PRINCIP NEDOLOČENOSTI

Z uporabo načela nedoločenosti oceni energijo osnovnega stanja : a) Harmonskega oscilatorja (HO)

b) Delca v linearnem potencialu

(kvarki, mezoni in močna interakcija)

a) 1D harmonski oscilator ($E_n = \hbar\omega(n+\frac{1}{2})$, $n=0,1,\dots$)

$$\hat{H} = \hat{T} + \hat{V} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$$

$$E = \langle 0|\hat{H}|0\rangle = \frac{\langle p^2\rangle}{2m} + \frac{1}{2} m\omega^2 \langle x^2\rangle$$

$\uparrow$ osnovno stanje

· IZBEREMO TEŽIŠČNI SISTEM : $\langle p\rangle = 0$, $\langle x\rangle = 0$

$$\delta p^2 = \langle p^2\rangle - \langle p\rangle^2 = \langle p^2\rangle \quad \text{in} \quad \delta x^2 = \langle x^2\rangle - \underset{0}{\underset{\shortparallel}{\langle x\rangle^2}}$$

· PREDPOSTAVIMO $\delta p \simeq \frac{\hbar}{2\delta x}$

$$E \simeq \frac{\hbar^2}{2m\,4\,\delta x^2} + \frac{1}{2} m\omega^2 \delta x^2 \quad ; \text{ MINIMUM?}$$

$$\frac{\partial E}{\partial \delta x^2} = -\frac{(\hbar c)^2}{8mc^2\,\delta x^4} + \frac{1}{2} m\omega^2 = 0 \Rightarrow \delta x^4 = \frac{(\hbar c)^2}{4\,m^2 c^2 \omega^2}$$

$$\delta x^2 = \frac{\hbar}{2m\omega} \quad \text{in} \quad E_{min} = \frac{\hbar^{\cancel{2}}\,\cancel{2m}\omega}{8\cancel{m}\cancel{\hbar}\,4} + \frac{1}{2}\cancel{m}\omega^{\cancel{2}}\frac{\hbar}{2\cancel{m}\cancel{\omega}}$$

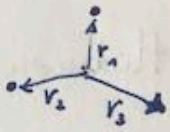
$$= \hbar\omega\frac{1}{2} \quad ✓$$

3b) Linearni potencial kvarkov u in d

$$m_u = m_d = 340\ \mathrm{MeV}/c^2$$

$$p = (u\,u\,d) \qquad Q(u) = \frac{2}{3}\,,\quad Q(d) = -\frac{1}{3}$$

$$k = 0{,}09\ \frac{\mathrm{GeV}^2}{\hbar c}$$



$$E_0 \simeq m_p c^2$$

$$\hat{H} = \sum_{i=1}^{3} \frac{\hat{p}_i^2}{2m} + k\left((\hat{r}_3 - \hat{r}_1) + (\hat{r}_3 - \cancel{\hat{r}_2}) + (\cancel{\hat{r}_2} - \hat{r}_1)\right)$$

$$\langle r_3 \rangle > \langle r_2 \rangle > \langle r_1 \rangle$$

$$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + 2k\left(\langle r_3 \rangle - \langle r_1 \rangle\right)$$

$$\langle p \rangle = 0 \quad , \quad \delta r \sim \langle r \rangle \quad \text{in} \quad \delta p_i\, \delta r_i = \frac{\hbar}{2}$$

$$\langle E \rangle \simeq \sum_i \frac{\delta p_i^2}{2m} + k\hbar\left(\frac{1}{\delta p_3} - \frac{1}{\delta p_1}\right)$$

$$\frac{\partial \langle E \rangle}{\partial \delta p_1} = \frac{\delta p_1}{m} + \frac{\hbar k}{\delta p_1^2} = 0 \qquad \delta p_1 = -\sqrt[3]{\hbar k m}$$

$$\frac{\partial \langle E \rangle}{\partial \delta p_2} = \frac{\delta p_2}{2m} = 0$$

