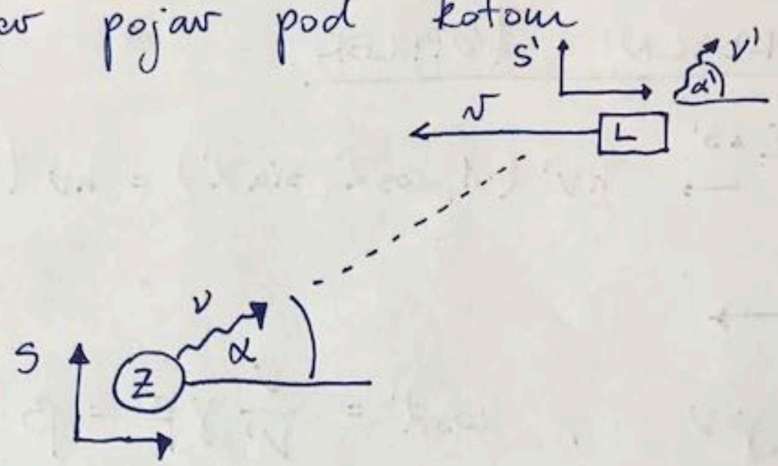
(62) Dopplerjev pojav pod kotom

$\beta = 0{,}6$

$\alpha = 30°$

$\nu = 100\,\mathrm{MHz}$



$$S: \quad p^\mu = h\nu\,(1,\ \underbrace{\cos\alpha,\ \sin\alpha,\ 0}_{\text{gibalna količina}})$$

energija

invarianta, masa fotona = 0

$$p^\mu p_\mu = h\nu\,(1 - \cos^2\alpha - \sin^2\alpha) = 0 = m_\gamma^2 c^4$$

$$S': \quad p'^\mu = h\nu\,(\gamma(1+\beta\cos\alpha),\ \gamma(\cos\alpha+\beta),\ \sin\alpha, 0)$$

$$= h\nu'\,(1,\ \cos\alpha',\ \sin\alpha',\ 0)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{36}{100}}} = \frac{10}{\sqrt{64}} = \frac{5}{4} = 1{,}25$$

- Kot pod katerim sprejme signal

$$\tan\alpha' = \frac{\sin\alpha'}{\cos\alpha'} = \frac{\sin\alpha}{\gamma(\cos\alpha+\beta)} = \frac{\frac{1}{2}}{\frac{5}{4}\left(\frac{\sqrt{3}}{2}+\frac{6}{10}\right)} \doteq 0{,}27$$

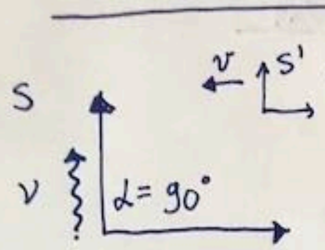
$$\Rightarrow \boxed{\alpha' = 15{,}3°}$$

- Frekvenca, ki jo izmerijo na ladji

$$\nu' = \nu\,\gamma\,(1+\beta\cos\alpha) = 100\,\mathrm{MHz}\ \frac{5}{4}\left(1+\frac{6}{10}\,\frac{\sqrt{3}}{2}\right)$$

$$\sim 190\ \mathrm{MHz}$$

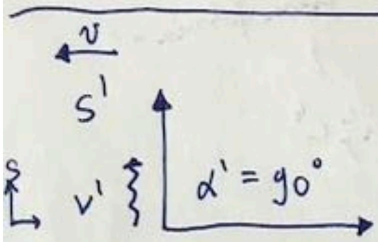
TRANSVERZALNI DOPPLER



$$h\nu'(1, \cos\alpha', \sin\alpha') = h\nu\left(\gamma(1+\beta \overset{=0}{\cos\alpha}),\ \gamma(\overset{=0}{\cos\alpha} + \beta),\ \underset{=1}{\sin\alpha}\right)$$

$$\nu' = \gamma\nu \quad , \quad \cos\alpha' = \frac{\nu}{\nu'}\gamma\beta = \beta$$

$$\sin\alpha' = \sqrt{1-\beta^2} = \frac{\nu}{\nu'} = \frac{1}{\gamma} \quad ✓$$



$$h\nu(1, \cos\alpha, \sin\alpha) = h\nu'\left(\gamma(1-\beta \overset{=0}{\cos\alpha'}),\ \gamma(\underset{=0}{\cos\alpha'} + \beta),\ \underset{=1}{\sin\alpha'}\right)$$

$$\nu = \gamma\nu'$$

$$\cos\alpha = \frac{\gamma\nu'}{\nu}\beta = \beta \quad , \quad \sin\alpha = \frac{\nu'}{\nu} = \frac{1}{\gamma}$$

