BARYOGENESIS

- We will discuss the basic ingredients, for more in-depth reading, see:
  - Book by: Kolb & Turner: The Early Universe
  - TASI lectures by Jim Cline
  - MITP lectures by Shaposhnikov, Morrissey

i) Basics and Sakharov conditions,

ii) GUT-genesis,

iii) Lepto-genesis. (We will not talk about EW baryogenesis or Affleck-Dine, many others)

- Experiments (BBN, CMB, direct observations) show that there is a net excess of baryons over anti-baryons.

$$\eta = \frac{n_B}{n_\gamma} \cong 6 \times 10^{-10}.$$

CMB, LSS, BBN : $\Omega_b h^2 = \frac{m_p n_B}{\rho_{cr}} = 0.0223 \pm 0.0002$

$\hookrightarrow T_\gamma \to n_\gamma$

$$\eta = \frac{n_B}{n_\gamma} = 6.1 \cdot 10^{-10}.$$

- This implies there are more B than $\bar{B}$: $n_B \neq n_{\bar{B}}$, otherwise they would have annihilated away already.

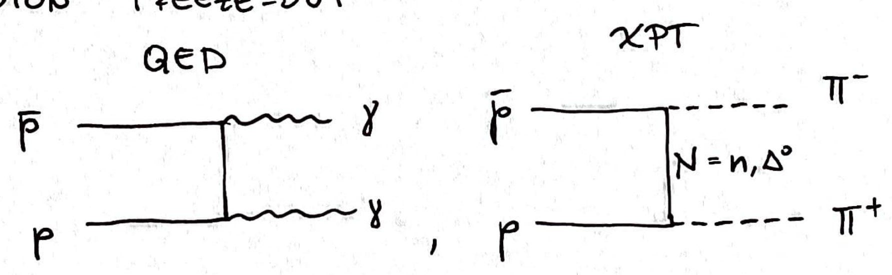
- How do we get to $\frac{n_B}{n_\gamma} \sim 10^{-10}$? After inflation, all the charges are diluted and $Q^{tot} = 0$ (the universe is charge neutral) and $B^{tot} = 0$, so: $n_B = n_{\bar{B}}$.

- Such a B-symmetric universe would give us a very small n_B, let's see why:

  * high $T \gg \text{GeV}$: g, l, ν, q, W, Z, h

  * below Λ_{QCD}, mesons and baryons form, we now have $p, n, \pi^\pm, \pi^0$, K's in the plasma.

PROTON - FREEZE-OUT



$$\langle \sigma v \rangle_\gamma \sim \frac{\alpha^2}{m_p} \sim 10^{-4}\,\text{GeV}^{-2}, \qquad \langle \sigma v \rangle_\pi \sim \frac{1}{m_\pi^2} \sim 10^{2}\,\text{GeV}^{-2}$$

In EQUILIBRIUM, the SAHA equation gives us

$$\left[\frac{n_p n_{\bar{p}}}{n_p^{(0)2}} - 1 \right] = 0 .$$

- Both interactions (QED & χPT) will freeze-out.

- $p-\bar{p}$ freeze-out

$$\Gamma = n_p^{(0)} \langle \sigma v \rangle_\pi = \frac{1}{m_\pi^2} \, 2 \left(\frac{m_p T}{2\pi} \right)^{3/2} e^{-m_p/T}$$

$$= H = \frac{1.7\sqrt{g_*}}{M_{Pl}} T^2 .$$

- We use the same logic as for the DM freeze-out, introducing $x = \frac{m_p}{T}$, which gives:

$$\frac{2}{m_\pi^2} \frac{m_p^{\cancel{3}}}{(2\pi)^{3/2}} x^{-3/2} e^{-x} \cong \frac{10}{M_{Pl}} \cancel{m_p^2} \, x^{-2}$$

$$x^{1/2} e^{-x} = \frac{5 m_\pi^2 (2\pi)^{3/2}}{m_p M_{Pl}} = 10^{-19}$$

This gives $x_f \simeq 46$, which is larger than DM, because we are freezing out via stronger interactions.

