MODERNA FIZIKA 1 – VAJE 2020–21

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- predstavnik letnika: izpiti, datumi, rezervacije
- režim: 3 izpiti 3 naloge = 100%, 90 minut

  1.a izpit ~ December

  1.b izpit ~ Januar

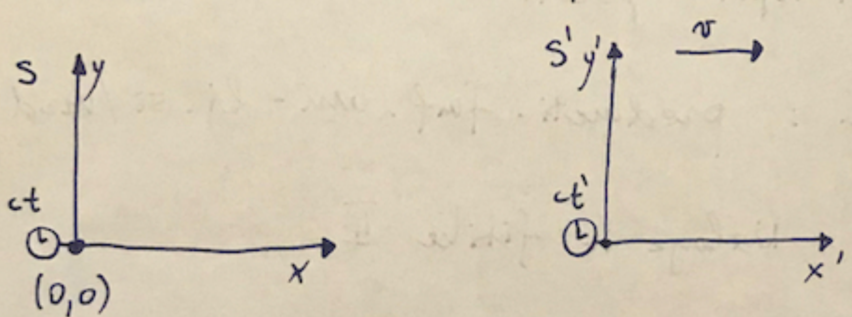
  2 in 3. izpit poleti
- info o predmetu: predmeti.fmf.uni-lj.si/modfiz1
- naloge po skripti: Naloge iz fizike II.
- rešeni izpiti z dodatnimi izpeljavami (Nemevšek et al.)

Posebna teorija relativnosti

- prostor in čas sta povezana preko linearne transformacije = Lorentzova transformacija

- dogodek : $A : (ct_A, x_A, y_A, z_A) = x^\mu_A$, $\mu = 0, 1, 2, 3$

- hitrost svetlobe je $c \cong 3 \cdot 10^8 \frac{m}{s}$ je enaka za vse inercialne (v = konst. $\beta = v/c < 1$) opazovalce



- iz teh postulatov sledi podaljšanje časa in skrčenje dolžin

URA

$$(ct')^2 = (vt')^2 + (ct)^2$$

$$t'^2 (1-\beta^2) = t^2$$

$$t' = \frac{t}{\sqrt{1-\beta^2}} = \gamma t$$

$$\beta < 1 \Rightarrow \gamma > 1 \cong \begin{cases} \gg 1 & \beta \sim 1 \quad \gamma \sim \frac{1}{\sqrt{1-(1-\varepsilon)^2}} \sim \frac{1}{\sqrt{2\varepsilon}} \gg 1 \quad \varepsilon \to 0 \\ 1 & \beta \sim 0 \quad \sim 1 + \frac{1}{2}\beta^2 \text{ for } \beta \ll 1 \end{cases}$$

IZPELJAVA PREKO L.T.

$$x^{\mu'} = \Lambda^{\mu}{}_{\nu} x^{\nu} \qquad \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$ct' = \gamma(ct - \beta x)$$
$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$

<u>URE</u> S: $A(0,0)$ $\qquad S'$: $A'(0,0)$

$B(ct, 0)$ $\qquad B'(ct', -vt') = (\gamma ct - 0, -\gamma\beta ct)$

$$\Rightarrow t' = \gamma t \quad , \quad \gamma > 1$$

1 Mion v letu

$\tau_\mu = 2.2\,\mu s$

$\beta = \frac{v}{c} = 0.994$

$= 1 - \varepsilon$

$\varepsilon = 0.006$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \sim \frac{1}{\sqrt{2\varepsilon}} \sim \frac{1}{\sqrt{0.012}} \sim \frac{10^2}{\underset{\overset{\shortparallel}{11}}{\sqrt{121}}} \sim 10(1-0$$

$$\sim 9 \quad (9.14)$$

$$\tau_\mu' = \gamma \tau_\mu \cong 22 \cdot 9\,\mu s \sim 20\,\mu s \quad (20.11\,\mu s)$$

$$l = v\tau_\mu' = 20\,\mu s \cdot 3 \cdot 10^{8}\,\frac{m}{s} \cdot \overset{1}{\beta} \cong 6\,km$$

$(6.014\,km)$

• Število razpadlih mionov $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

