2) REMINDER / NOTATION OF ΛCDM

The particle SM is valid at short distances, here we review ΛCDM, valid on scales > few Mpc.

SCALE FACTOR $a(t)$, today $t=t_0$ $a(t_0)=1$

$$\underbrace{d(t)}_{\text{proper physical distance}} = a(t)\, \underbrace{d_0}_{\text{comoving distance}}$$

recession velocity

Hubble law

$$\dot d = (a\dot{d_0}) = \frac{\dot a}{a} a d_0 = H d$$

linear in d

EXPANSION : $\dot a = \frac{da}{dt} > 0$, $H(t) = \frac{\dot a}{a}$ depends on the energy content.

REDSHIFT $\quad z+1 = \frac{\lambda_0}{\lambda_e} = \frac{a_0}{a_e} = \frac{1}{a}$

today : $a_0 = 1$, $z_0 + 1 = 1 \Rightarrow z_0 = 0$ (nearby objects have $z \simeq 0$)

early universe = high z and $z \approx \frac{1}{a}$.

- galaxy surveys (SDSS, 2DF) . $z \sim 1$
- furthest quasars $z \sim 6$ (Lyman α forest $z = 2-6$)
- fotons decouple at $z \sim 1100$. $a \sim 10^{-3}$

NEW IN 2022
Hubble finds the earliest star Earendel at $z \sim 6.2$

HUBBLE RATE / PARAMETER

$H(t)$ comes from the FRIEDMANN equation. It's value today $H_0 = H(t_0)$ is given by $H_0 = h \, 100 \frac{km}{s \, Mpc}$ where $h = 0.67 - 0.75$. (H_0 plot '21)

- EXPANSION RATE VS. ENERGY CONTENT is governed by the FRIEDMANN equation

$$H^2 = \frac{8\pi G}{3} \left(\underbrace{\rho_\gamma}_{\text{radiation}} + \underbrace{\rho_m}_{\text{matter}} + \underbrace{\rho_\Lambda}_{\text{c.c.}} + \underbrace{\rho_k}_{\text{curvature}} \right), \quad \rho = \frac{\text{energy}}{\text{Volume}}$$

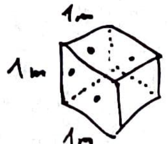
- $G = \frac{1}{M_{Pe}^2}$ (we use $\hbar = c = 1$ here) Newton's constant

$M_{Pe} = 1.2 \cdot 10^{19}$ GeV Planck mass

In a flat, Euclidean universe $\sum_i \rho = \rho_{cr}$, $k = 0$

$$\text{Today} \quad H_0^2 = \frac{8\pi G}{3} \rho_{cr}, \quad \rho_{cr} = \frac{3 H_0^2}{8\pi G} \approx h^2 \cdot 10^{-29} \frac{g}{cm^3}$$

$$\simeq \text{few} \frac{GeV}{m^3}$$



few protons (H-atoms) per 1 cubic meter

ENERGY DENSITY PARAMETERS

$$\Omega_i = \frac{\rho_i}{\rho_{cr}} \quad \text{or} \quad \omega_i = h^2 \Omega_i$$

often this cancels out

TYPES OF ENERGY DENSITY

RADIATION : ρ_γ or ρ_r : everything relativistic with $p \gg m$ (again $c=1$) is counted as radiation

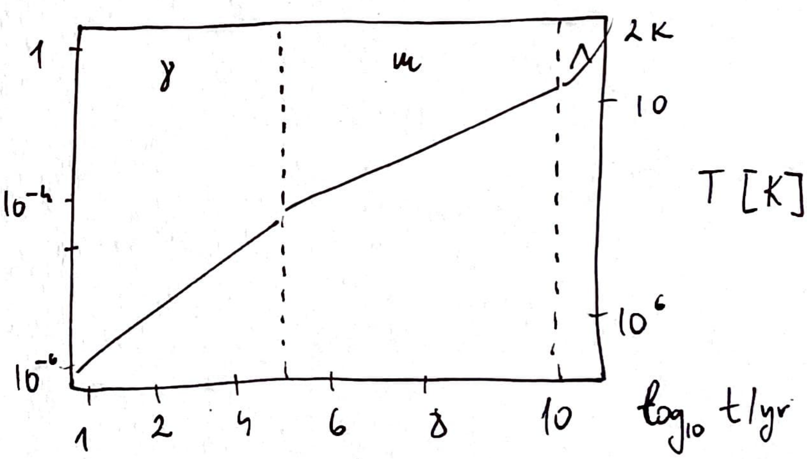
MATTER : ρ_m : whichever particle becomes non-relativistic with $p \ll m$ goes here.