- From DM f.o.: $\lambda = \frac{m_x^3 \langle \sigma v \rangle}{H(m_x)} \to \lambda_p = \frac{m_p^{\cancel{3}} M_{Pl}}{10 \cancel{m_p^2} m_\pi^2}$

$$Y_{p\infty} = \frac{x_f}{\lambda_p} \cong \frac{46 \cdot 10 \, m_\pi^2}{m_p M_{Pl}} \simeq 10^{-18} .$$

- We got: $Y_B = \frac{n_B}{s} = 10^{-18}$, while the observed value is $Y_B^{obs} = \frac{\eta n_\gamma}{s} = \frac{\eta}{7.04} \simeq 10^{-10}$, we're off by 8 orders of magnitude, also $n_B = n_{\bar{B}}$.

- How do we get to $\eta = 10^{-10}$? Start with a baryon asymmetric universe with $n_p > n_{\bar{p}}$ (or $n_B > n_{\bar{B}}$). Annihilation then proceeds as before until $n_{\bar{B}}$ gets depleted. We are then left with n_B only, which then scales as ordinary matter $\propto a^{-3}$.

- The aim here is to get to $\eta \neq 0$, starting from $B^{tot} = 0$ via some interactions. This dynamical mechanism is called BARYOGENESIS.

- To get to this result, there are 3 general conditions explained by Sakharov:

  1) B violation,

  2) CP violation,

  3) Out of equilibrium.

Let's go through these and explain the logic.

1) B-VIOLATION : Interactions need to violate when we start from $B^{tot}=0$. Otherwise B-conserving interactions (the SM for the most part) will erase Δn_B, i.e. quickly redistribute into $n_B = n_{\bar{B}}$.

TWO EXAMPLES OF B-VIOLATION :

- EXAMPLE #1: GRAND UNIFIED THEORIES (GUTs)

EXAMPLE #2: SPHALERONS IN THE SM (Advanced)

$\boxed{\#1}$: $$G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\left. Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L ,\ L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L ,\ \begin{matrix} u_R \\ d_R \end{matrix} ,\ e_R \right\} \times n_g = 3.$$

- In the SM, the W "transforms" a lepton in a neutrino and vice-versa : 

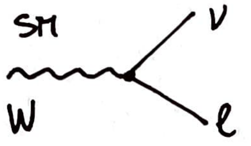
.

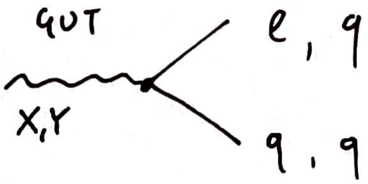
$$G_{GUT} = SU(5) \quad (\text{or } SO(10), E_6, \ldots) \text{ contains}$$

G_{SM} as a sub-group and $g_{1,2,3} \longrightarrow g_{GUT}$ @ M_{GUT}

$$SU(5) : \quad 5_f = \begin{pmatrix} L_L \\ \bar{d} \end{pmatrix} = \begin{pmatrix} \nu \\ e \\ \bar{d}_r \\ \bar{d}_s \\ \bar{d}_s \end{pmatrix} ,\ 10_F = \begin{pmatrix} 0 & \bar{u} & \bar{u} & u & d \\ & 0 & \bar{u} & u & d \\ & & 0 & & \\ & & & 0 & e_R \\ & & & & 0 \end{pmatrix}$$

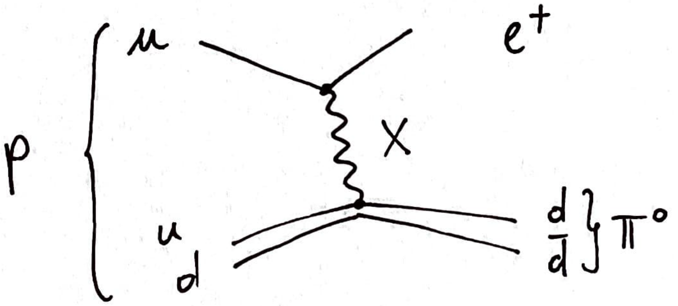
- Instead of 5 representations, we only have 2. However, now there are GUT gauge bosons that can "transform" a lepton to a quark:





$\mathcal{A} \propto g_2$, $\qquad \mathcal{A} \propto g_{GUT}$.