Za razliko od skrčenja dolžin, kjer se v y-smeri nič ne dogaja, se pri Dopplerju zaradi podaljšanja časa frekvenca spremeni tudi v y-smeri. Dobimo t.i. transverzalni Dopplerjev pojav.

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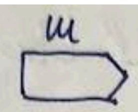

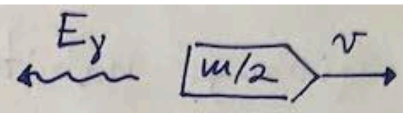

$$p^\mu = (mc^2, 0) \qquad p_1^\mu = E_\gamma(1, -1)$$

$$p_2^\mu = \frac{m}{2}c^2(\gamma, \gamma\beta)$$

$$\left.\begin{aligned} mc^2 &= E_\gamma + \frac{\gamma}{2}mc^2 \\ 0 &= -E_\gamma + \frac{1}{2}\gamma\beta mc^2 \end{aligned}\right) + \; , \; : mc^2 \; , \; \cdot 2$$

$$2 = \gamma(1+\beta) = \sqrt{\frac{1+\beta}{1-\beta}} \Rightarrow 4 - 4\beta = 1 + \beta$$

$$\beta = \frac{3}{5}$$

(23) · Gibanje nabitega delca v elektro-magnetnem polju

· lastni čas $(cd\tau)^2 = (cdt)^2 - \sum dx_i^2$

če smo na miru: $dx^2 = 0$ in $d\tau = dt$

sicer $dt = \gamma\, d\tau$ (podaljšanje časa)

· 4-hitrost u^μ iz $p^\mu = m u^\mu$ $\quad$ KLASIČNO $(\vec{G} = m\vec{v})$

· poenoten zapis enačb gibanja $\quad (\vec{F} = \frac{d\vec{G}}{dt})$

$$\frac{dp^\mu}{d\tau} = e F^{\mu\nu} u_\nu \quad , \quad u_\nu = g_{\nu\lambda} u^\lambda = (u^0, -u^1, -u^2, -u^3)$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ & 0 & -B_z & B_y \\ & \text{anti-simetrično} & 0 & -B_x \\ & & & 0 \end{pmatrix}$$

· vzemimo: $\vec{E} = (E_x, E_y, E_z) = (E, 0, 0)$ in $\vec{B} = 0$

$\mu = 0$: $\frac{dp^0}{d\tau} = m\frac{du^0}{d\tau} = eF^{01}u_1 = e\left(-\frac{E}{c}\right)(-u^1)$

OZIROMA : $m\dot{u}^0 = \frac{eE}{c} u^1$

$\mu = 1$: $\frac{dp^1}{d\tau} = m\frac{du^1}{d\tau} = eF^{10}u_0 = e\frac{E}{c}u^0$

Dobili smo sistem dveh diferencialnih enačb (linearnih)

$$\dot{u}^0 = \left(\frac{eE}{mc}\right) u^1 = \alpha u^1$$

$$\dot{u}^1 = \alpha u^0 \quad \Big/ \frac{d}{d\tau} \Rightarrow \ddot{u}^1 = \alpha \dot{u}^0 = \alpha^2 u^1$$

$$\Downarrow$$

$$\ddot{u}^1 = \alpha^2 u^1 \quad \text{in} \quad \ddot{u}^0 = \alpha^2 u^0$$

Kakšni so začetni pogoji za $u^\mu(\tau = 0)$?

$$u^\mu \equiv \frac{dx^\mu}{d\tau} = \gamma \frac{dx^\mu}{dt} = \gamma(c, \beta c)$$