$$m_p = 940\ \frac{\mathrm{MeV}}{c^2}$$

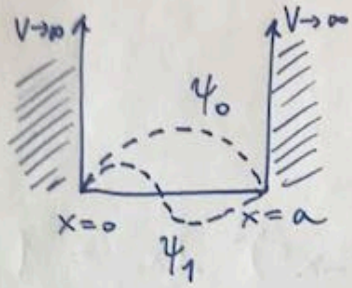
$$\frac{\partial \langle E \rangle}{\partial \delta p_3} = \frac{\delta p_3}{m} - \frac{\hbar k}{\delta p_3^2} = 0 \qquad \delta p_3 = \sqrt[3]{\hbar k m}$$

$$((0{,}3)^2)^2/0{,}3$$

$$E_{min} \simeq \frac{\cancel{2}}{\cancel{2}m}\sqrt[3]{(\hbar k m)^2} + 2\hbar k\left(\sqrt[3]{\hbar k m}\right)^{-1}$$

$$= \sqrt[3]{\frac{(\hbar k)^2}{m}} + 2\sqrt[3]{\frac{(\hbar k)^{3-2}}{\hbar k m}} = 3\sqrt{\frac{(\hbar c k)^2}{mc^2}} = 3\sqrt[3]{\frac{(0{,}09)^2\,\mathrm{GeV}^{4\cdot 3}}{0{,}3\ \mathrm{GeV}}} = 3 \cdot 0{,}3\ \mathrm{GeV} \simeq 0{,}9\ \mathrm{GeV}$$

(34) NESKONČNA POTENCIALNA JAMA



$$\hat{H} = \frac{\hat{p}^2}{2m} + \cancel{0} \qquad \hat{p} = -i\hbar\frac{d}{dx}$$

$$\hat{H}\psi = -\frac{\hbar^2}{2m}\psi'' = E_n\psi$$

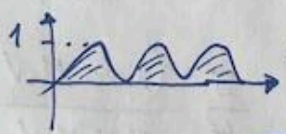
$$\psi'' = -\frac{2mE_n}{\hbar^2}\psi$$

$$\psi_n = A\sin\sqrt{\frac{2mE_n}{\hbar^2}}\,x + B\cos\sqrt{\frac{2mE_n}{\hbar^2}}\cdot x$$

$$\psi_n(0) = 0 \quad \Rightarrow \quad B = 0$$

$$\psi_n(a) = 0 \quad \Rightarrow \quad \sqrt{\frac{2mE_n}{\hbar^2}}\cdot a = n\pi \quad \Rightarrow E_n = \frac{n^2\pi^2\hbar^2}{2m\,a^2}$$

$$\int_0^a |\psi_n|^2\,dx = A^2 a\int_0^a \sin^2\left(\frac{n\pi}{a}x\right)\frac{dx}{a} = A^2\,\frac{a}{2} = 1$$



$\langle\sin^2 x\rangle = \frac{1}{2}$

Lastne funkcije: $\psi_n = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi x}{a}\right)$ $\qquad \hbar = \frac{h}{2\pi}$

in Lastne energije: $E_n = \frac{1}{2m}\left(\frac{n\pi\hbar}{a}\right)^2 = \frac{1}{2m}\left(\frac{nh}{2a}\right)^2$

$m = \mu g$ $\qquad E_1 = \frac{h^2}{8m\,a^2}$, $h = 6{,}6\cdot 10^{-34}\,Js$

$a = 1\,cm$ $\qquad \simeq \frac{49\cdot 10^{-68}\,J^{\cancel{2}}\cancel{s^2}}{8\cdot 10^{-9}\,\cancel{kg}\,10^{-4}\,\cancel{m^2}} = 6\cdot 10^{-55}\,J$

$E_n = 10^{-7}\,J \longrightarrow n = \sqrt{\frac{E_n}{E_1}} = \sqrt{\frac{10^{-7}\,\cancel{J}}{10^{-54}\,\cancel{J}}} = 4\cdot 10^{23}$ $\qquad 10^{-31}\,J$

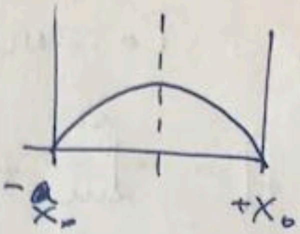
$\quad = n^2 E_1$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad =$

