rd way is

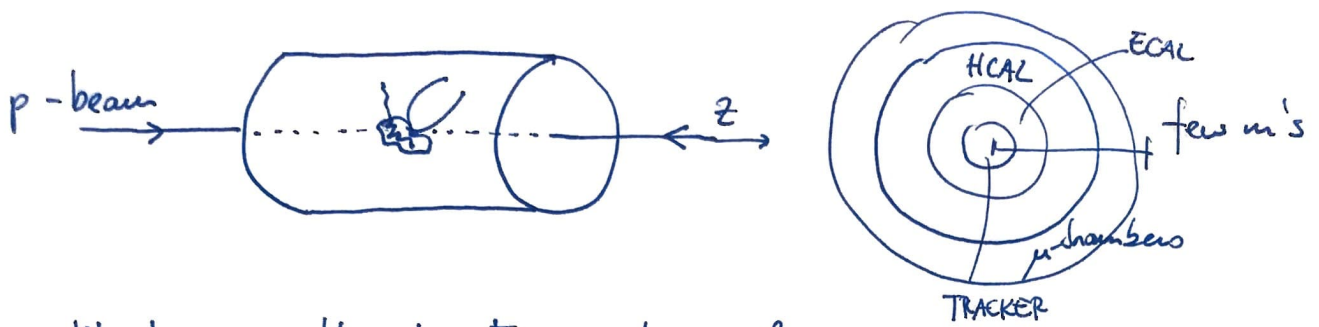


•) There is a range of "stable" particles we can collide, e^\pm , p^\pm (even μ 's).
LEP LHC, Tevatron

•) For $m_X \in (\text{GeV} \sim \text{TeV})$, the LHC is most suitable to detect since $\sqrt{s} = E_{\text{CMS}} \approx 14 \text{ TeV}$.

•) While DM may decay on τ_n scales, the typical collider lifetimes are $\sim \frac{\text{few meters}}{c} \ll \tau_n$, so DM if produced would be neutral (no charged tracks) and escapes detection, no ECAL/HCAL deposits.

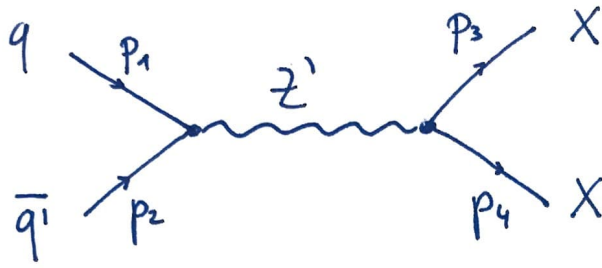
•) Typical detector ATLAS, CMS



-) We know that E is transferred along the z axis, so the total ΣE in \perp direction should vanish. The imbalance is called E_T^{miss} = missing transverse energy
-) A DM particle with $m_x \sim \text{TeV}$ or $E_x \sim \text{TeV}$ would take away a significant part of E and create a large imbalance $E_T^{\text{miss}} \sim m_x$ or E_x . While there are ν induced backgrounds in the SM, they are well known and predictable, \Rightarrow we can look for $S_{\text{DM}} / B_{\text{SM}}$.
-) To perform a reliable collider study, one has to properly define a DM model and simulate S and B events. Then one derives cuts (typically large E_T) to enhance $\frac{S}{B}$. We will do simple $\sigma_{\text{rel}} = N_{\text{events}}$ estimates instead.

SIMPLE COLLIDER ESTIMATE

•) Consider a simple toy model with a Z' & $X=DM$.



$$\hat{s} = (p_1 + p_2)^2$$

Let's define the coupling to be vectorial with

• g_1 to quarks and g_2 for X

$$j_{q\bar{q}}^{Z'} = g_1 \bar{q} \gamma^\mu q, \quad j_{XX}^{Z'} = g_2 \bar{X} \gamma^\mu X$$

Since we do not know $M_{Z'}$, let's consider its full mass dependence via a Breit-Wigner (BW)

•) Partonic $\hat{\sigma}_{q\bar{q} \rightarrow XX} = \underbrace{3\pi \left(\frac{g_1^2}{4\pi}\right)}_{SM-SM} \underbrace{\frac{\Gamma M}{(\hat{s} - M^2)^2 + (\Gamma M)^2}}_{\text{MEDIATOR}} \underbrace{\text{Br}(Z' \rightarrow XX)}_{DM \text{ } XX \text{ part}}$

•) The BW sets the dimension: $\begin{cases} \sim \frac{1}{\hat{s}} & \text{for massless like } \gamma \\ \sim \frac{1}{M^2} & \text{for massive, } \hat{s} < M^2 \end{cases}$

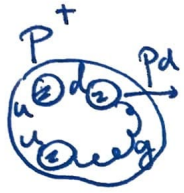
•) The DM part is in the

$$\text{Br}_{Z' \rightarrow XX} = \frac{1}{\Gamma_{\text{tot}}} \frac{g_2^2}{48\pi} M \underbrace{\left(1 - \frac{m_X^2}{\hat{s}}\right)^2}_{\text{phase space suppression}} \left(1 + \frac{m_X^2}{2\hat{s}}\right).$$

$\Gamma_{\text{tot}} \propto M$ so $[\text{Br}] = 0$

phase space suppression

b) Now that we have the partonic $\hat{\sigma}_{qq \rightarrow XX}$, we can convolute it with the Parton Distribution Functions PDFs of the proton $f_q(x, Q^2)$, which give the final $\sigma(pp \rightarrow XX)$ at the nucleus level.