C.C : $\rho_\Lambda = \text{const.}$

γ : $a \propto t^{1/2}$

m : $a \propto t^{2/3}$

Λ : $a \propto e^{\sqrt{\frac{\Lambda}{3}}t}$



Hubble rate comes from here:

γ : $\ln a = \ln c + \frac{1}{2}\ln t \; \Big/ \frac{d}{dt} \Rightarrow \frac{\dot a}{a} = H = \frac{1}{2t}$

m : $\ln a = \ln c + \frac{2}{3}\ln t \Rightarrow H = \frac{2}{3t}$

NUMBER DENSITY $\quad n = \frac{N}{V}$

- When N is conserved, we will get $n \propto a^{-3}$, so n of matter dilutes with time when a grows to 1.

- For NR (non-relativistic) species, we have:

  $E^2 = p^2 + m^2 \cong m^2 + \dots$, $E \simeq m$ and

  $\rho_m = m\, n$, which falls as a^{-3}.

- For radiation (relativistic species) $E \sim p$ $(m \sim 0)$

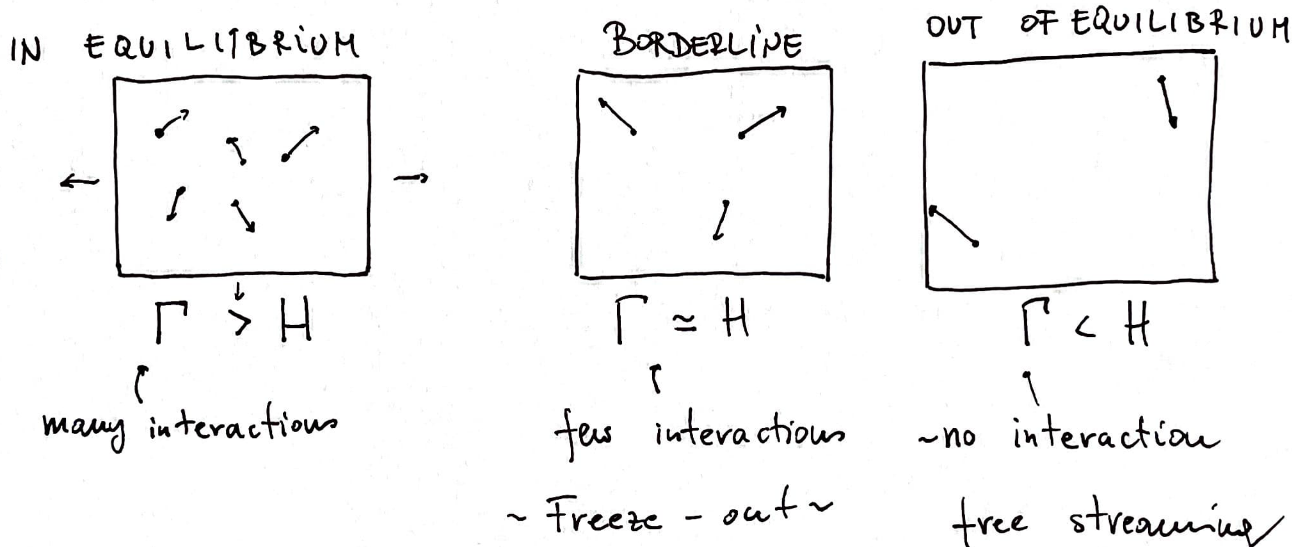
  $\rho_\gamma \simeq p\, n \propto \frac{1}{a} a^{-3} = a^{-4}$.

  Energy of photons redshifts with the expansion, so then ρ goes down faster. Thus the early universe is radiation dominated.

EQUILIBRATION

To justify our treatment of particles in the early universe as a TD (thermodynamic) system, the species have to form a "plasma". They have to interact often enough to exchange momenta (kinetic equilibrium) and their number densities (chemical equilibrium).

We will be comparing the thermal rates Γ with the Hubble rate H:



- The proof of equilibration is CMB, a perfect black body spectrum: $I_\nu = \dfrac{4\pi p^3}{e^{p/T} - 1}$.

- The measured value today is $T_0 = 2.75\,K.$

$= 2.4 \times 10^{-4}\,eV.$

- Note that $T_0 \ll m_e$, also $\lesssim m_\nu$.

