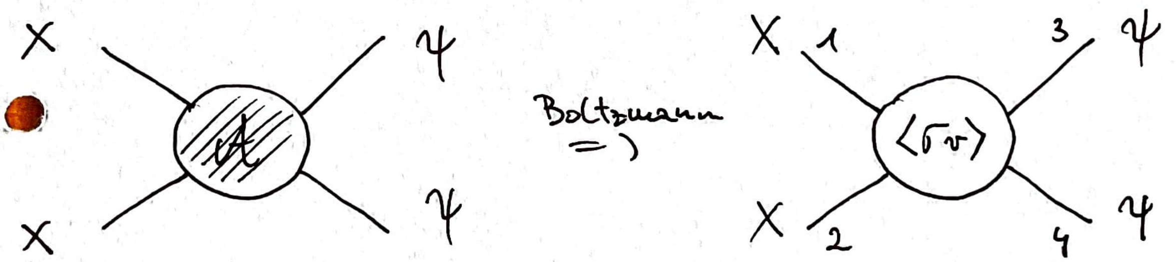
Dark Matter Production

- We will discuss three mechanisms for DM production
  1) Thermal freeze-out
  2) Freeze-in
  3) Relativistic decoupling & dilution

Freeze-out of WIMPs

- assume a stable X with $m_X \simeq 100\,\mathrm{GeV}$.
- at $T \simeq 100\,\mathrm{GeV}$, the SM is in equilibrium with $g_* \sim 100$
- assume X is weakly coupled to the SM w. Z_2



- Like for ν's, the rate Γ will go down below H and n_X will freeze-out. Let us calculate:

  i) T_{fo}

  ii) $n_X(T_{fo}) \rightarrow \Omega_X$ (today)

- Our aim is to estimate the decoupling temperature / freeze-out temperature and the remaining yield, or the DM relic abundance.

SIMPLIFIED BOLTZMANN EQUATION

this term depletes X via $XX \to \psi\bar{\psi}$

$$a^{-3}\frac{d}{dt}(n_X a^3) = \langle\sigma v\rangle n_X^{(0)2}\Bigg(\frac{n_\psi^2}{\underset{\substack{\| \\ 1}}{n_\psi^{(0)2}}} - \frac{n_X^2}{n_X^{(0)2}}\Bigg) = \langle\sigma v\rangle\left(n_X^{(0)2} - n_X^2\right)$$

this term creates X by $\psi\bar{\psi}$ annihilation

1 because ψ remains in eq. throughout the DM freeze-out

- In many analyses, one introduces the yield variable

$$Y = \frac{n_X}{T^3} \quad \text{or} \quad \frac{n_X}{s} \quad (\text{in case } g_* \text{ is changing during f.o.})$$

This is very useful because $n \propto a^{-3}$ and so is $s \propto a^{-3}$, or $\frac{d}{dt}(sa^3) = \text{const}$.

$$\Rightarrow a^{-3}\frac{d}{dt}\left(Y(aT)^3\right) = \cancel{a^{-3}}\cancel{a^3}T^3\dot{Y} = \langle\sigma v\rangle\left(Y^{(0)2}T^6 - Y^2T^6\right)$$

$$\frac{dY}{dt} = T^3\langle\sigma v\rangle\left(Y^{(0)2} - Y^2\right).$$

- At early times, high T, we are in equilibrium and the Saha equation is simply: $Y = Y^{(0)}$ for $\Gamma \gg H$ & $T \gg m_X$.

- To keep track of t, or better T, it is useful to introduce a dimensionless $x = \frac{m_X}{T}$.

- What do we expect?

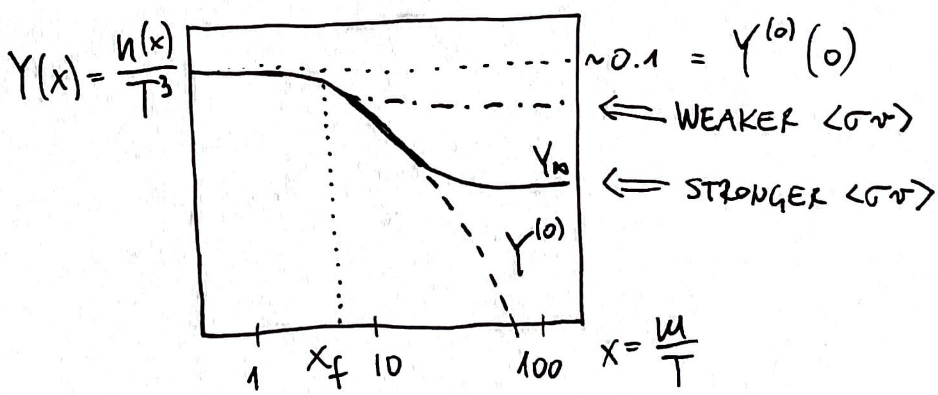
$x \ll 1$ = high $T \gg m_X$, early times, in equilibrium

$$Y(x \simeq 0) \cong Y_1^{(0)} \; n_X^{(0)} T^{-3} = \frac{g}{2\pi^2} \int_0^\infty \frac{y^2 \, dy}{e^{\sqrt{x^2+y^2}} \pm 1} = Y^{(0)}(x)$$

The equilibrium yield $Y^{(0)}$ goes to the $m=0$ distribution

$$Y^{(0)}(x \ll 1) \simeq \frac{g}{2\pi^2} \int_0^\infty \frac{y^2 \, dy}{e^{y} \pm 1} \quad \text{at early times.}$$

At late times, $Y^{(0)}(x \gtrsim 1)$ becomes Boltzmann suppressed, because XX keep annihilating but the reverse is small.



- If Γ (or $\langle \sigma v \rangle$) is large, Y will follow $Y^{(0)}$ for longer and will decrease more $\Rightarrow$ less DM.

- As soon as we hit $\langle \sigma v \rangle$ such that $\Gamma \sim H \Rightarrow$ FREEZE-OUT.
Stronger $\langle \sigma v \rangle \Rightarrow$ HIGHER x_f or lower T_{fo}.

