ATOM VODIKA V MOČNEM MAGNETNEM POLJU

$$\hat{H} = \hat{H}_0 + \underbrace{\hat{H}_{ls}}_{ls\text{ - sklopitev}} - \underbrace{\hat{\mu}_z B_z}_{\text{Zeemanov pojav}}$$

- Poglejmo si energijsko shemo za ψ_{100} & ψ_{21m_l}

A) Brez polja $B=0$, samo ls

$\boxed{n=1}$ $l=0 \quad m_l=0$ ... $l=0,\ s=\frac{1}{2}$ $\qquad \langle ls \rangle \propto \cancel{j(j+1) - \overset{0}{\overset{\shortparallel}{l(l+1)}} - s(s+1)} = 0$

$s=1/2 \quad m_s=\pm\frac{1}{2}$ $\qquad j=\frac{1}{2},\ m_j=\pm\frac{1}{2}$ $\qquad \Downarrow$

$\boxed{2\times}$ $\qquad \boxed{2\times}$ $\qquad$ ni razcepa

$\boxed{n=2}$ $l=0$ je identičen zgornjemu

2x $\quad l=0,\ m_l=0,\ m_s=\pm 1/2$ $\qquad l=1,\ s=1/2,\ j=3/2$ $\quad$ 4x

6x $\quad l=1,\ m_l=0,\pm 1$ .... $\qquad l=0,\ s=1/2,\ j=1/2$ $\quad$ 2x

$s=\frac{1}{2},\ m_s=\pm\frac{1}{2}$

skupaj dobimo 8 stanj $\boxed{8\times}$ $\qquad l=1,\ s=1/2,\ j=1/2$ $\quad$ 2x

$\boxed{8\times}$

B) Močno magnetno polje $\mu_B B \gg \Delta E_{ls} \sim 10^{-5}\,\text{eV}$

$$\hat{\mu} \to \hat{\mu}_z B_z \quad \text{in} \quad \Delta E_B = -\left(\langle \overset{-1}{\overset{\shortparallel}{g_e}} L_z \rangle + \overset{-2}{\overset{\shortparallel}{g_s}} \langle S_z \rangle\right) B$$

$$= \mu_B (m_l + 2m_s) B$$

v močnem B so $l\ m_l\ s\ m_s$ <u>dobra</u> kvantna števila

$$\langle \hat{H} \rangle \sim \langle \hat{H}_0 \rangle + \frac{\mu_B}{\hbar} \left(\langle L_z \rangle + 2 \langle S_z \rangle \right)$$

$$\Delta E_B = \mu_B (m_l + 2 m_s) B$$

$\boxed{n=1}$, $l=0$, $m_l=0$

$s = 1/2$, $m_s = \pm 1/2$

	ΔE
$m_s = 1/2$	$+\mu_B B$
$m_s = -1/2$	$-\mu_B B$

popoln razcep

$\boxed{n=2}$

$l=0$ ...

$m_s = 1/2$	1x	2x
$m_s = -1/2$	1x	

$l=1$ $m_l = 0, \pm 1$

$m_l = 1$ $m_s = 1/2$	1x	6x
$m_l = 0$ $m_s = 1/2$	1x	
$m_l = \pm 1$ $m_s = \mp 1/2$	2x	ostane degenerirano
$m_l = 0$ $m_s = -1/2$	1x	
$m_l = -1$ $m_s = -1/2$	1x	

$\boxed{8x}$

$$\Delta E = \mu_B \left((0, \pm 1) \pm 2 \tfrac{1}{2} \right) B$$

$$= \mu_B B \, (\pm 1, \pm 2, \underset{\substack{\downarrow \\ 2x\ \text{deg.}}}{0})$$

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Landé-jev giromagnetni faktor

$$\hat{H} = \hat{H}_0 + \hat{H}_{es} \underbrace{- \hat{\mu}_z B}_{+ \frac{\mu_B}{\hbar}(\hat{l}_z + 2\hat{S}_z)B} = \frac{\mu_B}{\cancel{\hbar}} g_{lsj} \cancel{\hbar} m_j B$$

$$\langle \hat{l}_z + 2\hat{S}_z \rangle \longrightarrow \frac{\langle \hat{j}^2 + \hat{s}\hat{j} \rangle}{\langle \hat{j}^2 \rangle} \langle \hat{j}_z \rangle$$

$$j = l + s \,,\quad j - s = l \,,\quad j^2 - 2sj + s^2 = l^2$$

$$\hat{s}\hat{j} = \frac{1}{2}(j^2 + s^2 - l^2) \,,\quad \langle \hat{s}\hat{j} \rangle = \frac{\hbar^2}{2}\big(j(j+1) + s(s+1) - l(l+1)\big)$$