The spectrum is fixed by $\frac{E_\gamma}{T_\gamma} = \text{const}$, $E_\gamma \propto \frac{1}{a}$

so. $T_\gamma = \dfrac{T_0}{a}$

we may drop γ and use $T(t) = \dfrac{T_0}{a(t)}$ as a proxy for a and time. We will be slightly more careful in relating T vs. a when more particles are present.

For now, we can take $T \propto a^{-1}$ at face value and relate H to T.

FRIEDMANN : $H^2 \propto \rho$ in radiation domination

γ : $\rho_\gamma \propto a^{-4} \Rightarrow H \propto a^{-2} \propto T^2$

EXAMPLE : $a_{eq} \sim 10^{-4} \rightarrow T_{eq} \simeq 10^4 T_0 \sim eV$

matter-radiation equality

EQUILIBRATION RATE Γ

In order to derive the n.f as a function of time, we need the interaction rate, given at some $a(t)$ or T. These are typically scatterings, annihilations and (inverse) decays, where:

thermally averaged cross-section

$$\Gamma = n \langle \sigma v \rangle .$$

number density

$[\Gamma] = [H] = [\frac{T^2}{M_{Pl}}] = 1$, $[n] = [T^3] = 3$

$\langle \sigma v \rangle$ has units of a cross-section $cm^2 \to GeV^{-2}$

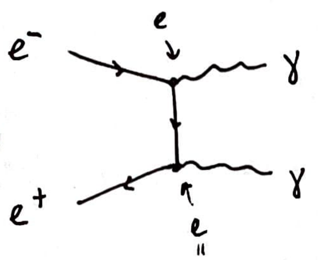
$[\langle \sigma v \rangle] = -2 \Rightarrow [\Gamma] = [n \langle \sigma v \rangle] = 3 - 2 = 1$ OK ✓

We will give a rigorous definition of $\langle \sigma v \rangle$ that enters into the Boltzmann equation. However, we can already use dimensional analysis to get a good approximation /understanding.

UNITS: Cross-section σ has units of area, cm^2. Rates Γ have units of t^{-1}, think $e^{-\Gamma t}$.

Thermal averaging will pick out the statistically most likely momenta with $p \simeq T$. The only question is how the interaction behaves with p.

FOR EXAMPLE, QED $e^+e^- \to \gamma\gamma$ equilibration



$\parallel$

coupling constant

$$\mathcal{A} \sim \bar{u}\gamma_\mu \frac{1}{\not{p}-m}\varepsilon\varepsilon \to p^0$$

$$\langle \sigma v \rangle \cong \frac{\alpha^2}{T^2} \quad , \quad \mathcal{A} \propto e^2/\cancel{q^2}\cdot\cancel{q^2}$$

$$|\mathcal{A}|^2 \propto \frac{e^4 \cancel{q^4}}{\cancel{q^4}} \quad \alpha = \frac{e^2}{4\pi}$$

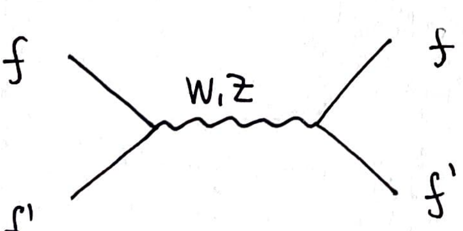
thermal averaging

$$\sigma \propto \frac{\alpha^2}{q^2}$$

This is essentially because A is massless $m_\gamma = 0$ and its propagator Δ is $\propto \frac{1}{q^2}$. $\quad \Delta(q^2) \propto \frac{1}{q^2}$

For massive propagators, we have $\Delta \propto \frac{1}{q^2 - M^2}$, so when T is below M, we can neglect q and get

(we take $T \gg m_f \sim m_e$)



$$\mathcal{A} \propto \frac{g^2}{M_W^2} \quad |\mathcal{A}|^2 \propto \frac{\alpha_W^2}{M_W^4} = G_F^2$$

so $\langle \sigma v \rangle \sim G_F^2 T^2$, because we get q^2 from kinematics.