- For $m_\chi \simeq 100$ GeV, $T_{fo} = \mathcal{O}(10\,\mathrm{GeV})$ ⇒ radiation dominated universe where $H^2 \propto T^4 \propto a^{-4}$

$$T = \frac{m}{x} \quad \text{or: } \boxed{x \propto a}$$

$$\Downarrow$$

$$H \propto a^{-2} \propto x^{-2}$$

$$\Downarrow$$

$$H(x) = c \cdot x^{-2} \quad / : H(x=1)$$

$$\ln x = \ln c + \ln a,$$

$$\frac{\dot{x}}{x} = \frac{\dot{a}}{a} = H \Rightarrow \dot{x} = Hx.$$

This gives the direct connection between time (derivatives) and the temperature, such that:

$$\dot{Y} = Y'\dot{x} = Y' H x = Y' \frac{H(x=1)}{x^{\cancel{2}}} \cancel{x},$$

$$\frac{dY}{dt} = \frac{dY}{dx}\frac{dx}{dt}$$

where we used: $\frac{H(x)}{H(x=1)} = \frac{\cancel{c}\, x^{-2}}{\cancel{c} \cdot 1} = x^{-2}$, or [redacted]

$$\text{or: } H(T) = \frac{H(T=m_x)}{x^2} = \frac{H(m_x)}{x^2}$$

We have:

$$Y' \frac{H(m_x)}{x} = T^3 \langle \sigma v \rangle \left(Y^{(0)2} - Y^2\right)$$

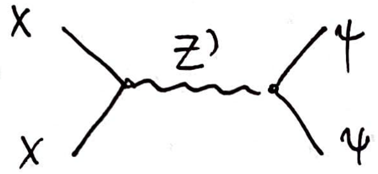
$$= \left(\frac{m_x}{x}\right)^3 \langle \sigma v \rangle \left(Y^{(0)2} - Y^2\right), \quad \lambda = \frac{m_x^3 \langle \sigma v \rangle}{H(m_x)}$$

Then, finally: $\boxed{Y' = -\frac{\lambda}{x^2}\left(Y^2 - Y^{(0)2}\right)}$

- We got to the simplified DM freeze-out equation, where λ characterizes the strength of annihilation: large λ means more annihilation and less DM remains.

- Let us calculate the freeze-out T_{fo} from $\Gamma = H$

i) $n_X^{(0)} = 2\left(\frac{m_X T}{2\pi}\right)^{3/2} e^{-m_X/T}$ (we are at $x \gtrsim 1$, so X is already NR, $g=2$)

ii) 

$\langle \sigma v \rangle \simeq \frac{\alpha_2^2}{M_W^4} m_X^2 \rightarrow \frac{\alpha_2^2}{M_W^2}$, $m_X \lesssim M_W$.

or: $\frac{\alpha_2^2}{m_X^2}$ $m_X \gg M_W$

Plugging these into Γ, we get

$$\Gamma = n_X^{(0)} \langle \sigma v \rangle = 2\left(\frac{m_X T}{2\pi}\right)^{3/2} e^{-m_X/T} \frac{\alpha_2^2}{M_W^2}$$

$$\simeq 10^{-4} x^{-3/2} e^{-x} \frac{m_X^3}{\underset{m_X}{\underset{\gtrsim}{M_W^2}}} \simeq 10^{-4} x^{-3/2} e^{-x} m_X .$$

Compare this to the Hubble rate

$$H \cong \sqrt{\frac{8\pi^2 g_*(m_X)}{30 M_{Pe}^2}}\, T^2 \simeq 100 \frac{m_X^2}{M_{Pe}} x^{-2}$$

Freeze-out : $\frac{\Gamma}{H} = 1 = 10^{-6} \frac{\cancel{m_X} M_{Pe}}{m_X^{\cancel{2}}} x^{-3/2+2} e^{-x}$

or : $e^x x^{-1/2} = 10^{-6} \frac{M_{Pe}}{m_X} \simeq 10^{10}$,

$x_f \sim 24$. (20-30)

- We ended up with $T_{fo} = \frac{m_X}{x_f} \simeq \frac{1}{20} m_X$, quite below m_X, which is self-consistent with $T < m_X$.

- Now that we have T_f, we can get the relic yield Y.

$$@\, x = 1 \quad \frac{dY}{dx} \sim Y \ll \lambda (Y^2 - Y_{eq}^2)$$

- We had: $\lambda = \dfrac{m_X^3 \langle \sigma v \rangle}{H(m_X)} \simeq \dfrac{m_X^3 \left(\frac{\alpha_2^2}{m_X^2}\right) M_{Pl}}{10^2\, m_X^2} \sim 10^{-4} \dfrac{M_{Pl}}{m_X} \sim \underline{\underline{10^{12}}}$

Thus $\lambda \ggg 1$ and $Y = Y^{eq}$ when $x < 1$.

- Near $\Gamma \sim H$, Y starts to differ from $Y^{(0)}$, which is dropping exponentially. Neglecting $Y^{(0)2}$, we get

$$Y' = -\frac{\lambda}{x^2} Y^2 \quad \text{or} \quad \frac{dY}{Y^2} = -\lambda \frac{dx}{x^2} \Rightarrow Y^{-1}\Big|_{x_f}^{x_\infty} = -\lambda \frac{1}{x}\Big|_{x_f}^{x \to \infty}$$

Thus the final freeze-out yield Y_∞ is:

$$\frac{1}{Y_\infty} - \frac{1}{Y_f} = -\frac{\lambda}{\infty} + \frac{\lambda}{x_f} \quad \text{and} \quad Y_f > Y_\infty$$

Our final estimate is:

$$\boxed{Y_\infty \simeq \frac{x_f}{\lambda}} \qquad \& \ x_f \sim 20\text{-}30.$$

- This is a pretty good approximation. For further analytic improvement, see [1204.3622] by Steigman, Dasgupta & Beacom.

- The final step of our calculation is to obtain the DM relic abundance Ω_X.

$$\Omega_X = \frac{\rho_X}{\rho_{cr}} = \frac{m_X \overset{\swarrow \text{ today}}{n_X}}{\rho_{cr}}$$

- After freeze-out: $n_{Xf} = Y_\infty T_1^3$, $T_1 \lesssim T_f$.

The interactions are turned off and $\frac{d(n_1 a^3)}{dt} = 0$, or

$$n_{Xf} a_1^3 = \underbrace{n_X}_{\text{today},\ a_0 = 1} a_0^3 \Rightarrow n_X = Y_\infty T_1^3$$

From entropy conservation, we get $g_*(T_1) T_1^3 = g_*(T_0) \underbrace{T_0^3}_{\text{CMB today}}$

$$\Rightarrow n_X = Y_\infty T_0^3 \frac{g_*(T_0)}{g_*(T_1)} \simeq \frac{1}{30} T_0^3 Y_\infty$$

- Let us remember that $Y_\infty = \frac{x_f}{X} = \frac{x_f H(m_X)}{m_X^3 \langle \sigma v \rangle}$

and: $H^2(m_X) = \frac{8\pi G}{3} \rho_\gamma(m_X) = \frac{8\pi^3 G}{90} g_*(m_X) m_X^4$.