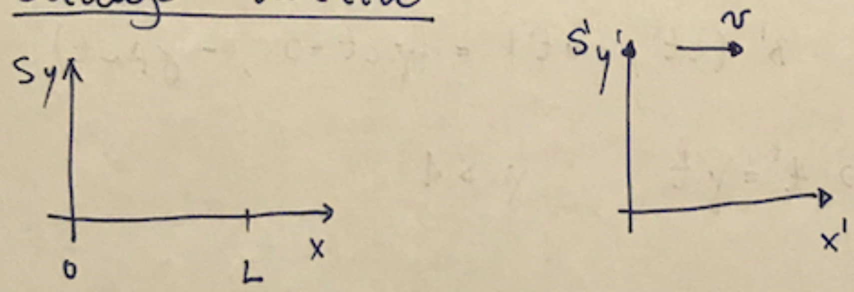
$$N(t) = N_0\, e^{-t/\tau_\mu'} = N_0\, e^{-\ell/(c\tau_\mu')} \quad , \ell = 2100\,\mathrm{m}$$

$$= 568 \cdot e^{-\cancel{\frac{2\cdot 10^3\,\mathrm{m\,s}}{3\cdot 10^8\,\mathrm{m}\; 2.2\,\mu s}}\,\frac{1}{3}} \sim 568 \cdot \frac{2}{3} \sim 400 \quad (400.6)$$

• Brez relativističnega podaljšanja , $\gamma = 1$, $\tau = 2\,\mu s$

$$N_{brez} = N_0\, e^{-\frac{9}{3}} \sim 568\, \underbrace{e^{-3}}_{\sim \frac{1}{2.7^3} \sim \frac{1}{25}} \sim 25 \quad (23.3)$$

<u>Skrčenje dolžin</u>



$A(0,0)$ $B(\bullet, L)$ $A'(0,0)$ $B'(0, L')$

Obratna Lorentzova transformacija $\beta \to -\beta$

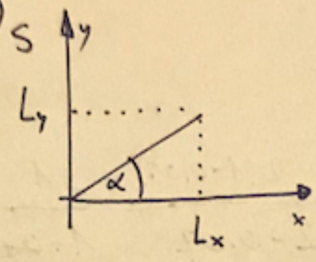
$$ct = \gamma(ct' + \beta x')$$
$$x = \gamma(x' + \beta ct)$$
$$y = y'$$
$$z = z'$$

$$B\left(\gamma(0 + \beta L'), \gamma L'\right)$$
$$= (ct, L)$$

$$L' = \frac{L}{\gamma}$$

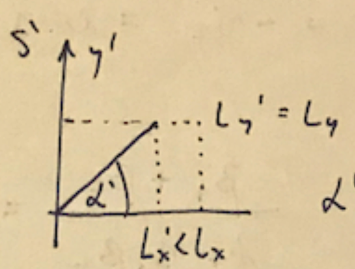
$$L_x' = \frac{L_x}{\gamma}$$
$$L_y' = L_y$$
$$L_z' = L_z$$

(2)



$$L_x' = \frac{L_x}{\gamma} < L_x$$

$$L_y' = L_y$$



$\alpha' > \alpha$

$$\mathrm{tg}\,\alpha = \frac{L_y}{L_x} \qquad \mathrm{tg}\,\alpha' = \frac{L_y'}{L_x'} = \frac{L_y}{L_x}\gamma = \gamma\,\mathrm{tg}\,\alpha > \mathrm{tg}\,\alpha$$

$$L_x = L\cos\alpha \qquad \alpha' = 30^\circ \quad \mathrm{tg}\,\alpha' = \frac{1}{\sqrt{3}}$$

$$L_y = L\sin\alpha \qquad \beta = 0.8 \quad \gamma = \frac{1}{\sqrt{1-0.64}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.6}$$

$$\mathrm{tg}\,\alpha = \frac{\mathrm{tg}\,\alpha'}{\gamma} = \frac{0.6}{\sqrt{3}} \Rightarrow \alpha = 19{,}1^\circ < \alpha'$$

$$L_x' = L'\cos\alpha' = \frac{L\cos\alpha}{\gamma}$$

$$L_y' = L'\sin\alpha' = L\sin\alpha \qquad L'^2 = L^2\left(\frac{\cos^2\alpha}{\gamma^2} + \sin^2\alpha\right)$$

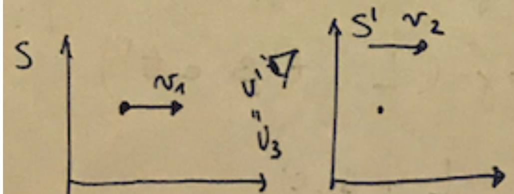
$$= 0.66\ \mathrm{m}$$

- Relativistično seštevanje hitrosti

$$ct' = \gamma(ct - \beta x) \qquad \frac{dx'}{c\,dt'} = \frac{\cancel{\gamma}(dx - \beta c\,dt)}{\cancel{\gamma}(c\,dt - \beta\,dx)}$$

$$x' = \gamma(x - \beta c t)$$

$$= \frac{v - \beta c}{c - \beta v} = \frac{v'}{c}$$



KLASIČNO : $v_3 = v_1 - v_2$ REL. : $\beta_3 = \dfrac{\beta_1 - \beta_2}{1 - \beta_1\beta_2}$