- Of course, when the temperature or momentum exchange goes above M_W, we get a "massless" theory that behaves like QED

$$\text{high } T \gg M_W : \quad \mathcal{A} \sim \frac{g_2^2}{\cancel{q^2}}\,\cancel{q^2} \quad |\mathcal{A}|^2 \propto \frac{g_2^4}{\cancel{q^4}}\,\cancel{q^4}$$

$$\Downarrow$$

$$\sigma \propto \frac{\alpha_2^2}{q^2}$$

and $\langle \sigma v \rangle \sim \dfrac{\alpha_2^2}{T^2}$

- Thus at very high T : $\Gamma \approx n \langle \sigma v \rangle$

$$\approx T^3 \frac{\alpha_2^2}{T^2} = \alpha_2^2 T.$$

- Comparing this to the Hubble rate in radiation domination $H \sim \dfrac{T^2}{\sqrt{M_{Pl}}}$ (here we're estimating with dimensional analysis and dropping $\frac{8\pi}{3} \to 1$)

$$\Gamma \overset{(*)}{=} H$$

$$\underline{\underline{\alpha_2^2 T}} = \underline{\underline{\frac{T^4}{M_{Pl}^2}}}$$

Hubble grows faster with high T, eventually $H > T$ and weak interactions go out of equilibrium.

$T_{out} = \alpha_2^2 M_{Pl}$ and similarly for α, α_s.

$$(*)\ H^2 = \frac{8\pi G}{3}\rho_\gamma = \frac{8\pi}{3M_{Pl}^2} * \frac{\pi^2}{15} T_\gamma^4$$

We will discuss a few cases, where Γ becomes similar to H. Some are well understood SM processes, like γ decoupling, recombination and e^- cooling. Some may be discovered in the future, like DM freeze-out or baryogenesis.

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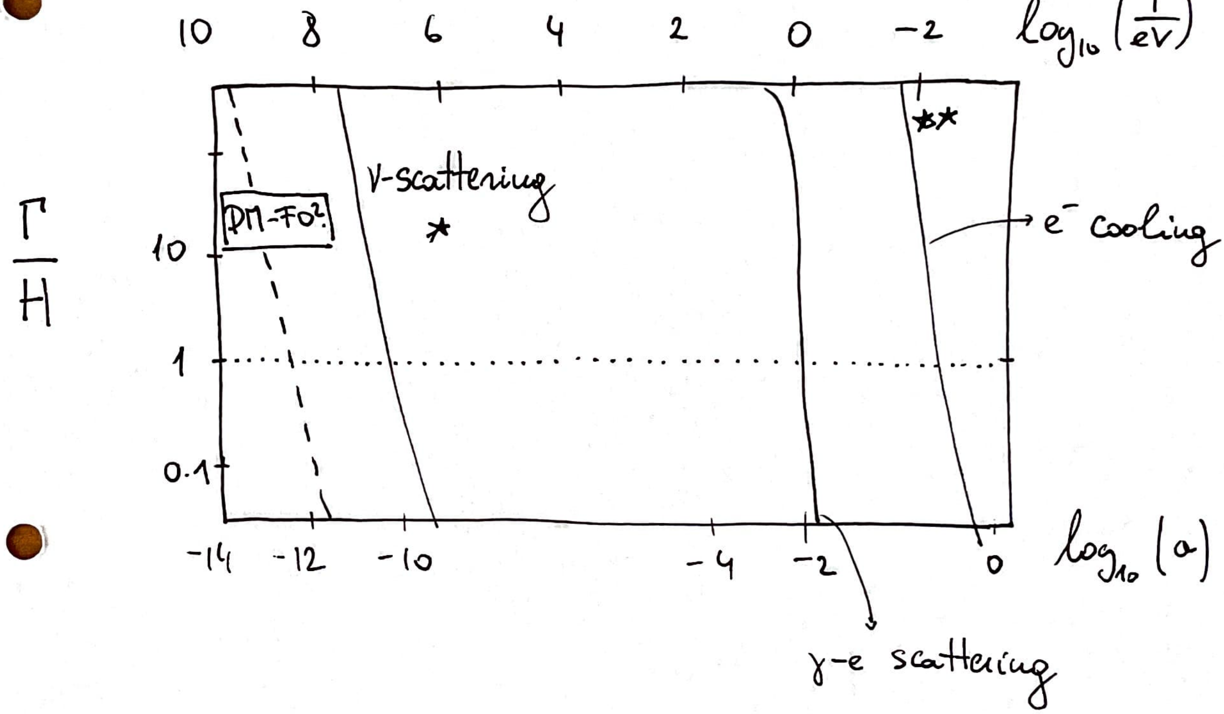


Going back in time to high T, different processes enter in equilibrium.

- $T \sim E_{Ry} \sim 13\,eV$: energetic photons destroy the atoms into nuclei and free electrons : $e^- - \gamma$ plasma forms

- $T \sim$ MeV : this is a typical nuclear binding energy scale. Above it, light nuclei H, D, He, Li get destroyed. The formation (when the universe cools) is called BBN (Big Bang Nucleosynthesis).