Combining: $\Omega_X = \frac{m_X}{\rho_{cr}} \cdot Y_\infty \underbrace{\frac{g_*(T_0)}{g_*(T_1)}}_{\simeq 1/30} T_0^3 = \frac{\cancel{m_X}}{\rho_{cr}} \frac{x_f}{\cancel{m_X^3} \langle \sigma v \rangle} \sqrt{\frac{4\pi^3 G}{45} g_*(m_X)} \times \cancel{m_X^2} \cdot T_0^3 \frac{1}{30}$

$$\Omega_X = \sqrt{\frac{4\pi^3 g_*}{45 M_{Pl}^2}} \frac{x_f T_0^3}{30 \rho_{cr} \langle \sigma v \rangle}$$

$$\Omega_{DM} h^2 \simeq 0.1 \left(\frac{x_f}{20}\right) \left(\frac{g_*}{100}\right)^{1/2} \frac{10^{-26} \frac{cm^3}{s}}{\langle \sigma v \rangle}.$$

- After this, somewhat lengthy derivation, a summary and some comments are in order.

FREEZE-OUT SUMMARY

1) Assume a stable DM candidate X with $m_X \sim M_W$, which is weakly coupled and can annihilate to SM $XX \to \psi\psi$.
This forces it to be in thermal equilibrium at high T, early t.

$$n_X = Y_X T^3 \quad , \quad Y_X \simeq Y_X^{(0)} \quad \text{at } \frac{m_X}{T} = x \ll 1.$$

2) These interactions become weaker in an expanding universe when $\Gamma \lesssim H$, which happens at $x_f \sim 20$.

3) At x_f, we have $Y \gg Y_{eq}$ and we calculate $Y_\infty \simeq \frac{x_f}{\lambda}$, where $\lambda \propto \langle \sigma v \rangle$. Afterwards n_X scales as a^{-3}, does not interact and we get n_{today} from entropy conservation

4) The final abundance $\Omega_X = \frac{m_X n_X (T_0)}{\rho_{cr}}$ is independent of m_X and scales as $\frac{x_f}{\lambda}$.

$$\boxed{\Omega_X h^2 = 0.1 \left(\frac{x_f}{20}\right)\left(\frac{g_*}{100}\right)^{1/2} \frac{10^{-26}\ \frac{cm^3}{s}}{\langle \sigma v \rangle}}$$

- The cross-section $\langle\sigma v\rangle$ is of typical electroweak size:

$$\langle\sigma v\rangle \simeq 10^{-26}\,\frac{cm^3}{s} = 10^{-26}\;0.3\cdot10^{-6}\,(10^{13}\,fm)^2 = 3\cdot10^{-7}\,fm^2$$

$$= 3\cdot10^{-7}\;4\cdot10^{-2}\,GeV^{-2} \sim 10^{-8}\,GeV^{-2},$$ which is

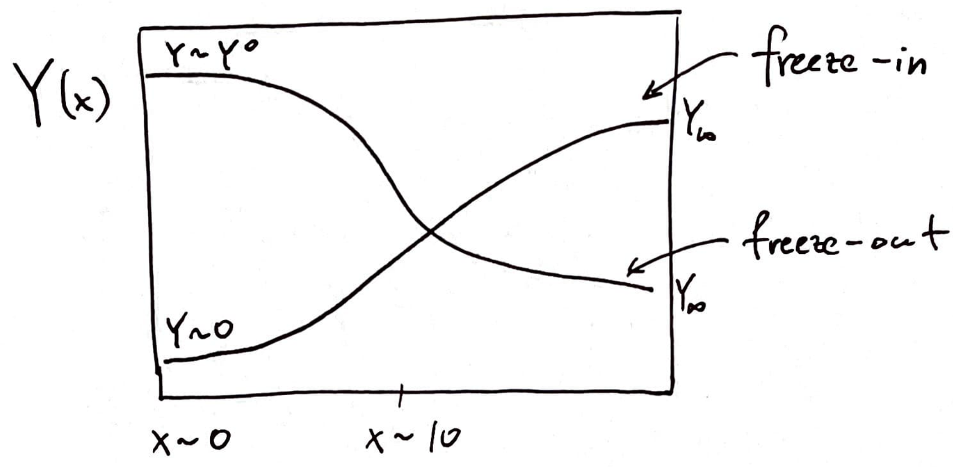
$$\sigma_w \simeq \frac{\alpha_2^2}{M_W^2} \sim 10^{-4}\cdot(10^2\,GeV)^{-2} \simeq 10^{-8}\,GeV^{-2},$$ hence the

WIMP = weakly interacting massive particle.

- MORE ON WIMPS:

  - detailed treatment of $\langle\sigma v\rangle$ in Gondolo & Gelmini '93,
  - three exceptions in freeze-out (co-annihilations $XY\to 44$, mass thresholds and up-scatterings) in Griest, Seckel '91,
  - refined analytics for x_f and Y_∞ (very nice read) in [1204.3622]

FREEZE - IN PRODUCTION OF DM

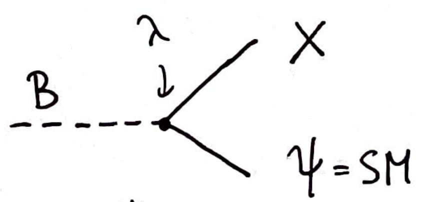


- In freeze-out scenarios we considered species that were in thermal equilibrium at early times / high T.

- FREEZE-IN are the opposite. We start with a very small yield of an out-of-equilibrium species and then slowly build up over time.

- X = DM, very weakly coupled, [0911.1120]
  ψ = SM, in thermal equilibrium,
  B = a heavy particle in eq. (bath, hence B).