- With both couplings to l & q, B and L is broken. Similar to β-decay, we get p-decay



$B=1 \Rightarrow B=0$

$\boxed{p \to e^+ \pi^0}$

$\Delta B = 1$

- The decay rate is similar to μ decay, because we have a heavy mediator M_X.

$$\Gamma_{p\to\pi^0 e^+} \sim g_{GUT}^4 \frac{m_p^5}{M_{GUT}^4}, \quad \Bigg\} \quad \underline{M_X \gtrsim 10^{15\text{-}16}\,\text{GeV}}$$

very high

Exp. (SUPER-K): $\tau_{p\to\pi^0 e^+} > 10^{32}\,\text{yr} - 10^{39}\,\text{s}$.

- At $T \gtrsim M_X$, these will be in equilibrium, their decays will produce $n_B - n_{\bar{B}} \neq 0$. ($\eta \sim 10^{-10}$)

$\boxed{\#2}$: B violation in the SM

Baryon number is violated in the SM in a highly non-trivial way via non-perturbative effects.

Conceptually this is because:

a) Vacua in non-abelian gauge theories have a non-trivial energy, more minima

b) Some symmetries get broken at the quantum level (via anomalies). This is true for B & L.

- Fully conserved charges (non-anomalous symmetries), like QED charge: $Q = \int j^0 d^3x$

$$j_f^\mu = Q \bar{f} \gamma^\mu f \, , \quad j_{SM}^\mu = \sum_f j_f^\mu = -\bar{e}\gamma^\mu e + \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d$$

Conservation: $\partial_\mu j^\mu = 0 \quad \Rightarrow \mu = 0 : \frac{d}{dt} j^0 = 0$

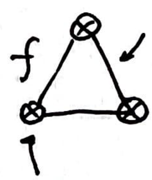
$$\boxed{\frac{dQ}{dt} = 0}$$

- We can construct similar currents for B & L

$$\text{BARYON \#}: \; j^\mu_B = \frac{1}{3}\bar{q}\gamma^\mu q + \cancel{\phi\,\bar{\ell}\gamma^\mu \ell}$$

$$\text{LEPTON \#}: \; j^\mu_L = 1\,\bar{\ell}\gamma^\mu \ell + 1\,\bar{\nu}\gamma^\mu \nu + \cancel{\phi\,\bar{q}\gamma^\mu q}$$

- The QED charge remains conserved at higher loops, however B and L get broken @ 1 loop by triangle anomalies



- Instead of getting $\partial_\mu j^\mu_L = \partial_\mu j^\mu_B = 0$, we have:

$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L = n_g \frac{\alpha_2}{8\pi} W\tilde{W},$$

\# of generations, 3

gauge field configuration

$$W\tilde{W} = \frac{1}{2}\varepsilon^{\mu\nu\lambda\rho} W^a_{\mu\nu} W^b_{\lambda\rho},$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + i\,\varepsilon_{abc} W^b_\mu W^c_\nu .$$