- $T \gtrsim$ GeV $\sim \Lambda_{QCD}$ we go from p, n, π, K to a dense plasma of q & g with many d.o.f.s.

- $T \gg 100$ GeV the entire SM is ~massless, including W, Z, h and eventually top.

What do we observe? Most relevant examples

Y_p , C_ℓ , P_k

Y_p: primordial He abundance

C_ℓ: CMB multipoles

P_k: matter power spectrum

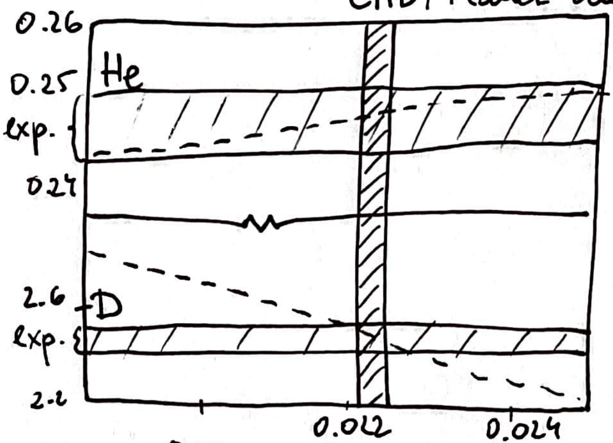
Which parameters are determined?

Ω_b , Ω_{DM} , N_{eff} , σ_8 , Ω_Λ .

BBN

T ~ MeV

CMB, Planck data



$$Y_p^{BBN} = \frac{\rho^{He}}{\rho^p + \rho^n}$$

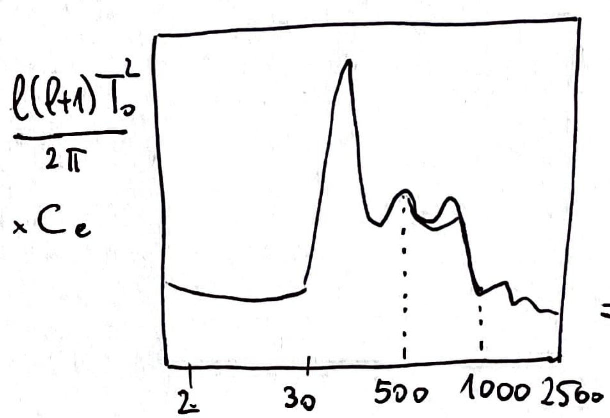
$$y_{DP} = 10^5 \frac{\rho_D}{\rho^p + \rho^n}$$

$\omega_b \sim 0.022$, $\Omega_b \sim 0.05 = 5\%$

We will learn how to predict and calculate Y_p^{BBN}.

After BBN, the p^+ and e^- scatter and cool down, with γ, e^- & p^+ interacting. At $T \sim 13.6$ eV or $z \sim 1100$ the ionitation level goes to zero (recombination) H atoms are formed and travel freely. At the same time (roughly) γs decouple $\Rightarrow$ CMB.

CMB : perfect black body formed at fixed z, we can decompose the 2D $(\theta, \varphi) \to \ell$ multipoles