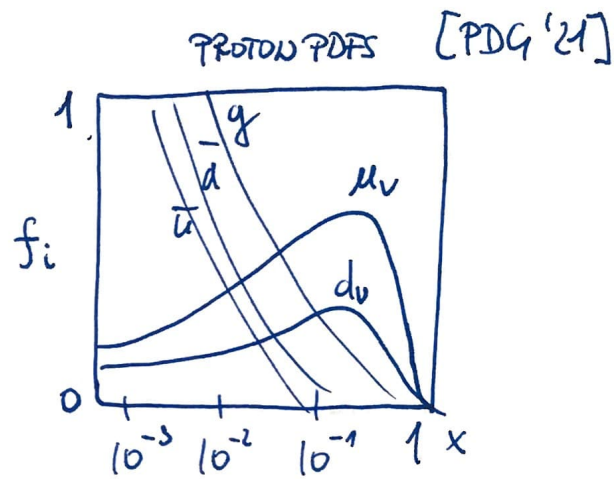
$p_i = x_i P$ partonic momentum fraction
 \uparrow parton momentum

P ... momentum of

the proton $\sqrt{s} = (P_1 + P_2)^2$

* light valence f_q peak at $x \sim 0.1$

* $\int f_u \sim 2 \int f_d$



c) Putting it all together, we have

$$\sigma_{pp \rightarrow XX} = \frac{1}{9} \int_{\frac{m_X^2}{s}}^1 dx_1 \int_{\frac{m_X^2}{x_1 s}}^1 dx_2 \sum_{q, q'} \hat{\sigma}_{qq' \rightarrow XX} (f_q(x_1) f_{\bar{q}'}(x_2) + (1 \leftrightarrow 2))$$

d) That's it! From $\mathcal{L} \cdot x \sigma_{pp \rightarrow XX} = \text{Events}$

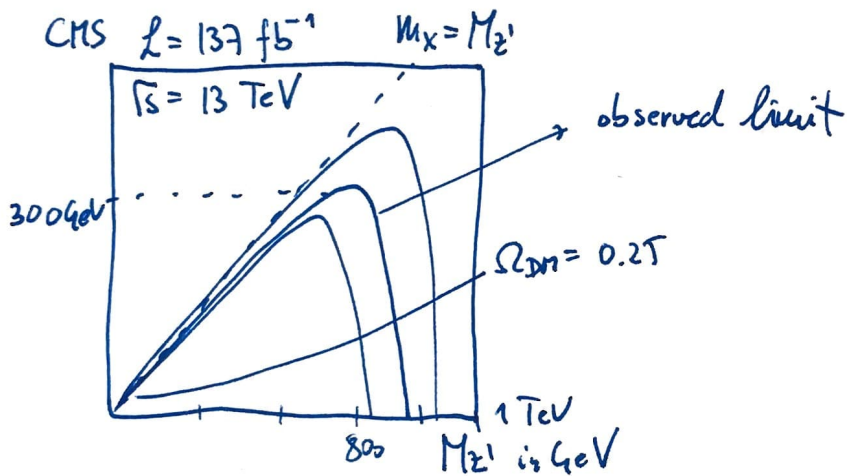
luminosity measured at

the LHC, $\mathcal{L} \approx 100 \text{ fb}^{-1}$ \propto amount of statistics collected.

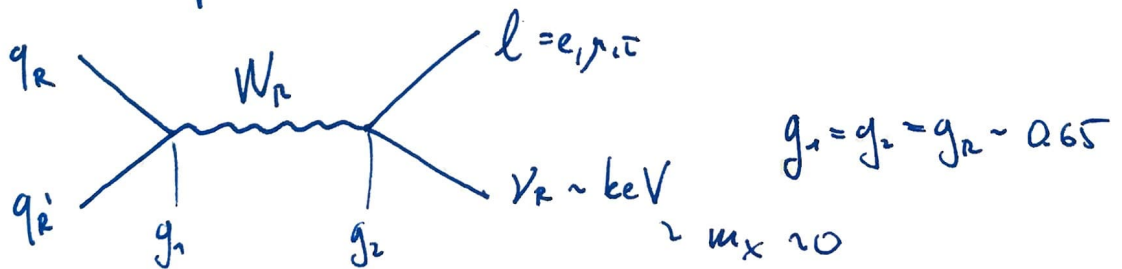
•) Finally, we have to devise cuts to enhance the signal rate (say $E_T \geq 100$ GeV, initial jets, γ 's, W 's), which typically reduces the rate by $\epsilon \sim 1\% - 10\%$.

$$N_{obs} = \epsilon \cdot L \cdot \sigma$$

Current searches exist in a myriad of final states, here we show [CMS-PAS-EXO-19-003]



•) Another example is ν_R as DM in the LRSM



$$M_{W_R} > \begin{cases} 4.6 \text{ TeV } \mu^- & [1801.05813] \\ 4.5 \text{ TeV } e^- \end{cases}$$