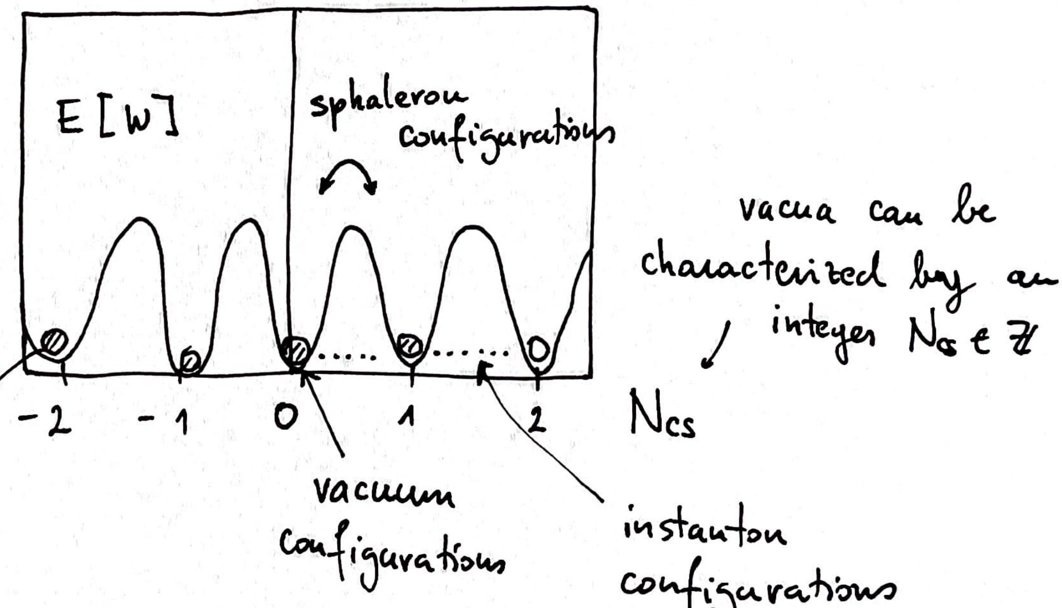
- The RHS is in general non-zero, symmetries are broken by the gauge field configurations in $W\tilde{W}$.

- Incidentally, B-L is not anomalous $\partial_\mu(j_B^\mu - j_L^\mu) = 0$ and is conserved also at loop level (like Q, except that it is global, not local $U(1)$).

B+L on the other hand gets broken

$$\partial_\mu (j_B^\mu + j_L^\mu) = \frac{n_g \alpha_2}{4\pi} W\tilde{W}$$

B+L violation, non-trivial vacuum configuration.



- $W\tilde{W}\big|_{vac}$ are characterized by N_{cs} (similar to $FF = \frac{1}{2}(\vec{E}^2 + \vec{B}^2)$ in QED)
- Besides the vacua, we have configurations of W_μ^-

$W^{a\ \mathrm{inst}}_{\mu\nu}$ & $W^{a\ \mathrm{sph}}_{\mu\nu}$ that interpolate between different N_{cs} and describe tunneling between vacua.

INSTANTONS $W^{a\ \mathrm{inst.}}_{\mu\nu}$ ['t Hooft] @ $T=0$.

$$\Delta N_{cs} = N_{cs}(t\to\infty) - N_{cs}(t\to-\infty)$$

$$= \frac{\alpha_2^2}{8\pi}\int d^4x\, W\tilde{W}^{\mathrm{inst}},$$

This goes directly to the breaking of B

$$\Delta B = \int_{-\infty}^{\infty} dt\, \partial_0 \int j^0_B\, d^3x = \int_x j^{B0} (t\to\infty) - \int_x j^{B0}(t\to-\infty)$$

$$= \Delta B = n_g \frac{\alpha_2^2}{8\pi}\int W\tilde{W} = 3\,\Delta N_{cs}.$$

$\Rightarrow$ The SM breaks ΔB by 3 units. The rate is

$$\frac{1}{V}\Gamma^{\Delta B=3}_{\mathrm{inst}} \sim v^4 e^{-\frac{2\pi}{\alpha_2}\Delta N_{cs}} \sim \underbrace{10^{-160}\, v^4}_{\text{unobservably small}}$$

SPHALERONS $W^{a\ \mathrm{sph}}_{\mu\nu}$ [Manton, Klinkhamer] @ $T>0$

• Sphaleron $W_{\mu\nu}^{a\,sph}$ configurations describe tunneling between different vacua at high T

Lower T : $\gamma_{sph} \sim (\alpha_2 T)^4 \left(\frac{E_{sph}}{T}\right)^7 e^{-E_{sph}/T}$, $E_{sph} = \frac{8\pi v}{g_2}$

High $T \gg v$: $\gamma_{sph} \sim 18\, \alpha_2^5 \, T^4$ ... FAST.

2) CP VIOLATION : This symmetry converts between particles and anti-particles. If the interactions of particles f and and antiparticles $\bar{f}$ would be the same, then starting from $B^{tot} = 0$, the total B would equilibrate to 0 since $B(f) = -B(\bar{f})$. CPT still remains conserved.

Let's see how CP works in a specific model.