measured to very small scales

$\frac{\pi}{\ell} < 0.1°$ high ℓ

$\Rightarrow$ requires Ω_{DM}, $\Delta N_{eff} \lesssim 1$

$\hphantom{\Rightarrow requires} \ \ \ ^2_{0.25}$

MATTER PERTURBATIONS

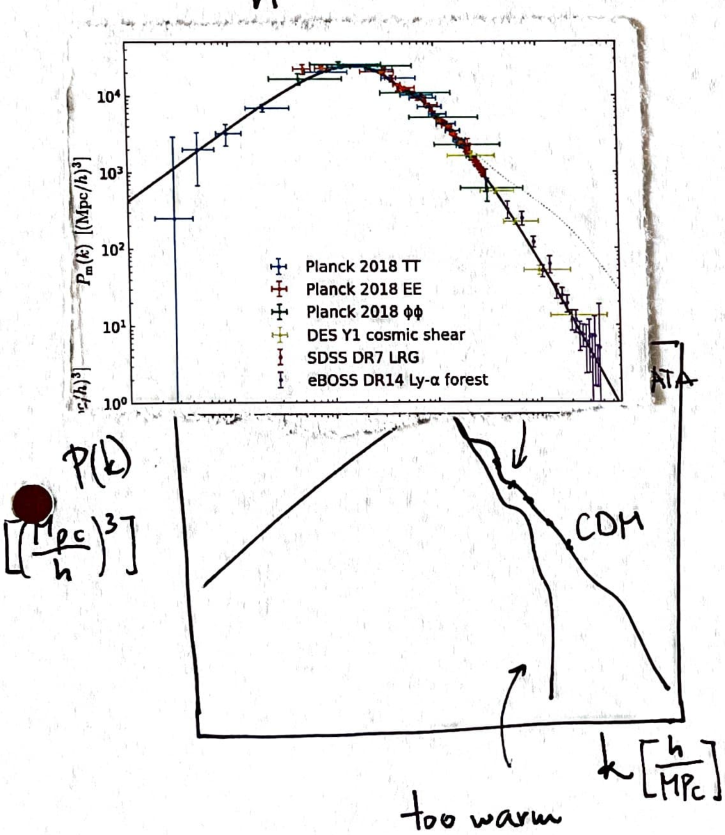
After decoupling universe waits for some time before structures start to form at $z \sim$ few.

Measurements of galaxies (SDSS, 2DF) and clusters are mapped in 3D (θ, φ, z), from where

$$\delta = \frac{n - \bar{n}}{\bar{n}} \xrightarrow{\text{FOURIER}} \tilde{\delta}(k) \Rightarrow \langle \tilde{\delta}(k), \tilde{\delta}(k') \rangle =$$

$$= (2\pi)^3 \, \delta(k - k') \, P(k)$$

$P(k)$ = MATTER POWER SPECTRUM



$\Rightarrow \Omega_b \sim 5\%$

$\Omega_{DM} \sim 25\%$

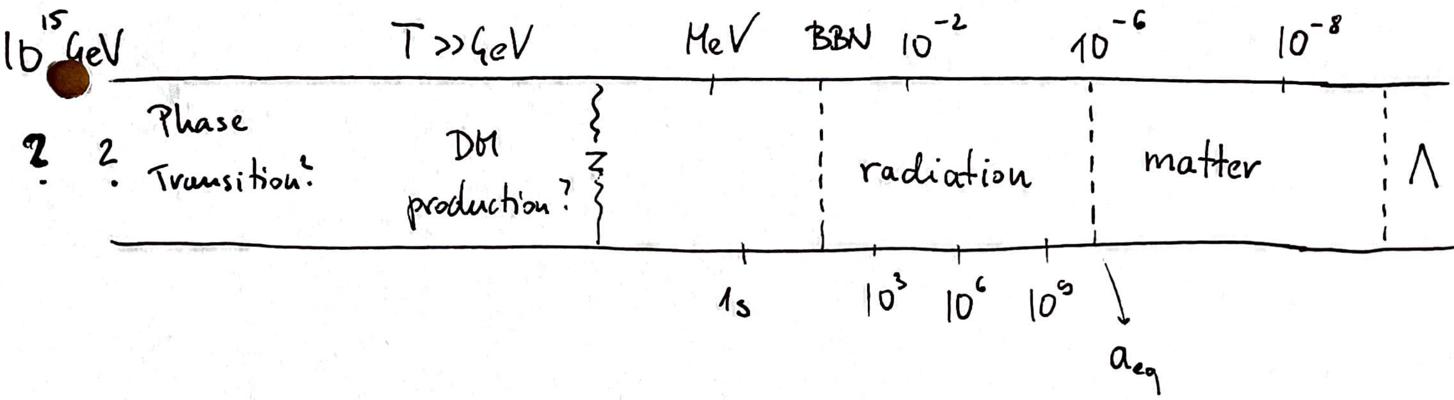
+ needs to be cold, we'll discuss candidates

Computing the features of $P(k)$ is beyond this course, but Ch 8 of Dodelson does it and CLASS is a great tool to use.

Finally at $z < 1/3$ the universe seems to be expanding at an accelerated pace by Λ

SN '98 - '99 give $\Omega_\Lambda \sim 0.7$, $\Omega_{tot} = 1$ (flat)

SUMMARY



TODAY : $\Omega_\gamma \sim 10^{-5}$, $\Omega_\nu \sim \Omega_\gamma$

we will compute both shortly

$\Omega_b \sim 5\%$

$\Omega_{DM} \sim 25\%$ ... we will discuss models for cold DM at length

$\Omega_\Lambda \sim 70\%$