$\sim (\beta_1 - \beta_2)(1 + \beta_1\beta_2)$

za $\beta_1, \beta_2 \ll 1$

(3) $v_1 = -v_2 = 0.99c$

$$\beta_3 = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2} = \frac{2 \cdot 0.99}{1 + (0.99)^2} \sim \frac{2(1-0.01)}{2 - 0.02 + 10^{-4}} = \frac{1}{1 + \cancel{0.01}\, 5 \cdot 10^{-5}}$$

$$(1-0.01)^2 \sim 1 - 0.02$$

$$\sim 1 - 5 \cdot 10^{-5} = 0{,}99995$$

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(4) $L^* = 100\,m$

$v = 0{,}5c$

$v_k = 0{,}9c$

$t' = ?$

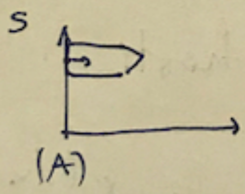
klasično $v_k' = v_k - v$, $t_L = v_k'^{-1} L$

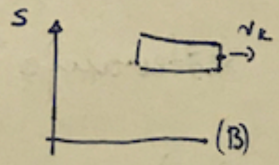
relativistično $\beta' = \frac{\beta_k - \beta}{1 - \beta\beta_k}$

$$t' = \frac{L^*}{c\beta'} = \frac{L(1 - \beta\beta_k)}{v_k - v}$$

$$= \frac{100\cancel{m}\,(1 - 0.45)}{3 \cdot \cancel{10^8}\,\frac{\cancel{m}}{\cancel{s}}\; 0.4}\,\mu s = \frac{0.55}{1.2}\,\mu s \sim 0.46\,\mu s$$

z DOGODKI





ZEMLJA A: $(0, 0)$

B: $(ct, v_k t)$

OBRATNA L.T.

$ct = \gamma\,(ct' + \beta L)$

$v_k t = \gamma\,(L + \beta c t')$ / :

LADJA A': $(0, 0)$

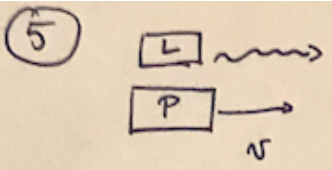
B': (ct', L^*)

$$\beta_k = \frac{L + \beta c t'}{ct' + \beta L}$$

$$ct'\beta_k + \beta\beta_k L = L + \beta c t'$$

$$t' = \frac{L(1 - \beta_k\beta)}{v_k - v} \longleftarrow ct'(\beta_k - \beta) = L(1 - \beta\beta_k)$$

5



$v = 0.6c$

$t_s = 1250\,s$

$t_z = ?$

$t_L = ?$

Z

DIREKTNO

$$L = c t_s \qquad t_z = \frac{L}{v} = \frac{c t_s}{v} = \frac{1250\,s}{0.6} \sim 2000\,s$$

$$t_L = \frac{t_z}{\gamma} = \frac{t_s}{\gamma\beta}$$

$$\cancel{(\gamma\beta)^2 = \frac{\beta^2}{1-\beta^2}}$$

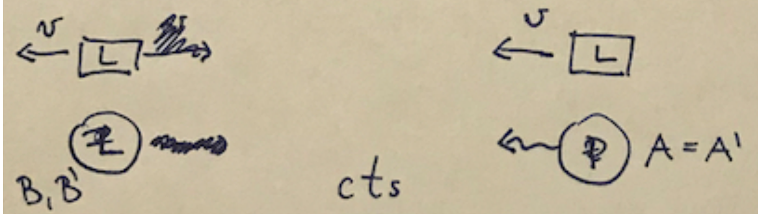
$$\cancel{-1 + \frac{1}{1-\beta^2} = \frac{1-1+\beta^2}{1-\beta^2} = \gamma^2 - 1}$$

$$\cancel{t_L^2 = \frac{t_s^2}{\gamma^2 - 1}}$$

$$t_L \approx \frac{2000\,s}{1.25} \sim \underline{1600\,s} \qquad \gamma = \frac{1}{\sqrt{1-\underbrace{0.6^2}_{0.36}}} = \frac{1}{\sqrt{0.64}} = \frac{1}{0.8}$$

$$= \frac{10}{8} = 1.25$$

2 DOGODKI



$$S: \; A(0, \overset{c t_s}{\overset{\shortparallel}{L}})$$

$$B(c t_z, \underset{\underset{L - \beta c t_z}{\shortparallel}}{0})$$

$$S': \; A'(c t_1', 0)$$

$$B'(c t_2', -L')$$

$$c t_1' = \gamma(0 - \overset{= \beta c t_z}{\beta L})$$

$$c t_2' = \gamma(c t_z - 0)$$

$$c\Delta t = \gamma(c t_z - \beta^2 c t_z)$$

$$= \frac{t_z}{\gamma} = t_L$$