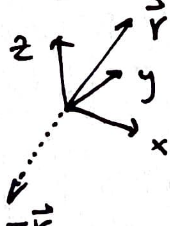
FERMIONS

$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$

$P : \psi \to \gamma_0 \psi(t,-x) = \begin{pmatrix} \psi_R(t,-x) \\ \psi_L(t,-x) \end{pmatrix}$

$C : \psi \to i\gamma_2 \psi^* = \psi^c = \begin{pmatrix} i\sigma_2 \psi_R^* \\ -i\sigma_2 \psi_L^* \end{pmatrix}$

P is an inversion of space $\vec{x} \to -\vec{x}$



$x^\mu = (ct, \vec{x})$

$\downarrow P$

$(t, -\vec{x})$

SCALARS $\phi \in \mathbb{C}$ GAUGE BOSONS A_μ are vectors

$P: \phi(t,\vec{x}) \to \phi(t,-x)$ $P: A_0 \to A_0(t,-\vec{x}),\ A_i \to -A_i(t,-x)$

$C: \phi(t,x) \to \phi^*(t,x)$ $C: A_\mu \to -A_\mu$

- With these transformations, it is easy to check whether CP is conserved. Gauge interactions have real couplings and typically conserve CP (in the interaction basis). However, Yukawa terms are $\mathbb{C}$

$$\mathcal{L}_Y = y_{ij}\,\overline{\psi}_{Li}\,\phi\,\psi_{Rj} + y_{ij}^*\,\overline{\psi}_{Rj}\,\phi^+\,\psi_{Li}$$

$\Downarrow$ we do the CP

$$y_{ij}\,\overline{\psi}_{Rj}\,\phi^+\,\psi_{Li} + y_{ij}^*\,\overline{\psi}_{Li}\,\phi\,\psi_{Rj}$$

$\cancel{CP}$ if $y_{ij} \neq y_{ij}^*$, which has to hold after all possible field redefinitions.

EXAMPLE, SM : $u_L^{(f)} \longrightarrow U_L u_L^{(m)}$ mass basis

$d_L^{(f)} \longrightarrow D_L d_L^{(m)}$

flavor (interaction) basis

$V_{CKM} = U_L^+ D_L$ * contains only one CP phase, even though it's 3×3, $\mathbb{C}$.

3) OUT OF EQUILIBRIUM

The out of equilibrium condition is needed to ensure that reverse processes do not wash out the created asymmetry. If we start with $B^{tot}=0$ and a certain interaction creates $B \neq 0$, then the inverse will erase it.

Options for out-of equilibrium:

- EXPANSION OF THE UNIVERSE $H(T)$ (freeze-out)
- PHASE TRANSITIONS (1^{st} order, irreversible)

• Before we move on to concrete models, let us discuss the role of the chemical potentials and how they behave.

CHEMICAL POTENTIALS

- Let's have a look at fermionic (BARYONS) distributions and remember $\mu_{\bar{f}} = -\mu_f$

$$n_f - n_{\bar{f}} = g \int_p \frac{1}{e^{(E-\mu)/T}+1} - \frac{1}{e^{(E+\mu)/T}+1}$$

$$= g \int_p \frac{\mathrm{sh}\left(\frac{\mu}{T}\right)}{\mathrm{ch}\left(\frac{E}{T}\right) \underset{\uparrow \text{ for bosons}}{\pm} \mathrm{ch}\left(\frac{\mu}{T}\right)} \xrightarrow{\mu = 0} 0 .$$

- We can separate the relativistic $E \sim p$ and NR regime. The latter is valid at $\underbrace{T \lesssim m}_{\text{NR}}$, the former $\underset{\text{REL.}}{T > m}$.

FERMIONS

$$n_f - n_{\bar{f}} = g \begin{cases} \frac{T^2}{6} \mu = \frac{T^3}{6} \frac{\mu}{T} & , \ T > m \\ 2\, \mathrm{sh}\left(\frac{\mu}{T}\right) \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} & , \ T < m \end{cases}$$

- The chemical potential μ measures the difference between n_f and $n_{\bar{f}}$. In equilibrium, e.g. $e^+ e^- \leftrightarrow \gamma\gamma$

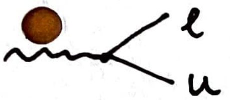
$$n_\gamma = n_\gamma^{(0)} \quad , \quad \text{SAHA} : \quad \frac{n_e n_{\bar{e}}}{n_e^{(0)2}} - 1 = 0$$

GUT - genesis

- Let us consider a gauge boson $X_\mu\,(3, 2, -5/6)$ under $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$.