$\mathcal{L} \ni \lambda \bar{X} B \psi,$



(toy model)

$\Downarrow$

Decay rate $\Gamma_B = \frac{\lambda^2}{8\pi} m_B .$

- The simplest estimate we can do is :

$$Y_x = \frac{n_x}{s} \simeq \underbrace{\gamma_B^{-1}\Gamma_B^0}_{} t \simeq \frac{m_B}{T}\Gamma_B^0 H^{-1},$$

decay rate in the medium, when B is boosted
So $\langle E \rangle \sim T = \gamma m$

$\tau \equiv \gamma \tau^0$, so : $\Gamma = \gamma^{-1}\Gamma^0$

where we used $H = \frac{1}{2t}$ (we're neglecting $O(1)$ numbers)

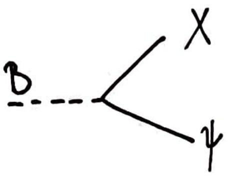
$$\boxed{Y_x \simeq \frac{(m_B \lambda)^2 M_{Pl}}{T^3}}$$

- This is enough to understand the principle. Freeze-in is dominated by low T (late times) and <u>grows</u> with λ (which has to be small enough though to justify the initial out-of-equilibrium), and shuts off at high T. This is the opposite of F-O, where $\Omega \propto \frac{1}{\langle \sigma v \rangle}$, and one starts with a large $Y \sim Y^{(0)}$, which then drops exponentially until $\Gamma \lesssim H$.

- Let us look at a more rigorous derivation, using the $1 \to 2$ simplified Boltzmann eq.

FREEZE-IN SIMPLIFIED

- neglect scatterings, inverse decays and $\bar{f} \to f$, all of these are ok because our Y is tiny, plus $n_\psi = n_\psi^{(0)}$



$$a^{-3}\frac{d}{dt}\left(n_X a^3\right) \equiv \int_{\Pi_{X,B,\psi}} |\mathcal{A}|^2 f_B$$

$$\simeq 2 g_B \int_{\Pi_B} \Gamma_B m_B f_B$$

- For simplicity, we drop the subscripts B and evaluate

$$a^{-3}\frac{d}{dt}\left(n_X a^3\right) = \cancel{2} g m \Gamma \int_0^\infty \frac{4\pi p^2}{(2\pi)^3 \cancel{2}E} dp \; e^{-E/T}, \quad E^2 - p^2 = m^2 \Rightarrow p\,dp = E\,dE$$

$$= \frac{g m \Gamma}{2\pi^2} \int_m^\infty \frac{\cancel{E}\,dE}{\cancel{E}} \sqrt{E^2-m^2}\, e^{-E/T}, \quad x = \frac{E}{T}, \text{[redacted]}$$

$$= \frac{g m \Gamma}{2\pi^2} \int_{\frac{m}{T}}^\infty \sqrt{x^2 - \left(\frac{m}{T}\right)^2}\, e^{-x}\, dx \cdot T^2, \quad \frac{m}{T} = y,$$

$$= \frac{g m^3 \Gamma}{2\pi^2} \underbrace{\int_1^\infty \sqrt{y^2-1}\, e^{-xy}\, dy}$$

$$= \frac{g m^3 \Gamma}{2\pi^2} \frac{K_1(x)}{x} = \frac{g m^2 \Gamma}{2\pi^2} T\, K_1\!\left(\frac{m}{T}\right).$$

- To solve for the abundance, we introduce the usual yield $Y = \frac{n}{s}$ and work in radiation domination

• So, we have: $Y_x = \dfrac{n_x}{s}$, $\underbrace{\dfrac{d}{dt}(sa^3)=0}_{\text{entropy conservation}}$, $\underbrace{H \propto \dfrac{1}{t} \propto T^2}_{\text{radiation domination}}$.

$$H = \frac{1}{2t}$$

$$\Downarrow$$

$$2 \ln T = - \ln t$$

$$2 \frac{dT}{T} = - \frac{dt}{t}$$

$$\Downarrow$$

$$\frac{dT}{dt} = - HT$$

$$a^{-3} \frac{d(n_x a^3)}{dt} = \cancel{a^{-3}} s \cancel{a^3} \frac{dY_x}{dt} = s \frac{dY_x}{dT} \frac{dT}{dt} ,$$

Boltzmann becomes

$$- s \frac{dY_x}{dT} H \cancel{T} = \frac{g m^2 \Gamma}{2\pi^2} \cancel{T} K_1\left(\frac{m}{T}\right)$$

$$\int_0^{Y_x} dY_x = - \frac{g m^2 \Gamma}{2\pi^2} \int_{T_{max}}^{T_{min}} \frac{K_1\left(\frac{m}{T}\right)}{sH} dT ,$$

$$H \simeq 1.7 \sqrt{g_*} \frac{T^2}{M_{pe}} , \quad s = \frac{2\pi^2}{45} g_* T^3 , \quad x = \frac{m}{T}$$

$$dx = - \frac{m}{T^2} dT$$

$$= \frac{45 \, g \, \Gamma M_{pe}}{1.7 \; 4 \pi^4 m^2 g_*^{3/2}} \underbrace{\int_{x_{min} \simeq 0 \; (T \to \infty)}^{x_{max} = \infty} K_1(x) x^3 dx}_{\frac{3\pi}{2}}$$

$$\boxed{Y_x = \frac{135}{1.7 \; 8\pi^4} \frac{g_B}{g_*^{3/2}} \left(\frac{M_{pe} \Gamma_B}{m_B^2}\right)}$$

, as initially expected $Y_x \propto \Gamma_B$

• Rescaling the Y to today, we get the relic abundance

$$\Omega_x = \frac{\rho_x}{\rho_{cr}} = \frac{m_x n_x}{\rho_{cr}} = \frac{m_x Y_x s_0}{\rho_{cr}} = \frac{10^{27}}{g_*^{3/2}} \left(\frac{m_x \Gamma_B}{m_B^2}\right)$$

• To get $\Omega_x = \Omega_{DM} \sim 1/4$, we need

$$\lambda = 10^{-13} \sqrt{\frac{m_B}{m_x}} \left(\frac{g_*(m_B)}{100}\right)^{3/4} \left(\frac{g_{Bich}}{10^2}\right)^{-1/2} .$$

- ## Thermal freeze-in of sterile neutrinos

Neutrino mass $\Longleftrightarrow$ Dark matter

$L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}$, ν_s ... sterile neutrino with Majorana mass

