$$\langle p \rangle = \int \underset{\text{soda}}{\psi} \; \underset{\text{liha}}{\frac{d\psi}{dx}} = 0$$

$$\langle p^2 \rangle = 2m \langle E \rangle = \cancel{2m} \cdot \frac{5}{2\cdot 4} \frac{\hbar^2}{\cancel{m} x_0^2}$$

$$\Rightarrow \delta p \, \delta x = \sqrt{\frac{5\hbar^2}{2\cancel{x_0^2}} \cdot \frac{\cancel{x_0^2}}{7}} = \frac{\hbar}{2} \sqrt{\frac{10}{7}} = \frac{\hbar}{2} \cdot \underbrace{1{,}2}$$

20% nad Heisenbergovo mejo

(42) $\psi = A(\psi_1 + 3\psi_3)$, $\hat{H}\psi_n = E_n \psi_n$ $\quad A = \frac{1}{\sqrt{10}}$

$$\int |\psi|^2 = A^2 \int \psi_1^2 + 6\psi_1\psi_3 + 9\psi_3^2$$

$$= A^2 (1 + 6\cdot 0 + 9 \cdot 1) = 10 A^2 = 1$$

Energija iz razvoja po lastnih stanjih.

$$\psi = \sum c_n \psi_n, \quad \langle E \rangle = \sum_{n,m} \int c_n^* \psi_n^* \hat{H} c_m \psi_m$$

$$= \sum_{n,m} E_m c_n^* c_m \underbrace{\int \psi_n^* \psi_m}_{\delta_{nm}}$$

$$= \sum_n E_n |c_n|^2$$

$$\Rightarrow \langle E \rangle_\psi = \left(\frac{1}{\sqrt{10}}\right)^2 E_1 + \left(\frac{3}{\sqrt{10}}\right)^2 E_3 = \frac{1}{10}(1 + 81)E_1 = 8{,}2\, E_1$$

potencialna jama: $E_n = n^2 E_1$

· Zanima nas ortogonalno stanje $\psi_\perp$ : $\int \psi \psi_\perp = 0$

$$\psi_\perp = A_\perp (\psi_1 + b\psi_3)$$

$$\int A_\perp A (\psi_1 + 3\psi_3)(\psi_1 + b\psi_3)$$

$$= A_\perp A (1 + 3b) = 0 \,, \quad b = -\frac{1}{3} \,, \quad A_\perp^2 \left(1 + \frac{1}{9}\right) = A_\perp^2 \frac{10}{9} = 1$$

$$\Rightarrow \psi_\perp = \frac{1}{\sqrt{10}} (3\psi_1 - \psi_3)$$

$$\langle E \rangle_{\psi_\perp} = \frac{9}{10} E_1 + \frac{9}{10} E_1 = \frac{18}{10} E_1 = 1{,}8\, E_1$$

$$E_1 = \frac{(hc)^2}{8mc^2 a^2} = 2{,}35\,eV$$

$$\langle E \rangle_\psi = 19{,}3\,eV \,, \quad \langle E \rangle_{\psi_\perp} = 4{,}2\,eV$$

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Ad. 42 : ČASOVNI RAZVOJ

$$\hat{H}\psi_n = E_n \psi_n \qquad i\hbar \frac{d\psi}{dt} = \hat{H}\psi$$

lastne energije   lastna stanja

$$i\hbar \frac{d\psi_n}{dt} = \hat{H}\psi_n = E_n \psi_n$$

$$\psi_n = \psi_n(t=0) \cdot e^{-i\hbar^{-1} E_n t}$$

$$t=0: \psi = \sum c_n \psi_n \quad \Rightarrow \quad \psi(t) = \sum_n c_n e^{-i E_n t/\hbar} \psi_n$$

- Energija se ohranja ⇒ ne sme biti odvisna od časa: $\langle \psi(t) | \hat{H} | \psi(t) \rangle =$

$$= \int \sum_n c_n^* e^{iE_n/\hbar t} \psi_n^* \hat{H} \sum_m c_m e^{-iE_m/\hbar t} \psi_m$$

$$= \sum_{n,m} c_n^* c_m E_m \, e^{-i(E_m-E_n)/\hbar t} \underbrace{\int \psi_n^* \psi_m}_{\delta_{nm}}$$

$$= \sum_n |c_n|^2 E_n \, e^{-i\overset{=0}{(E_n-E_n)}/\hbar t}$$

$$= \langle E \rangle$$

- Kako je z ostalimi operatorji?