- It can decay to ℓu or to $q\,d$, e.g.

$X \to e^- u$ or $X \to \bar{u}\,\bar{d}$ $\quad (Q(X) = -\tfrac{1}{3})$



We need: $\Gamma(X \to \ell u) \neq \Gamma(\bar{X} \to \bar{\ell}\bar{u})$

- Let us define a dimensionless constant ε_B, which measures the asymmetry of baryon numbers

$$\varepsilon_B = \frac{\frac{1}{3}\left(\Gamma_{X\to eu} - \Gamma_{\bar{X}\to \bar{e}\bar{u}}\right) - \frac{2}{3}\left(\Gamma_{X\to \bar{q}\bar{d}} - \Gamma_{\bar{X}\to qd}\right)}{\Gamma_X^{tot}}.$$

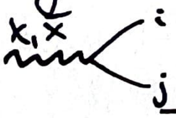
$-\frac{2}{3}$ because we got 2 anti-quarks

$\frac{1}{3}$ because we get one u quark and $B(\ell) = 0$

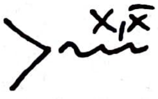
- The Boltzmann equation for the net BARYON:

$$a^{-3}\frac{d}{dt}\left(n_B a^3\right) = \underbrace{(n_X + n_{\bar{X}})\,\Gamma_X\,\varepsilon_B}_{} - \underbrace{\sum_{i,j} n_i n_j \langle \sigma v\rangle_{ij\to X\bar{X}}\, Br_{X\to ij}}_{}$$

production of asymmetry in $X, \bar{X}$ decays



INVERSE DECAYS, which wash out the asymmetry



- When T drops below m_X, washout is exponentially suppressed. We proceed as usual: $Y_B = \frac{n_B}{s}$

$$\Rightarrow \frac{dY_B}{dt} \simeq 2\, Y_X^{(0)}\, e^{-\Gamma_X t}\, \Gamma_X\, \varepsilon_B\,, \quad n_X \sim n_X^{(0)} e^{-\Gamma_X t}$$

$$\int_0^{Y_{B\infty}} dY_B = 2\, Y_X^{(0)}\, \frac{\cancel{\Gamma_X}}{-\cancel{\Gamma_X}}\, e^{-\Gamma_X t}\Big|_0^{\infty} \varepsilon_B = 2\, Y_X^{(0)}\, \varepsilon_B$$

from zero   Here we create B : BARYO-GENESIS

$$Y_X^{(0)} = n_X^{(0)}/s = \frac{g\,\frac{\zeta(3)}{\pi^2}\,\cancel{T^3}}{\frac{2\pi^2}{45}\, g_*(T)\,\cancel{T^3}} = \text{finite number}$$

$$Y_{B\infty} = 2\, Y_X^{(0)}\, \varepsilon_B\,, \quad n_B = Y_{B\infty} \cdot s$$

while: $$\frac{n_B}{s} \sim \eta = 7\, Y_{B\infty} \sim 14\, Y_X^{(0)}\, \varepsilon_B$$

- So, with known $g_*(T)$ and $\varepsilon_B \neq 0$ given by difference of X vs $\bar{X}$ decays, we get $\eta \neq 0$, starting with 0 baryon number.