$\Delta E_n = ?$

 $\Delta E_n = ((n+1)^2 - n^2)E_1 \sim 2nE_1 = 10^{24}\cdot 10^{-55}\,J$

(37) $2x_0 = a = 1\,nm$

$\psi(x) = A\,(x_0^2 - x^2)$



$\langle E\rangle = ?$

i) Normalizacija $\int_{-x_0}^{x_0} \psi^2\, dx = A^2 \int_{-x_0}^{x_0} (x_0^2 - x^2)^2\, dx = 1$

$$= A^2 \left(x_0^4\, 2x_0 - 2x_0^2 \frac{2x_0^3}{3} + \frac{2x_0^5}{5} \right) = A^2 \frac{2x_0^5}{15}(15 - 10 + 3)$$

$$\Rightarrow A = \sqrt{\frac{15}{16x_0^5}} \quad , \quad \langle E\rangle = \langle \hat{H}\rangle = \frac{\langle p^2\rangle}{2m}$$

$$\langle E\rangle = \int_{-x_0}^{x_0} \frac{-\hbar^2}{2m}\, \psi\, \psi''\, dx = -\frac{\hbar^2}{2m} \int_{-x_0}^{x_0} A^2 (x_0^2 - x^2)(-2)\, dx$$

$$= \frac{\hbar^2}{m} \frac{15}{16x_0^5} \left(x_0^2 \cdot 2x_0 - \frac{2x_0^3}{3} \right) = \frac{\hbar^2}{m} \frac{5}{8x_0^2} (3 - 1)$$

$$= \frac{5}{4} \frac{(\hbar c)^2}{mc^2\, x_0^2} = \frac{8}{\cancel{4}} \frac{(200\ MeV\ fm)^2}{\underset{0{,}1}{\cancel{0{,}5}}\ MeV\ \cancel{\tfrac{1}{4}}\ nm^2} = \frac{4 \cdot 10^4\ MeV\ \cancel{fm^2}}{10^{+12-1}\ \cancel{fm^2}}$$

$$= 0{,}4\ eV$$

ii) Nedoločenost

liha

$$\langle x\rangle = \int_{-x_0}^{x_0} \psi^2\, x\, dx = 0 \quad , \quad \langle x^2\rangle = \int_{-x_0}^{x_0} \psi^2 x^2\, dx \neq 0$$

lih interval

soda

$$\langle x^2\rangle = A^2 \int (x_0^4 x^2 - 2x_0^2 x^4 + x^6)\, dx = \frac{15}{16x_0^5} \cdot 2x_0^7 \left(\frac{1}{3} - \frac{2}{5} + \frac{1}{7} \right)$$

$$= \frac{\cancel{15}\, x_0^2}{8 \cdot \cancel{15} \cdot 7} (\underbrace{35 - 42 + 15}_{8}) = \frac{x_0^2}{7}$$

$\langle p \rangle = \int \psi \frac{d\psi}{dx} = 0$

soda   liha

$\langle p^2 \rangle = 2m \langle E \rangle = \cancel{2m} \cdot \frac{5}{2\cdot 4} \frac{\hbar^2}{\cancel{m} x_0^2}$

$\Rightarrow \delta p \, \delta x = \sqrt{\frac{5\hbar^2}{2\cancel{x_0^2}} \cdot \frac{\cancel{x_0^2}}{7}} = \frac{\hbar}{2}\sqrt{\frac{10}{7}} = \frac{\hbar}{2} \cdot 1{,}2$

20% nad Heisenbergovo mejo

(42) $\psi = A(\psi_1 + 3\psi_3)$, $\hat{H}\psi_n = E_n \psi_n$ $\quad A = \frac{1}{\sqrt{10}}$

$\int |\psi|^2 = A^2 \int \psi_1^2 + 6\psi_1\psi_3 + 9\psi_3^2$ $\quad \Uparrow$

$= A^2 (1 + 6\cdot 0 + 9\cdot 1) = 10 A^2 = 1$

Energija iz razvoja po lastnih stanjih.