$$\langle p \rangle = \sum_{nm} \int \psi_n^* \hat{p} \psi_m = 0$$

$\langle p^2 \rangle = 2m \langle E \rangle$ ni odvisno od časa

$$\langle x \rangle_\psi = \frac{1}{10} \int_0^a \left(\psi_1 e^{iE_1/\hbar t} + 3\psi_3 e^{iE_3/\hbar t}\right) \hat{x} \left(\psi_1 e^{-iE_1/\hbar t} + 3\psi_3 e^{-iE_3/\hbar t}\right)$$

$$\psi_1 = \sqrt{\tfrac{2}{a}} \sin \tfrac{\pi x}{a} \qquad \psi_3 = \sqrt{\tfrac{2}{a}} \sin \tfrac{3\pi x}{a}$$

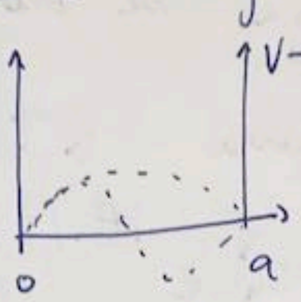
$= \frac{a}{2}$ in neodvisno od časa

$$\langle x^2 \rangle_\psi = \frac{1}{10} \int_0^a \left(\psi_1 e^{iE_1/\hbar t} + 3\psi_3 e^{iE_3/\hbar t}\right) x^2 \left(\psi_1 e^{-iE_1/\hbar t} + 3\psi_3 e^{-iE_3/\hbar t}\right)$$

$$= \frac{1}{10} \left( \int_0^a x^2 \left(\psi_1^2 + 9\psi_3^2 + 6\psi_1\psi_3 \cos\left(\frac{E_3-E_1}{\hbar} t\right)\right)\right)$$

$$= \frac{a^2}{120\pi^2} \left(10\pi^2 - 12 + 27 \cos\left(\frac{E_3-E_1}{\hbar} t\right)\right).$$

(46) Določi nedoločenost vzbujenih stanj potencialne jame




$$\psi_n = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi x}{a}\right)$$

$$\langle x\rangle = \int_0^a x\,\psi_n^2\,dx \quad , \quad \frac{n\pi x}{a} = t, \quad dx = \frac{a}{n\pi}\,dt$$

$$= \int_0^{n\pi} \frac{2}{a}\left(\frac{a}{n\pi}\right)^2 t\,\sin^2 t\,dt = \frac{\cancel{2}a}{(n\pi)^2}\int_0^{n\pi} t\,\frac{1-\cos 2t}{\cancel{2}}\,dt$$

$$= \frac{a}{(n\pi)^2}\left(\underbrace{\frac{t^2}{2}\Big|_0^{n\pi}}_{\frac{(n\pi)^2}{2}} - \underbrace{\frac{\sin 2t}{2}\,t\Big|_0^{n\pi}}_{\emptyset} + \underbrace{\int_0^{n\pi}\frac{\sin 2t}{2}\,dt}_{\emptyset}\right) = \frac{a}{2}.$$

$$\langle x^2\rangle = \frac{\cancel{2}}{a}\left(\frac{a}{n\pi}\right)^3\int_0^{n\pi} t^2\,\frac{1-\cos 2t}{\cancel{2}}\,dt = \frac{a^2}{(n\pi)^3}\left(\underbrace{\frac{t^3}{3}\Big|_0^{n\pi}}_{\frac{(n\pi)^3}{3}}\right.$$

$$\left. - \overset{0}{\frac{\sin \cancel{2}t}{2}}\,2t\Big|_0^{n\pi} + \int_0^{n\pi} t\sin 2t\,dt\right)$$

$$= \frac{a^2}{3} + \frac{a^2}{(n\pi)^3}\left(-\frac{\cos 2t}{2}\,t\Big|_0^{n\pi} + \frac{1}{2}\int_0^{n\pi}\cos 2t\,dt\right)$$

$$= \frac{a^2}{3} + \frac{a^2}{(n\pi)^{\cancel{3}2}}\left(-\frac{1}{2}\cancel{n\pi}\right) = \frac{a^2}{3}\left(1 - \frac{3}{2(n\pi)^2}\right)$$

$$\langle p\rangle = \frac{2}{a}(-i\hbar)\int_0^{n\pi}\sin t\,\cos t\,dt\left(\frac{a}{n\pi}\right) = \emptyset$$

$$\langle p^2\rangle = \frac{2m}{\hbar^2}\langle E\rangle = \left(\frac{\hbar n\pi}{a}\right)^2$$

$$\delta x^2 = \langle x^2\rangle - \langle x\rangle^2 = \frac{a^2}{3}\left(1-\frac{3}{2(n\pi)^2}\right) - \frac{a^2}{4}$$

$$\delta x = a^2\left(\frac{1}{12}-\frac{1}{2(n\pi)^2}\right)^{1/2}$$

$$\delta p = \frac{n\pi\hbar}{a}$$

$$\delta x\,\delta p = \frac{\hbar}{2}\sqrt{\frac{(n\pi)^2}{3}-\frac{1}{2}}$$

$$= \frac{\hbar}{2}\begin{cases} 1{,}13 & ;\ n=1 \\ 3{,}34 & ;\ n=2 \end{cases}$$