- In full chemical equilibrium $\sum \mu_i = 0$

With sphalerons active : $\sum (\mu_{Bi} + \mu_{Li}) =$

$$= \sum (3\mu_{qi} + \mu_{\ell i}) = 0$$

- As we mentioned, the sphaleron rate is :

$$\frac{\Gamma}{\underbrace{V}_{\propto T^3}} = \gamma \sim \alpha_2^5 T^4 \Rightarrow \Gamma_{sph} \simeq \alpha_2^5 T.$$

This we compare to the usual H in radiation

$$\Gamma_{sph} = H \sim \frac{T^2}{M_{Pl}} \quad (\text{we drop } 1.7\sqrt{g_*(T)})$$

$$\alpha_2^5 \cancel{T} = \frac{T^{\cancel{2}}}{M_{Pl}} \Rightarrow T_{sph}^{IN} \sim \alpha_2^5 M_{Pl} = 10^{-8} M_{Pl}$$

$$\sim 10^{11}\,\text{GeV EARLY.}$$

- Sphalerons enter in equilibrium very early and keep it until $T_{sph}^{OUT} = 133\,\text{GeV}$ (drop is sharp $e^{-v/T}$).

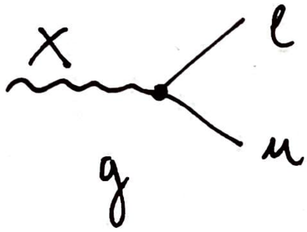
between T^{IN} & T^{OUT} : $\sum_{i=1}^{3} (3\mu_{qi} + \mu_{\ell i}) = 0.$

- Now we consider specific scenarios : GUT-genesis.

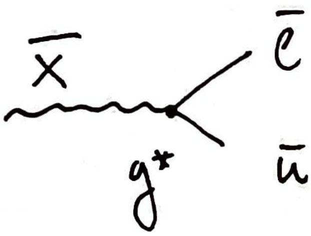
- We found out that η can be computed from a model where $\Gamma_X \neq \Gamma_{\bar{X}}$. But how does this happen precisely in a certain model?

CONNECTING ε_B TO MODEL PARAMETERS

- Let's start with tree-level



$\mathcal{A} \propto g$, $\Gamma_X \propto |g|^2$



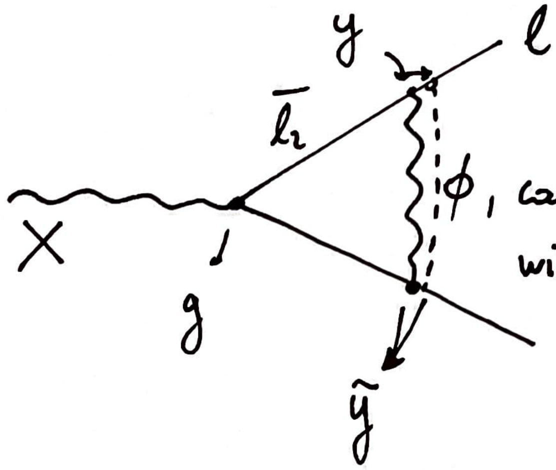
$\mathcal{A} \propto g^*$, $\Gamma_{\bar{X}} \propto |g^*|^2 = |g|^2$

No ASYMMETRY gets created at tree level. The same is true for $\Gamma_{X \to \bar{q}\bar{q}} = \Gamma_{\bar{X} \to qd}$.

- We need an additional diagram with complex couplings. Typically this happens at one loop level. There, we get WEAK CP phases that change sign from $X \to \bar{X}$ and

STRONG CP phases that remain the same

@ ONE LOOP :



ϕ, can also be a scalar, with a Yukawa $\in \mathbb{C}$.

$F \in \mathbb{C}$, loop functi

X decay : $\mathcal{A}_{X \to \ell u} = g + g_2 y \tilde{y} \overbrace{(A + iB)}^{F}$

$$\mathcal{A}_{\bar{X} \to \ell u} = g^* + \underbrace{g_2^* y^* \tilde{y}^*}_{\text{weak CP phases}} \underbrace{(A + iB)}_{\text{strong CP phase}} , \quad A, B \in \mathbb{R}$$

- The final result for the baryon asymmetry parameter @ one loop will be

$$\varepsilon_B = \underbrace{[\text{loop factor}]}_{10^{-2}\text{ number}} \frac{B}{|g|^2} \operatorname{Im}\left(g g_2^* y^* \tilde{y}^*\right)$$

- The g_i violate B, the $y \in \mathbb{C}$ violate CP and dropping inverse term in Boltzmann = OUT OF EQ.
All the conditions are satisfied.

[redacted]