ϕ ... SM Higgs doublet $\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$

constant $v = 246$ GeV

$$\mathcal{L} = \mathcal{L}_{SM} + \underbrace{y_D \bar{L} \phi \nu_s}_{\text{DIRAC mass}} + \underbrace{M_s \nu_s^T C \nu_s}_{\text{MAJORANA mass}}$$

$$\mathcal{L}_{mass} \sim (\bar{\nu}_L \ \bar{\nu}_s^c) \begin{pmatrix} 0 & m_D \\ m_D & M_s \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_s^c \end{pmatrix} \Rightarrow$$

SEESAW

$m_s \sim M_s$

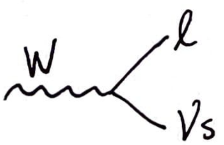
$m_\nu \sim - m_D^T M_s^{-1} m_D$

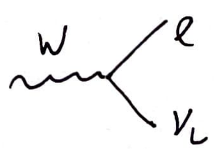
- ν_s are very weakly coupled and can be heavier than the SM neutrinos ν_L. Because of the off-diagonal term, ~~the~~ the mass states are rotated

$$\nu_L^{(m)} \sim \nu_L + \Theta \nu_s \,, \quad \nu_s^{(m)} \sim \nu_s - \Theta \nu_L$$

where Θ is small $\quad \Theta \sim \frac{m_D}{M_s} \cong \left(\frac{m_D^2}{m_s^2}\right)^{1/2} \cong \sqrt{\frac{m_\nu}{m_s}}$.

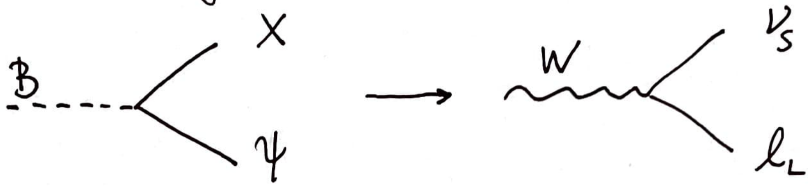
- For $m_\nu \sim 10^{-4}$ keV and $m_s \sim$ keV $\Rightarrow \Theta^2 \lesssim 10^{-4}$, small.

- This means 

$= \Theta \times$ 

$$\Gamma_s = \underbrace{\langle \sigma v \rangle}_{G_F^2 T^2 \theta^2} \underbrace{n_s}_{\sim T^3} \simeq G_F^2 T^2 \theta^2 T^3 \sim \theta^2 G_F^2 T^5 .$$

- To get an estimate for the production, we translate the general toy model into this case



$m_B = M_W$, $m_X = m_{\nu_s}$, $\Gamma_B = \Gamma_{W \to \nu_s l} = \frac{\alpha_W}{4} \theta^2 M_W$

$g_*(m_B) = g_*(M_W) \simeq 100$

$$\Omega_{\nu_s} h^2 \cong 0.1 \left(\frac{m_{\nu_s}}{\text{keV}}\right) \left(\frac{\theta}{10^{-7}}\right)^2 .$$

- This is a very naive picture, to properly study sterile ν DM, one has to take into account oscillations and T-dependent potentials, resonance effects.

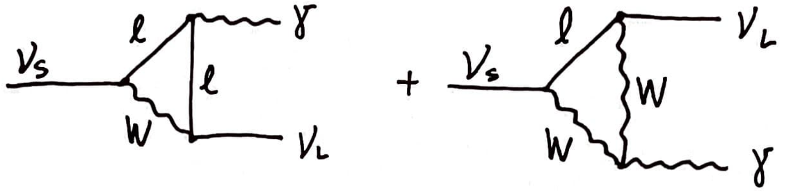
$$\underset{\text{vacuum mixing}}{\theta_0} \longrightarrow \theta(T) \simeq \frac{\theta_0}{1 + \left(\frac{T}{T_0}\right)^6} , \quad T_0 \sim 100\,\text{MeV}$$

$$\Downarrow$$

$$\Omega_{\nu_s} h^2 = 0.1 \left(\frac{\theta^2}{3 \cdot 10^{-9}}\right) \left(\frac{m_{\nu s}}{3\,\text{keV}}\right)^{1.8}$$

↳ This is the Dodelson - Widrow mechanism.

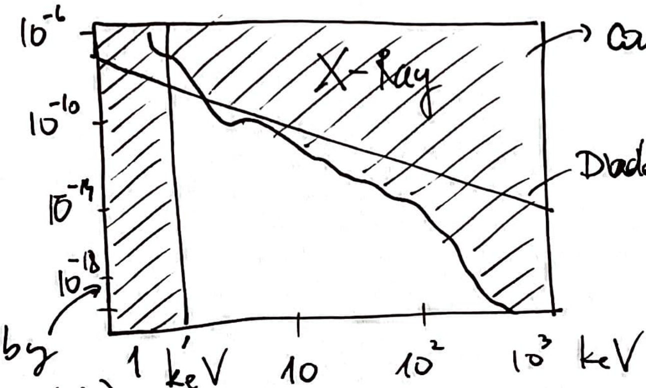
- The ν_s scenario is strongly constrained by indirect searches for X-rays. At loop level, ν_s is destabilized



, $E_\gamma \sim \frac{m_{\nu_s}}{2}$.

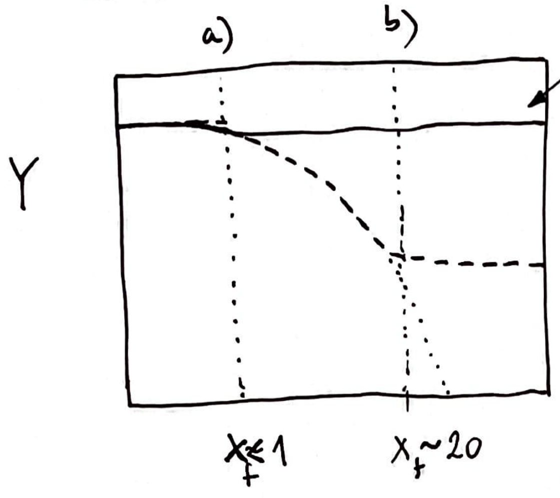
X-ray

with : $\tau_{\nu_s \to \nu\gamma} = \frac{1}{\Gamma_{\nu_s \to \nu\gamma}} = 10^{30}\,\mathrm{s} \left(\frac{10^{-8}}{\Theta^2}\right)\left(\frac{\mathrm{keV}}{m_{\nu_s}}\right)^5$

- 

- There are caveats that can save the DW picture, either by a primordial lepton number asymmetry or via additional hidden interactions of ν_s.