V našem primeru elektron ob $t=0$ miruje,

zato: $\quad u^\mu(0) = (c, 0) = (u^0, u^1)$

Rešitev zgornjega sistema je $u^0 = \tilde{A} e^{\alpha\tau} + \tilde{B} e^{-\alpha\tau}$

$$= A \,\mathrm{ch}\, \alpha\tau + B \,\mathrm{sh}\, \alpha\tau$$

$$\dot{u}^0 = \alpha(A \,\mathrm{sh}\, \alpha\tau + B \,\mathrm{ch}\, \alpha\tau)$$

$$= \alpha u^1$$

$$u^1 = A \,\mathrm{sh}\, \alpha\tau + B \,\mathrm{ch}\, \alpha\tau$$

$$u^0(\tau = 0) = A = c$$

$$u^1(\tau = 0) = B = 0$$

$$u^0(\tau) = c \,\mathrm{ch}\, \alpha\tau$$

$$u^1(\tau) = c \,\mathrm{sh}\, \alpha\tau$$

Z integracijo $u^0(\tau)$ dobimo zvezo med lastnim časom τ in laboratorijskim časom t

$$u^0 = \frac{d(ct)}{d\tau} \Rightarrow ct = \int_0^\tau u^0(\tau')\,d\tau'$$

$$= \frac{1}{\alpha} c \,\mathrm{sh}\, \alpha\tau' \Big|_0^\tau$$

$$\boxed{\alpha t = \mathrm{sh}\, \alpha\tau} \quad , \alpha = \frac{eE}{mc}$$

lab. čas   lastni čas $\alpha \ll 1 : \cancel{\alpha} t \sim \cancel{\alpha}\tau$

Podobno dobimo prepotovano pot z integracijo u^1

$$u^1 = \frac{dx}{d\tau} \Rightarrow \int_0^L dx = L = \int_0^\tau u^1 d\tau' = \frac{c}{\alpha}(\mathrm{ch}\,\alpha\tau - 1)$$

$$= \frac{c}{\alpha}\left(\sqrt{\mathrm{sh}^2\alpha\tau + 1} - 1\right) = \frac{c}{\alpha}\left(\sqrt{(\alpha t)^2+1} - 1\right)$$

Tako smo dobili prepotovano pot z lab. časom

$$L(t) = \frac{c}{\alpha}\left(\sqrt{(\alpha t)^2+1} - 1\right).$$

$$\alpha t = \frac{eEtc}{m_e c^2} = \frac{1{,}75\,\cancel{\mathrm{keV/m}}\, 10^{-6}\cancel{s} \cdot 3\cdot 10^8 \cancel{\frac{m}{s}}}{511 \cdot \cancel{10^3 \mathrm{eV}}} = 1{,}03$$

in $L = 126\,\mathrm{m}$

Nalogo lahko rešimo tudi z uporabo laboratorijskega časa t

$$\mu = 1 \quad : \quad m\frac{du^1}{d\tau} = e\frac{E}{c}u^0 = \frac{eE}{c}\frac{d(ct)}{d\tau}$$

$$u^1 = \frac{eE}{mc}\,c = \alpha c t = \gamma v$$

oziroma: $\gamma\beta = \alpha t \quad /^2$

$$(\gamma\beta)^2 = \gamma^2 - 1 = \frac{\beta^2}{1-\beta^2} = (\alpha t)^2$$

$$\beta(t) = \frac{\alpha t}{\sqrt{1+(\alpha t)^2}}.$$

Pot dobimo z direktno integracijo po t

$$L = \int_0^t \frac{dx}{dt'}\,dt' = \frac{1}{\alpha}\int_0^t \frac{c\alpha t'\;\alpha dt'}{\sqrt{1+(\alpha t')^2}} = \frac{c}{\alpha}\int_1^{\sqrt{1+(\alpha t)^2}} \frac{\cancel{u}\,du}{\cancel{u}}$$

$$1+(\alpha t')^2 = u^2 \qquad = \underbrace{\frac{c}{\alpha}\left(\sqrt{1+(\alpha t)^2} - 1\right)}_{\text{dobili smo isti izraz}}$$

$$\cancel{2}\alpha^2 t'\,dt' = \cancel{2}u\,du$$

Tretji način je z uporabo $eU = eEL = T$

kjer $T = (\gamma - 1)mc^2$ in $\gamma = \sqrt{1+(\alpha t)^2}$

$$\Rightarrow L = \frac{T}{eE} = \frac{mc^2}{eE}\left(\sqrt{1+(\alpha t)^2} - 1\right) = \frac{c}{\alpha}\left(\sqrt{1+(\alpha t)^2} - 1\right)$$