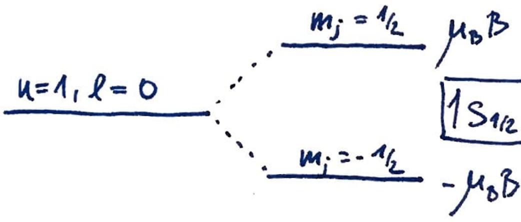
$$\Rightarrow g_{lsj} = \frac{j(j+1)(1+\frac{1}{2}) - \frac{1}{2} l(l+1) + \frac{1}{2}(\frac{1}{2}+1)}{j(j+1)} \,,\quad \text{za } s = \frac{1}{2}$$

$$= \frac{3}{2} - \frac{l(l+1) - 3/4}{j(j+1)}$$

Poglejmo kaj se zgodi z atomom vodika.

$n=1$, $l=0$, $m_l=0$, $s=1/2$, $m_s = \pm 1/2$ OSNOVNO STANJE

$$j = l + s = 0 + \frac{1}{2} = \frac{1}{2} \,,\quad m_j = \pm \frac{1}{2}$$



$$g_{1\frac{1}{2}\frac{1}{2}} = \frac{3}{2} + \underbrace{\frac{0 \cdot 1 + 3/4}{3/2}}_{1/2} = 2 \quad \Rightarrow \quad \Delta E_B = \mu_B \cdot 2 m_j B = \pm \mu_B B$$

PRVO VZBUJENO STANJE : $n=2,\ l=0,1$

$n=2,\ l=0$ isto kot prej $\Delta E_B = \pm \mu_B B$: $\boxed{2S_{1/2}}$ $\boxed{2\times}$

$n=2,\ l=1,\ m_l = 0, \pm 1,\ s = 1/2,\ m_s = \pm 1/2$

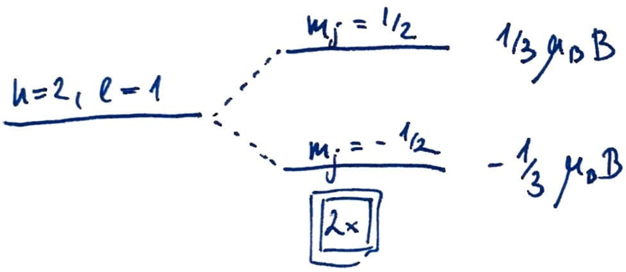
$$j^{max} = l+s = 1+\frac{1}{2} = \frac{3}{2},\quad j^{min} = |l-s| = 1-1/2 = 1/2$$

$\boxed{j=1/2}$

$$g_{1\,1/2\,1/2} = \frac{3}{2} - \frac{l(l+1)-3/4}{2j(j+1)} = \frac{3}{2} - \frac{2-3/4}{\cancel{2}\,\cancel{\frac{1}{2}}\,3/2} = \frac{3}{2} - \frac{8-3}{6}$$

$$= \frac{9-5}{6} = \frac{4}{6} = \frac{2}{3} \Rightarrow \Delta E_B^{j=1/2} = \mu_B B \cancel{\frac{2}{3}}\left(\pm \frac{1}{\cancel{2}}\right)$$

$$= \pm \frac{1}{3} \mu_B B$$

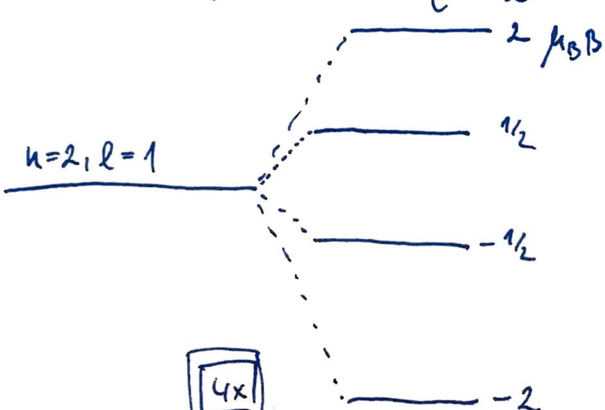


$\boxed{2p_{1/2}}$ $n=2,\ l=1,\ j=1/2$

$\boxed{j=\frac{3}{2}}$

$$g_{1\,1/2\,3/2} = \frac{3}{2} - \frac{2-3/4}{2\,\frac{3}{2}\,\frac{5}{2}} = \frac{3}{2} - \frac{8-3}{30} = \frac{9-1}{6} = \frac{4}{3}$$

$$\Delta E_B^{j=3/2} = \mu_B B \,\frac{4}{3} \times \begin{cases} \pm \frac{1}{2} \\ \pm \frac{3}{2} \end{cases} = \mu_B B \begin{cases} \pm \frac{2}{3} \\ \pm 2 \end{cases}$$