THERMAL OVERPRODUCTION & ENTROPY DILUTION

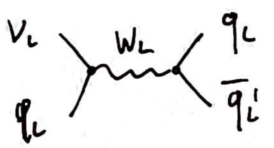


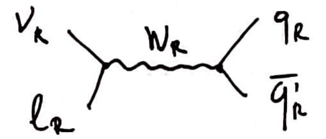
a) RELATIVISIC FREEZE-OUT (like ν in the SM, $T_f \gg m_\nu$) happens when $x \ll 1$, as opposed

b) to the WIMP, which freezes out at $x_f \sim 20$, so $T_f \sim \frac{m_f}{20}$.

- Because Y is higher, it may lead to overproduction.

For an explicit example consider that ν_s have some weaker interaction $SU(2)_R$; ν_R (LEFT-RIGHT MODEL)

SM: 

LRSM: 

$$G_F^2 T^5 = \sqrt{g_*} \frac{T^2}{M_{Pe}}$$

$$\Rightarrow T_\nu \simeq 1\,\mathrm{MeV},$$

$$G_F^2 T^5 \left(\frac{M_W}{M_{W_R}}\right)^4 = \sqrt{g_*} \frac{T^2}{M_{Pe}}$$

$\parallel$ WEAKER, $M_{W_R} \geq (5-6)\,\mathrm{TeV}$

$$T_f \sim 400\,\mathrm{MeV} \left(\frac{M_{W_R}}{5\,\mathrm{TeV}}\right)^{4/3}.$$

- This is very generic: weaker interactions decouple earlier.

- Repeating the same arguments as for ν_L, and taking $m_{\nu_R} \geq \mathrm{keV}$ from stellar constraints, we get

$$Y_{\nu_R}\Big|_{T_f} = \frac{n_{\nu_R}}{s}, \quad n_{\nu_R} = 2 \cdot \frac{3}{2\pi^2} \zeta(3) T_f^3, \quad s = \frac{2\pi^2}{45} g_{*s}(T_f) T_f^3,$$

- We then rescale to today $n_{\nu_R} = Y_{\nu_R}(T_f)\, s_0$,

where $s_0 = s(\text{today}) = \frac{2\pi^2}{45}\left(2 + 2\cdot 3\cdot \frac{7}{8}\cdot \frac{4}{11}\right)T_\gamma^3 = \frac{2\pi^2}{45}\frac{43}{11}T_\gamma^3$

$\simeq 3000\ \text{cm}^{-3}$

- The final relic energy density is OVER-ABUNDANT:

$$\Omega_{\nu_R} = 3.3\left(\frac{m_{\nu_R}}{\text{keV}}\right)\left(\frac{70}{g_*(T_f)}\right) \simeq 12\cdot\Omega_{DM}$$

ENTROPY DILUTION via LATE DECAYS

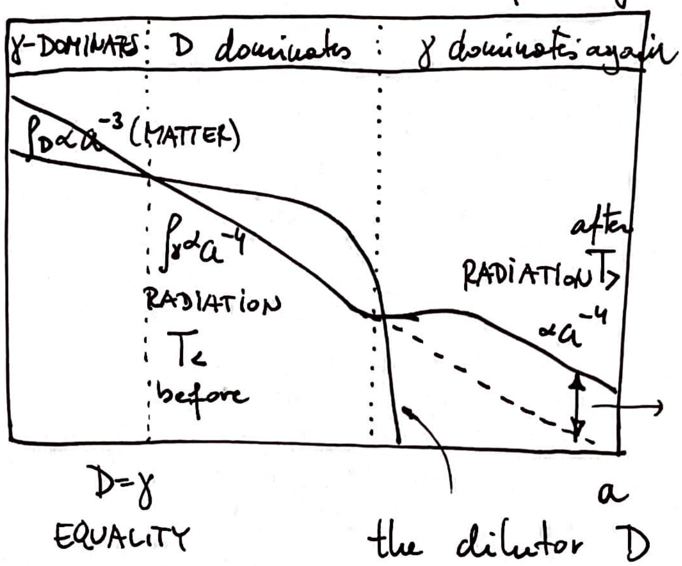
[book by Kolb & Turner, paper by Scherrer, Turner '85]

- The issue of overproduction (DM, monopoles, other relics) can be addressed and resolved by a process which is non-adiabatic. This means $sa^3 \neq$ const. and some entropy gets produced. Physically this happens by a massive particle D that dominates ρ and decays into radiation.

This happens after DM has frozen-out and if D decays into the SM ($e^\pm$, q, γ...) and not into DM, this effectively dilutes the n_X. Let's calculate the amount of entropy dilution in this fascinating case.

@ time of decay τ_D

ρ



photon plasma heats up

the dilutor D decays away and releases entropy.

Before Decay, we have matter dominance by D

$$H_<^2 = \frac{8\pi G}{3}\rho_m = \frac{8\pi G}{3} m_D Y_D s = \frac{16\pi^3}{135 M_{Pl}^2} m_D Y_D g_* T_<^3$$

$$= \Gamma_D^2 \leftarrow \Gamma_D \text{ is the } \tau_D^{-1} \text{ of the dilutor D.}$$

$H = \tau_D^{-1}$ is the assumption / approximation of instantaneous decay at the $t = \tau_D$.

photon T before decay : $$T_< = \left(\frac{\Gamma_D^2 M_{Pl}^2}{Y_D m_D g_*}\right)^{1/3}.$$

After Decay all the energy goes into radiation

$$\Gamma_D = H_> = 1.7\, g_*^{1/2} \frac{T_>^2}{M_{Pl}} \quad \text{or} : \quad T_> = \sqrt{\frac{\Gamma_D M_{Pl}}{g_*^{1/2}}}.$$

- We thus get the amount of entropy dilution

$$\frac{S_>}{S_<} = \left(\frac{T_>}{T_<}\right)^3 = \frac{g_*^{1/4}\, Y_D\, m_D}{\sqrt{\Gamma_D M_{Pl}}} > 1 \quad \text{(need 12 or so)}$$

- The amount of dilution depends on the yield Y_D and the mass of D. The more massive, the more energy gets released and transferred into radiation.
- For a relativistic freeze-out, we may get the maximal yield $Y_D = \frac{n_D}{s} = \frac{3\zeta(3)\cancel{T^3}\cdot 45}{2\pi^2\, 2\pi^2\, g_*\cancel{T^3}} = \frac{135\,\zeta(3)}{4\pi^4}$.
- Finally, $\Gamma_D \propto \tau_D^{-1}$ and the longer we wait, the more D will dominate over radiation and more dilution will get. In practice BBN limits $\tau_D \lesssim 1\,\text{sec}$.
- Plugging in the above expressions we get from

$$\Omega_{\nu_R} \simeq 3\left(\frac{m_{\nu_R}}{\text{keV}}\right) \Longrightarrow \Omega_{\nu_R} \sim 0.23\left(\frac{m_{\nu_R}}{\text{keV}}\right)\left(\frac{2\,\text{GeV}}{m_D}\right)\sqrt{\frac{1\,\text{sec}}{\tau_D}}.$$

OVER-PRODUCTION

ENTROPY DILUTION

CORRECT YIELD / DM RELIC DENSITY ✓

Misalignment Production : Axion DM

- The final production mechanism that we will consider, is the axion, a (pseudo) scalar field ϕ.