$\psi = \sum c_n \psi_n$, $\langle E \rangle = \sum_{n,m} \int c_n^* \psi_n^* \hat{H} c_m \psi_m$

$= \sum_{n,m} E_m c_n^* c_m \underbrace{\int \psi_n^* \psi_m}_{\delta_{nm}}$

$= \sum_n E_n |c_n|^2$

$\Rightarrow \langle E \rangle_\psi = \left(\frac{1}{\sqrt{10}}\right)^2 E_1 + \left(\frac{3}{\sqrt{10}}\right)^2 E_3 = \frac{1}{10}(1 + 81) E_1 = 8{,}2 E_1$

potencialna jama: $E_n = n^2 E_1$

· Zanima nas ortogonalno stanje $\psi_\perp : \int \psi \psi_\perp = 0$

$$\psi_\perp = A_\perp (\psi_1 + b\psi_3)$$

$$\int A_\perp A\, (\psi_1 + 3\psi_3)(\psi_1 + b\psi_3)$$

$$= A_\perp A\, (1 + 3b) = 0 \quad , \quad b = -\frac{1}{3} \quad , \quad A_\perp^2 \left(1 + \frac{1}{9}\right)$$

$$= A_\perp^2 \frac{10}{9} = 1$$

$$\Rightarrow \psi_\perp = \frac{1}{\sqrt{10}} (3\psi_1 - \psi_3)$$

$$\langle E \rangle_{\psi_\perp} = \frac{9}{10} E_1 + \frac{9}{10} E_1 = \frac{18}{10} E_1 = 1{,}8\, E_1$$

$$E_1 = \frac{(hc)^2}{8mc^2 a^2} = 2{,}35\, eV$$

$$\langle E \rangle_\psi = 19{,}3\, eV \quad , \quad \langle E \rangle_{\psi_\perp} = 4{,}2\, eV$$

Ad. 42 : ČASOVNI RAZVOJ

$$\hat{H}\psi_n = E_n \psi_n \qquad i\hbar \frac{d\psi}{dt} = \hat{H}\psi$$

lastne energije — lastna stanja

$$i\hbar \frac{d\psi_n}{dt} = \hat{H}\psi_n = E_n \psi_n$$

$$\psi_n = \psi_n(t=0) \cdot e^{-i\hbar^{-1} E_n t}$$

$$t=0 : \psi = \sum c_n \psi_n \quad \Rightarrow \quad \psi(t) = \sum_n c_n e^{-i E_n t/\hbar} \psi_n$$

- Energija se ohranja => ne sme biti odvisna od časa: $\langle \psi(t) | \hat{H} | \psi(t) \rangle =$

$$= \int \sum_n c_n^* e^{iE_n/\hbar t} \psi_n^* \hat{H} \sum_m c_m e^{-iE_m/\hbar t} \psi_m$$

$$= \sum_{n,m} c_n^* c_m E_m e^{-i(E_m-E_n)/\hbar t} \underbrace{\int \psi_n^* \psi_m}_{\delta_{nm}}$$

$$= \sum_n |c_n|^2 E_n e^{-i\overset{=0}{(E_n-E_n)}/\hbar t}$$

$$= \langle E \rangle$$

- Kako je z ostalimi operatorji?

$$\langle p \rangle = \sum_{nm} \int \psi_n^* \hat{p} \psi_m = 0$$

$\langle p^2 \rangle = 2m \langle E \rangle$ ni odvisno od časa

$$\langle x \rangle_\psi = \frac{1}{10} \int_0^a \left(\psi_1 e^{iE_1/\hbar t} + 3\psi_3 e^{iE_3/\hbar t}\right) \hat{x} \left(\underset{\sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}}{\psi_1} e^{-iE_1/\hbar t} + 3\underset{\sqrt{\frac{2}{a}} \sin \frac{3\pi x}{a}}{\psi_3} e^{-iE_3/\hbar t}\right)$$

$= \frac{a}{2}$ in neodvisno od časa

$$\langle x^2 \rangle_\psi = \frac{1}{10} \int_0^a \left(\psi_1 e^{iE_1/\hbar t} + 3\psi_3 e^{iE_3/\hbar t}\right) x^2 \left(\psi_1 e^{-iE_1/\hbar t} + 3\psi_3 e^{-iE_3/\hbar t}\right)$$

$$= \frac{1}{10} \left(\int_0^a x^2 \left(\psi_1^2 + 9\psi_3^2 + 6\psi_1\psi_3 \cos\left(\frac{E_3-E_1}{\hbar} t\right)\right)\right)$$

$$= \frac{a^2}{120\pi^2} \left(10\pi^2 - 12 + 27 \cos\left(\frac{E_3-E_1}{\hbar} t\right)\right).$$