$\boxed{2p_{3/2}}$ $n=2,\ l=1,\ j=3/2$

* 8 stanj : $2+2+4=8$

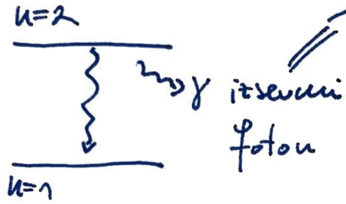
64 Sevalni prehodi v neskončni potencialni jami

$n=2 \rightarrow n=1$, $\delta E = ?$ $\qquad a = 0{,}3 \text{ nm}$

$$\delta E \, \underset{\tau}{\underset{\parallel}{\delta t}} = \frac{\hbar}{2}, \quad \delta E = \frac{\hbar}{2\tau}$$

$$E_\gamma = E_2 - E_1$$

• Dipolno sevanje



$$\frac{1}{\tau} = \frac{2}{3}\alpha E_{12}^3 \left(\frac{x_{12}}{\hbar c}\right)^2, \quad E_{12} = E_2 - E_1$$

$$x_{12} = \langle 1 | x | 2 \rangle$$

dipolni prehodni matrični element

• Za jamo: $E_n = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{n\pi\hbar}{a}\right)^2 \Rightarrow E_{12} = \frac{\pi^2(\hbar c)^2}{2mc^2a^2}\underbrace{(2^2-1)}_{3}$

$$x_{12} = \int_0^a \psi_1 x \psi_2 \, dx, \quad \psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \quad \frac{\pi x}{a} = t, \quad dx = \frac{a}{\pi} dt$$

$$= \frac{2}{a}\int_0^\pi \sin t \cdot t \cdot \sin 2t \cdot dt \cdot \left(\frac{a}{\pi}\right)^2$$

$$= \frac{2a}{\pi^2}\int_0^\pi \sin t \cdot t \cdot 2\sin t \cos t \, dt = \frac{4a}{\pi^2}\int_0^\pi t(1-\cos^2 t)\cos t \, dt$$

• Imamo dva integrala: $\int_0^\pi t \cos t \, dt$ in $\int_0^\pi t \cos^3 t \, dt$

$$\int_0^\pi \underbrace{\cos t}_{dv}\ \overset{u}{t}\ \underline{dt} = \sin t\ t\Big|_0^\pi - \int_0^\pi \sin t\, dt = +(-1) - 1 = -2 .$$

$$\int_0^\pi \sin t\, dt = \cos t\Big|_0^\pi$$

$du = dt, \quad v = \sin t$

In še drugi preostali integral

$$\int_0^\pi \underbrace{\cos^3 t}_{dv}\ \overset{u}{t}\ \underline{dt} = \sin t\left(1 - \frac{\sin^2 t}{3}\right)\Big|_0^\pi - \int_0^\pi \left(\sin t - \frac{1}{3}\sin^3 t\right) dt = *$$

$$v = \int \cos^3 t\, dt = \int 1 - \sin^2 t \cdot d(\sin t) = \sin t - \frac{\sin^3 t}{3}$$

$$* = \cos t\Big|_0^\pi + \frac{1}{3}\int_0^\pi \overset{1-\cos^2 t}{\sin^2 t}\ \underbrace{\sin t\, dt}_{-d(\cos t)} = -2 - \frac{1}{3}\int_1^{-1} (1 - c^2)\, dc$$

$$= -2 - \frac{1}{3}\left((-2) + \frac{2}{3}\right) = -2 + \frac{1}{\cancel{3}}\left(2 - \frac{2}{3}\right) = \frac{-18 + 6 - 2}{9} = -\frac{14}{9} .$$

$$\Rightarrow X_{12} = \frac{4a}{\pi^2}\left(-2 + \frac{14}{9}\right) = -\frac{16a}{9\pi^2} .$$

$$\Rightarrow \delta E = \frac{2}{3}\alpha\left(\frac{3\pi^2\ (\hbar c)^2}{2mc^2\ \cancel{a^2}}\right)^3 \cdot \left(-\frac{\overset{2^4}{16}\ \cancel{a}}{9\pi^2 \hbar c}\right)^2$$

$$= \underbrace{\frac{2^6 \pi^2}{3^2}}_{0{,}5}\ \alpha\ \underbrace{\left(\frac{\hbar c}{a\ mc^2}\right)^4}_{\left(\frac{0{,}2\ \cancel{\mathrm{keV\,nm}}}{0{,}3\ \cancel{\mathrm{nm}}\ 511\ \cancel{\mathrm{keV}}}\right)^4 \cdot 511\,\mathrm{keV}} mc^2 \cong 10^{-6}\,\mathrm{eV} = \mu\mathrm{eV}.$$