- Here, the DM is in the form of a coherent scalar field, whose oscillations contribute to energy density that scales as matter.

- We are dealing with a field in an expanding FLRW universe, so no Boltzmann, but instead

$$S = \int_V \sqrt{-g}\left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V + \underbrace{\dots}_{}\right) = \int_V \mathcal{L}$$

additional interactions of ϕ, not important for e.o.m.

$$g_{\mu\nu} \overset{\text{diag}}{=} (-1, a^2, a^2, a^2)$$

$$\sqrt{-g} = \sqrt{-\det g} = \sqrt{-(-a^6)} = a^3, \quad g^{\mu\nu} = \text{diag}(-1, a^{-2}, a^{-2}, a^{-2})$$

- The axion potential appears when $T \lesssim \Lambda_{QCD}$ and is approximately

$$V(\phi) \cong \underset{\substack{\uparrow \\ \text{axion mass}}}{m^2} \underset{\substack{\uparrow \\ \text{scale of PQ symmetry breaking}}}{f_a^2} \left(1 - \cos\left(\frac{\phi}{f_a}\right)\right),$$

- The axion comes as an accidentally light particle, a Goldstone boson of a global Peccei-Quinn (PQ) symmetry. This symmetry would exist at very high scales $f_a \sim 10^{10}$ GeV (bounds from stellar cooling) and

$$m_a f_a \simeq m_\pi f_\pi .$$

- We can expand the potential for $\phi \ll f_a$ as

$$V_0(\phi) \simeq \frac{1}{2} m^2 \phi^2 - \frac{1}{4} \left(\frac{m}{f_a}\right)^2 \phi^4 + \dots$$

$$\mathcal{L} = - a^3 \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} a^3 a^{-2} (\vec{\nabla}\phi)^2 - V(\phi) a^3$$

- The extremization of the action leads to Eul-Lagrange:

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} - \frac{\delta \mathcal{L}}{\delta \phi} = 0 \Rightarrow \partial_0 \frac{\delta \mathcal{L}}{\delta \partial_0 \phi} + \partial_i \frac{\delta \mathcal{L}}{\delta \partial_i \phi} - \frac{\delta \mathcal{L}}{\delta \phi} = 0$$

OR :

$$\frac{d}{dt}\left(- \dot{\phi} a^3\right) + a \nabla^2 \phi - \frac{dV}{d\phi} a^3 = 0 \quad \Big| :(-a^3)$$

$$\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} - \frac{\nabla^2 \phi}{a^2} + \frac{dV}{d\phi} = 0 , \quad \frac{dV}{d\phi} \simeq m^2 \phi^2$$

- This gives us the axion equation of motion

$$\boxed{\ddot{\phi} + 3H \dot{\phi} - \frac{\nabla^2 \phi}{a^2} + m^2 \phi^2 = 0}$$

- This is a damped harmonic oscillator, when $\nabla^2\phi$ can be neglected. This is a good approximation if PQ breaking happens above the scale of inflation.

$$H_I > f_a \quad : \quad \ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

- In the early universe H dominates and freezes the ϕ

$$3H\dot{\phi} \simeq 0 \;, \quad \dot{\phi} \simeq 0 \;, \quad \phi = \phi_0$$

- When H starts to drop, ϕ is released and begins to oscillate, which produces the energy density.

- The ρ and pressure P can be obtained from $T_\mu^{\ \nu} = \begin{pmatrix} -\rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} \Rightarrow \begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 + \overset{\to 0}{\cancel{\frac{1}{2a^2}(\nabla\phi)^2}} \,, \\ P = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 . \end{cases}$$

- Let us solve $\phi(t)$ in radiation domination, where

$$\text{RADIATION} \quad : \quad H = \frac{1}{2t} \;, \quad H^2 = \frac{8\pi G}{3}\frac{\pi^2}{30} g_* T^4$$

$$\ddot{\phi} + \frac{3}{2t}\dot{\phi} + m^2\phi = 0 \;, \quad \text{solution} : \phi(t) = \phi_0 \,\Gamma\left(\tfrac{5}{4}\right)\left(\frac{2}{mt}\right)^{1/4} J_{1/4}(mt).$$

Boundary conditions : $\dot{\phi}(0) = 0$, $\phi(0) = \phi_0$

$$\ddot{\phi} + \frac{3}{2t}\dot{\phi} + m^2\phi = 0 \quad ; \quad t \text{ small} : \dot{\phi} = 0$$

$$\phi(t) = \phi_0\, \Gamma\left(\tfrac{5}{4}\right) \underbrace{\left(\frac{2}{mt}\right)^{1/4} J_{1/4}(mt)}_{\text{drops with } t \text{ and oscillates}}$$

- the only energy scale here is $m \simeq \frac{m_\pi f_\pi}{f_a}$ and it sets the frequency of oscillations, e.g.: $m = \mu eV \Leftrightarrow t \approx 1ns$, via

$\hbar = c = 1$: $\frac{\hbar c}{c} = \frac{200\, eV\, n\cancel{m}\, s}{3 \cdot 10^8\, \cancel{m}} \approx 10^{-6} eV\, ns \Rightarrow \mu eV \cdot ns = 1$