- za $n \gg 1$ : $\delta x\,\delta p \simeq \frac{\hbar n}{2\sqrt{3}}$

## 50 Harmonski oscilator

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \ , \quad \hat{H}\psi_n = E_n\psi_n \ , \quad E_n = \hbar\omega\left(n+\frac{1}{2}\right)$$

$$\psi_n = \frac{1}{\sqrt{2^n n!\sqrt{\pi}\,a}}H_n(y)\,e^{-y^2/2} \ , \quad y = \frac{x}{a} \ , \quad a = \sqrt{\frac{\hbar}{m\omega}}$$

$$H_n(y) = (-1)^n e^{y^2}\frac{d^n}{dy^n}e^{-y^2} \ , \quad \int_{-\infty}^{\infty}\psi_n\psi_m\,dx = \delta_{nm}$$

| SODI | LIHI |
| --- | --- |
| $H_0 = 1$ | $H_1 = 2y$ |
| $H_2 = 4y^2-2$ | $H_3 = 8y^3-12y$ |
| $H_4 = 16y^4-48y^2+12$ | $H_5 = 32y^5-160y^3+120y \ \ldots$ |

(50) $\psi(x, t=0) = A(2y^2 + iy)e^{-y^2/2} = B(2y^2+iy)\psi_0$

$$\psi = \sum_n c_n \psi_n \qquad \psi_0 = \frac{1}{\sqrt{\sqrt{\pi}a}} e^{-y^2/2}$$

$$\psi_1 = \sqrt{2}\, y\, \psi_0$$

$$= B\left(\sqrt{2}\psi_2 + \psi_0 + \frac{i}{\sqrt{2}}\psi_1\right)$$

$$\psi_2 = \frac{1}{\sqrt{2}}(2y^2-1)\psi_0$$

$$\int \psi^* \psi = 1$$

$$\Downarrow$$

$$B^2\left(1 + \frac{1}{2} + 2\right) = \frac{7}{2}B^2 = 1 \qquad \sqrt{2}\psi_2 + \psi_0 = 2y^2$$

$$\Rightarrow \psi = \sqrt{\frac{2}{7}}\left(\psi_0 + \frac{i}{\sqrt{2}}\psi_1 + \sqrt{2}\psi_2\right)$$

$$\langle E\rangle = \hbar\omega \sum |c_n|^2 \left(n + \frac{1}{2}\right) = \hbar\omega \frac{2}{7}\left(1\,\frac{1}{2} + \frac{1}{2}\cdot\left(1+\frac{1}{2}\right) + 2\left(2+\frac{1}{2}\right)\right)$$

$$= \sum |c_n|^2 E_n$$

$$= \hbar\omega \frac{2}{7}\left(\frac{2}{4} + \frac{3}{4} + \frac{20}{4}\right) = \frac{25}{14}\hbar\omega$$

$$\psi(x,t) = \sum c_n e^{-iE_n t/\hbar} \psi_n(x)$$

$$= \sqrt{\frac{2}{7}}\left(\psi_0 e^{-i\omega t/2} + \frac{i}{\sqrt{2}}\psi_1 e^{-i3\omega t/2} + \sqrt{2}\psi_2 e^{-i5\omega t/2}\right)$$

$$= \sqrt{\frac{2}{7\sqrt{\pi}a}}\, e^{-y^2/2}\left(e^{-i\omega t/2} + i\,y\, e^{-i3\omega t/2} + (2y^2-1)e^{-i5\omega t/2}\right)$$

---

• V splošnem koeficiente $c_n$ dobimo s skalarnim produkt[om]

$$\psi = \sum_m c_m \psi_m \;\Big/ \int \psi_n^* \;\Rightarrow\; \int \psi_n^* \psi\, dx = \sum c_m \underbrace{\int \psi_n^* \psi_m}_{\delta_{nm}} = c_n$$

$$\boxed{c_n = \int \psi_n^* \psi\, dx}$$