- At late times the Bessel can be expanded and we get
  $t \gg m^{-1}$

$$\phi(t) \cong \phi_0\, \Gamma\left(\tfrac{5}{4}\right) \frac{1}{\sqrt{\pi}} \left(\frac{2}{mt}\right)^{3/4} \sin\left(mt + \frac{\pi}{8}\right)$$

$$\Downarrow$$

$$\dot{\phi}(t) \simeq \phi_0\, \Gamma\left(\tfrac{5}{4}\right) \frac{1}{\sqrt{\pi}} \left(\frac{2}{mt}\right)^{3/4} m \cos\left(mt + \frac{\pi}{8}\right) + \mathcal{O}\left((mt)^{-3/4-1}\right)$$

drops down faster than the first term

- While $(mt)^{-3/4}$ is decaying, we can average over the oscill.

$$\langle \phi^2 \rangle = \phi_0^2\, \Gamma\left(\tfrac{5}{4}\right)^2 \frac{1}{\pi} \left(\frac{2}{mt}\right)^{3/2} \frac{1}{2} \quad , \quad \langle \dot{\phi}^2 \rangle = m^2 \langle \phi^2 \rangle .$$

- The energy density $\langle \rho \rangle$ and pressure $\langle P \rangle$ are then:

$$\langle \rho \rangle = \frac{1}{2}\langle \dot{\phi}^2 \rangle + \frac{1}{2}\langle \phi^2 \rangle m^2 = m^2 \langle \phi^2 \rangle = \frac{\Gamma(5/4)^2}{2\pi} m^2 \phi_0^2 \left(\frac{2}{mt}\right)^{3/2}$$

$$\langle P \rangle = \frac{1}{2}\langle \dot{\phi}^2 \rangle - \frac{1}{2}\langle \phi^2 \rangle m^2 \simeq \emptyset . \quad \text{PRESSURE - less.}$$

- We see that the primordial axion classical field behaves similar to matter. It is pressureless $P \approx 0$ and we will also see that similar to $\rho_{DM} \approx m_{DM} n_{DM}$, where $n_{DM} \propto a^{-3}$, the $\rho_\phi = \langle \rho \rangle$ also scales as a^{-3}.

$$\langle \rho \rangle = \rho_\phi = \frac{\Gamma(5/4)^2}{2\pi} m^2 \phi_0^2 \left(\frac{2}{mt}\right)^{3/2} \propto t^{-3/2},$$ and we are in RADIATION domination, so $H \propto t^{-1}$ $\frac{\dot{a}}{a} = \frac{1}{2t}$; $\frac{da}{a} = \frac{dt}{2t}$ and $a \propto t^{1/2} \Rightarrow \rho_\phi \propto (a^{2})^{-3/2} = a^{-3}$, which scales like non-relativistic matter $\Rightarrow$ a good DM candidate.

- RELIC DENSITY of the axion DM field Ω_ϕ

To calculate Ω_ϕ, we will identify the time t_*, when the oscillations begin and identify n_*. Then we proceed just like with freeze-out (in) and observe that $\frac{n_*}{s_*} = \frac{n_0}{s_0}$, to get to n_0, the number density today.

- Onset of oscillations, when: $H = m$ or $\frac{1}{2t_*} = m$.

- The n_* at the onset comes from $\rho_\phi = m n_\phi$, such that

$$n_* = \frac{\rho_\phi(t_*)}{m} = m\langle\phi^2\rangle(t_*) = \frac{\Gamma(5/4)^2}{2\pi} m\phi_0^2 \underbrace{\left(\frac{2}{mt_*}\right)^{3/2}}_{4^{3/2}=8}, \quad mt_* = \frac{1}{2},$$

$$= \frac{4}{\pi}\Gamma\left(\frac{5}{4}\right)^2 m\phi_0^2 .$$

This corresponds to the temperature T_*, which we get from the usual Friedmann equation:

$$H(T_*) = m = \sqrt{\frac{8\pi^3 g_*}{90 M_{Pl}^2}}\, T_*^2$$

Temperature at the onset of oscillations T_*:

$$T_* = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4}\sqrt{m M_{Pl}} \quad \Big\} \text{ a geometric mean of the axion \& Planck mass}$$

- We can now calculate the $Y = \frac{n}{s}$ yield at T_*:

$$\frac{n_*}{s_*} = \frac{\cancel{2}4\,\Gamma(5/4)^2 m\phi_0^2 \cdot 45}{\cancel{2}\pi^3 g_* T_*^3} = \frac{90\,\Gamma(5/4)^2 m\phi_0^2}{\pi^3 g_* (m M_{Pl})^{3/2}}\left(\frac{8\pi^3 g_*}{90}\right)^{3/4}$$

$$\propto m^{-1/2}.$$

- To get the relic density we first rescale to today

$$\frac{n_0}{s_0} = \frac{n_*}{s_*} \Rightarrow n_0 = \frac{n_*}{s_*} s_0, \quad s_0 = \frac{2\pi^2}{45}\underbrace{\frac{43}{11}}_{\text{photons and colder neutrinos}} T_0^3$$

- also $\rho_{cr} = \frac{\rho_\gamma}{\Omega_\gamma}$, $\rho_\gamma = \frac{2\pi^2}{30} T_0^4$

- Now we get to the final relic density, and write $\phi_0 = \Theta_0 f_a$ ← dimensionless "angle"

$$\Omega_\phi = m\, n_\phi = m \frac{n_*}{s_*} \frac{\cancel{2\pi^2}\, 43}{\cancel{45}_{3}\, 11} \frac{\cancel{T_0^3}\, \Omega_\gamma\, \cancel{30}^{2}}{\cancel{2\pi^2}\, \cancel{T_0^4}}$$

$$= \frac{86}{33} \frac{m}{T_0} \frac{n_*}{s_*}$$

$$= 0.26\, \Theta_0^2 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{1/2} \left(\frac{f_a}{10^{17}\,\mathrm{GeV}}\right)^2 \left(\frac{10}{g_*}\right)^{1/4}$$

$$= 0.26\, \Theta_0^2 \left(\frac{m}{\cancel{\mu}\mathrm{eV}}\right)^{1/2} \left(\frac{f_a}{10^{10}\,\mathrm{GeV}}\right)^2 \left(\frac{10}{g_*}\right)^{1/4}.$$

This is the final result. We got to the axion relic density, which is $\propto \sqrt{m}$ and $\propto f_a^2$.

The Θ_0 is called the mis-alignment angle and is an arbitrary initial parameter, which clearly has to be $\Theta_0 \neq 0$ for this mechanism to work.

- Beyond this simple analysis, one has to worry about inflation and other means of producing axions from topological defects. Fascinating, but beyond these lectures :)